Exercise 4.4. In both parts below, the productions in a CFG G are given.
In each part, show first that for every string $x \in L(G)$, $n_a(x) = n_b(x)$; then find a string $x \in \{a, b\}^*$ with $n_a(x) = n_b(x)$ that is not in $L(G)$.

a. $S \to SabS \mid SbaS \mid \Lambda$

b. $S \to aSb \mid bSa \mid abS \mid baS \mid Sab \mid Sba \mid \Lambda$

Exercise 4.9. Suppose that $G_1 = (V_1, \{a, b\}, S_1, P_1)$ and $G_2 = (V_2, \{a, b\}, S_2, P_2)$ are CFGs and that $V_1 \cap V_2 = \varnothing$.

a. It is easy to see that no matter what G_1 and G_2 are, the CFG $G_u = (V_u, \{a, b\}, S_u, P_u)$ defined by $V_u = V_1 \cup V_2$, $S_u = S_1$ and $P_u = P_1 \cup P_2 \cup \{S_1 \to S_2\}$ generates every string in $L(G_1) \cup L(G_2)$. Find grammars G_1 and G_2 (you can use $V_1 = \{S_1\}$ and $V_2 = \{S_2\}$) and a string $x \in L(G_u)$ such that $x \notin L(G_1) \cup L(G_2)$.

b. As in part (a), the CFG $G_c = (V_c, \{a, b\}, S_c, P_c)$ defined by $V_c = V_1 \cup V_2$, $S_c = S_1$ and $P_c = P_1 \cup P_2 \cup \{S_1 \to S_1 S_2\}$ generates every string in $L(G_1)L(G_2)$.
Find grammars G_1 and G_2 (again with $V_1 = \{S_1\}$ and $V_2 = \{S_2\}$) and a string $x \in L(G_c)$ such that $x \notin L(G_1)L(G_2)$.

Exercise 4.9. (continued)

c. The CFG $G^* = (V, \{a, b\}, S, P)$ defined by $V = V_1$, $S = S_1$ and $P = P_1 \cup \{S_1 \to S_1S_1 \mid \Lambda\}$ generates every string in $L(G_1)^*$. Find a grammar G_1 with $V_1 = \{S_1\}$ and a string $x \in L(G^*)$ such that $x \notin L(G_1)^*$.

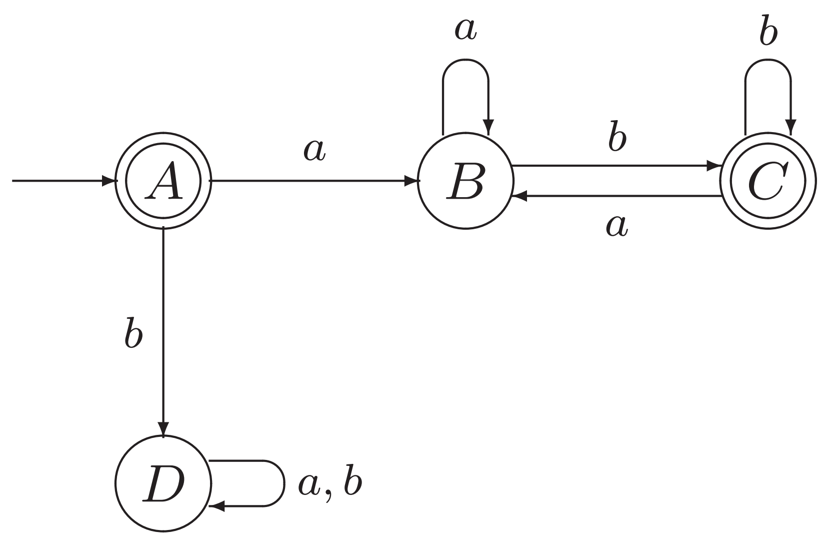
Exercise 4.26. In each part, draw an NFA (which might be an FA) accepting the language generated by the CFG having the given productions.

a.

$$S \to aA \mid bC \qquad A \to aS \mid bB \qquad B \to aC \mid bA \qquad C \to aB \mid bS \mid \Lambda$$

Exercise 4.27.

Find a regular grammar generating the language $L(M)$, where M is the FA shown below:



Exercise 4.22.

Show that if G is a context-free grammar in which every production has one of the forms

$$A \to aB, \quad A \to a \quad \text{and } A \to \Lambda$$

(where A and B are variables and a is a terminal), then $L(G)$ is regular.

Suggestion: construct an NFA accepting $L(G)$, in which there is a state for each variable in G and one additional state F, the only accepting state.

Exercise 4.28.

Draw an NFA accepting the language generated by the grammar with productions

$$S \to abA \mid bB \mid aba \qquad A \to b \mid aB \mid bA \qquad B \to aB \mid aA$$

Exercise 4.29.

Each of the following grammars, though not regular, generates a regular language. In each case, find a regular grammar generating the language.

a. $S \to SSS \mid a \mid ab$

b. $S \to AabB \quad A \to aA \mid bA \mid \Lambda \quad B \to Bab \mid Bb \mid ab \mid b$

Exercise 4.34.

Show that the CFG with productions

$$S \to a \mid Sa \mid bSS \mid SSb \mid SbS$$

is ambiguous.

Exercise 4.36.

In each case below, decide whether the grammar is ambiguous or not, and prove your answer.

b. $S \to SS \mid bS \mid a$

c. $S \to SaS \mid b$

e. $S \to TT \qquad T \to aT \mid Ta \mid b$

f. $S \to aSa \mid bSb \mid aAb \mid bAa \qquad A \to aAa \mid bAb \mid a \mid b \mid \Lambda$

g. $S \to aT \mid bT \mid \Lambda \qquad T \to aS \mid bS$

Exercise 4.38.

In each case below, show that the grammar is ambiguous, and find an equivalent unambiguous grammar.

a. $S \to SS \mid a \mid b$

b. $S \to ABA \qquad A \to aA \mid \Lambda \qquad B \to bB \mid \Lambda$

c. $S \to aSb \mid aaSb \mid \Lambda$

d. $S \to aSb \mid abS \mid \Lambda$

Exercise.

Let G be a context-free grammar with start variable S and the following productions:

$$S \to aSbS \quad | \quad bSaS \quad | \quad \Lambda$$

a. Show that $L(G) = \mathit{AEqB} = \{x \in \{a,b\}^* \ \ | \ \ n_a(x) = n_b(x)\}$

b. Is G ambiguous? Motivate your answer.