Fundamentele Informatica 3

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8. Recursively Enumerable Languages
8.3. More General Grammars
8.4. Context-Sensitive Languages and The Chomsky Hierarchy

A slide from lecture 5

Definition 8.1. Accepting a Language and Deciding a Language

A Turing machine T with input alphabet Σ accepts a language $L \subseteq \Sigma^*$,
if $L(T) = L$.

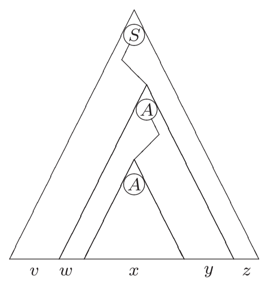
T *decides* L,
if T computes the characteristic function $\chi_L : \Sigma^* \rightarrow \{0,1\}$

A language L is *recursively enumerable*,
if there is a TM that accepts L,

and L is *recursive*,
if there is a TM that decides L.

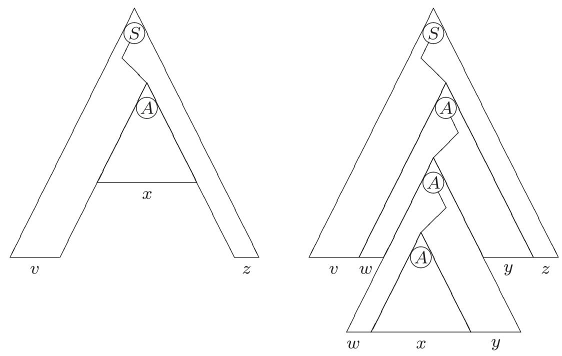
A slide from lecture 1

FI2: Pumping Lemma for CFLs



A slide from lecture 1

FI2: Pumping Lemma for CFLs



8.3. More General Grammars

reg. languages	reg. grammar	FA	reg. expression
determ. cf. languages		DPDA	
cf. languages	cf. grammar	PDA	
re. languages	unrestr. grammar	TM	

Definition 8.10. Unrestricted grammars

An unrestricted grammar is a 4-tuple $G = (V, \Sigma, S, P)$, where V and Σ are disjoint sets of variables and terminals, respectively, S is an element of V called the start symbol, and P is a set of productions of the form

$$\alpha \rightarrow \beta$$

where $\alpha, \beta \in (V \cup \Sigma)^*$ and α contains at least one variable.

Notation as for CFGs:

$$\alpha \Rightarrow_G^* \beta$$

$$L(G) = \{x \in \Sigma^* \mid S \Rightarrow_G^* x\}$$

but. . .

Example 8.12. A Grammar Generating $\{a^n b^n c^n \mid n \geq 1\}$

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$$S \to SABC \mid LABC$$

$$BA \to AB \quad CB \to BC \quad CA \to AC$$

$$LA \to a \quad aA \to aa \quad aB \to ab \quad bB \to bb \quad bC \to bc \quad cC \to cc$$

Example 8.11. A Grammar Generating $\{a^{2^k} \mid k \in \mathbb{N}\}$

$\{a, a^2, a^4, a^8, a^{16}, \ldots\} = \{a, aa, aaaa, aaaaaaaa, aaaaaaaaaaaaaaaa, \ldots\}$

Example 8.11. A Grammar Generating $\{a^{2^k} \mid k \in \mathbb{N}\}$

$\{a, a^2, a^4, a^8, a^{16}, \ldots\} = \{a, aa, aaaa, aaaaaaaa, aaaaaaaaaaaaaaaa, \ldots\}$

$$S \to LaR$$

$$L \to LD \quad Da \to aaD \quad DR \to R$$

$$L \to \Lambda \quad R \to \Lambda$$

Example.

An Unrestricted Grammar Generating $XX = \{xx \mid x \in \{a,b\}^*\}$

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$$S \to aAS \mid bBS \mid M$$

$$Aa \to aA \quad Ab \to bA \quad Ba \to aB \quad Bb \to bB$$

$$AM \to Ma \quad BM \to Mb \quad M \to \Lambda$$

Theorem 8.13.
For every unrestricted grammar G, there is a Turing machine T with $L(T) = L(G)$.

Proof.
1. Move past input
2. Simulate derivation in G on the tape of a Turing machine
3. Equal

Theorem 8.13.
For every unrestricted grammar G, there is a Turing machine T with $L(T) = L(G)$.

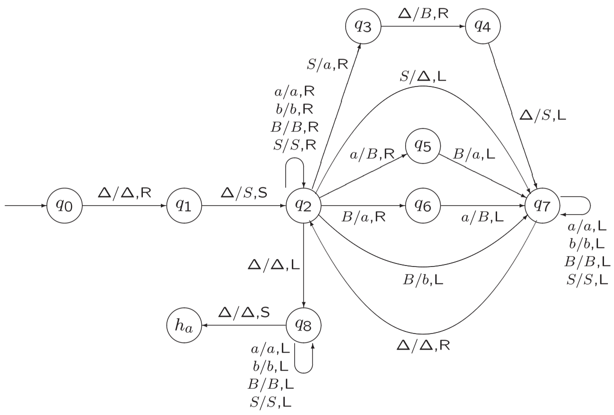
Proof.
1. Move past input
2. Simulate derivation in G on the tape of a Turing machine:
   Write S on tape
   Repeat
   a. Select production $\alpha \to \beta$
   b. Select occurrence of α (if there is one)
   c. Replace occurrence of α by β
   until b. fails (caused by ...)
3. Equal

Example.
(The second part of) the construction from Theorem 8.13 to obtain a TM simulating a derivation in the unrestricted grammar with productions

$$S \to aBS \mid \Lambda \quad aB \to Ba \quad Ba \to aB \quad B \to b$$

See next slide
N.B.:
In next slide, we simulate application of arbitrary production by
- first moving to arbitrary position in current string (at q_2)
- only then selecting (and applying) a possible production

This implementation of the construction must be known for the exam



8.4. Context-Sensitive Languages and the Chomsky Hierarchy

reg. languages	reg. grammar	FA	reg. expression
determ. cf. languages		DPDA	
cf. languages	cf. grammar	PDA	
cs. languages	cs. grammar	LBA	
re. languages	unrestr. grammar	TM	

Definition 8.16. Context-Sensitive Grammars
A *context-sensitive grammar* (CSG) is an unrestricted grammar in which no production is length-decreasing.
In other words, every production is of the form $\alpha \to \beta$, where $|\beta| \geq |\alpha|$.

A language is a context-sensitive language (CSL) if it can be generated by a context-sensitive grammar.

Example 8.12. A Grammar Generating $\{a^n b^n c^n \mid n \geq 1\}$

$$S \to SABC \mid LABC$$

$$BA \to AB \quad CB \to BC \quad CA \to AC$$

$$LA \to a \quad aA \to aa \quad aB \to ab \quad bB \to bb \quad bC \to bc \quad cC \to cc$$

Not context-sensitive.

Example 8.17. A CSG Generating $L = \{a^n b^n c^n \mid n \geq 1\}$

$$S \to SABC \mid \mathcal{A}BC$$

$$BA \to AB \quad CB \to BC \quad CA \to AC$$

$$\mathcal{A} \to a \quad aA \to aa \quad aB \to ab \quad bB \to bb \quad bC \to bc \quad cC \to cc$$

Example.

An Unrestricted Grammar Generating $XX = \{xx \mid x \in \{a,b\}^*\}$

$$S \to aAS \mid bBS \mid M$$

$$Aa \to aA \quad Ab \to bA \quad Ba \to aB \quad Bb \to bB$$

$$AM \to Ma \quad BM \to Mb \quad M \to \Lambda$$

Not context-sensitive.

Exercise 8.24.

Find a context-sensitive grammar generating the language

$$XX - \{\Lambda\} = \{xx \mid \ x \in \{a,b\}^* \text{ and } x \neq \Lambda\}$$

Definition 8.18. Linear-Bounded Automata
A *linear-bounded automaton* (LBA) is a 5-tuple $M = (Q, \Sigma, \Gamma, q_0, \delta)$ that is identical to a nondeterministic Turing machine, with the following exception.

There are two extra tape symbols [and], assumed not to be elements of the tape alphabet Γ.
The initial configuration of M corresponding to input x is $q_0[x]$, with the symbol [in the leftmost square and the symbol] in the first square to the right of x.
During its computation, M is not permitted to replace either of these brackets or to move its tape head to the left of the [or to the right of the].

Theorem 8.19.
If $L \subseteq \Sigma^*$ is a context-sensitive language, then there is a linear-bounded automaton that accepts L.

Proof. . .

Theorem 8.19.
If $L \subseteq \Sigma^*$ is a context-sensitive language, then there is a linear-bounded automaton that accepts L.

Proof. Much like the proof of Theorem 8.13, except
- two tape tracks instead of move past input
- reject also if we (want to) write on]

Theorem 8.19.
If $L \subseteq \Sigma^*$ is a context-sensitive language, then there is a linear-bounded automaton that accepts L.

Proof.
1. Create second tape track
2. Simulate derivation in G on track 2
3. Equal

Theorem 8.19.
If $L \subseteq \Sigma^*$ is a context-sensitive language, then there is a linear-bounded automaton that accepts L.

Proof.
1. Create second tape track
2. Simulate derivation in G on track 2:
   Write S on track 2
   Repeat
   a. Select production $\alpha \to \beta$
   b. Select occurrence of α on track 2 (if there is one)
   c. Try to replace occurrence of α by β
   until b. fails (caused by . . .)
   or c. fails (caused by . . .); then reject
3. Equal

Theorem 8.19.
If $L \subseteq \Sigma^*$ is a context-sensitive language, then there is a linear-bounded automaton that accepts L.

Proof. Much like the proof of Theorem 8.13, except
- two tape tracks instead of move past input
- reject also if we (want to) write on]

Alternative proof.
Simulate derivation of string x from S in reverse order
c.f., bottom-up parsing
Then one tape track is sufficient

Exercise 8.27.

Show that if L is any recursively enumerable language, then L can be generated by a grammar in which the left side of every production is a string of one or more variables.