From lecture 2:

FA $M_i = (Q_i, \Sigma, q_i, A_i, \delta_i) \qquad i = 1, 2$

Product construction

Construct FA $M = (Q, \Sigma, q_0, A, \delta)$ such that

– $Q = Q_1 \times Q_2$

– $q_0 = (q_1, q_2)$

– $\delta(\ (p, q), \sigma) = (\ \delta_1(p, \sigma), \delta_2(q, \sigma)\ )$

– A as needed

Theorem 2.15 (Parallel simulation).

– $A = \{(p, q) \mid p \in A_1$ or $q \in A_2\}$, then $L(M) = L(M_1) \cup L(M_2)$

– $A = \{(p, q) \mid p \in A_1$ and $q \in A_2\}$, then $L(M) = L(M_1) \cap L(M_2)$

– $A = \{(p, q) \mid p \in A_1$ and $q \notin A_2\}$, then $L(M) = L(M_1) - L(M_2)$

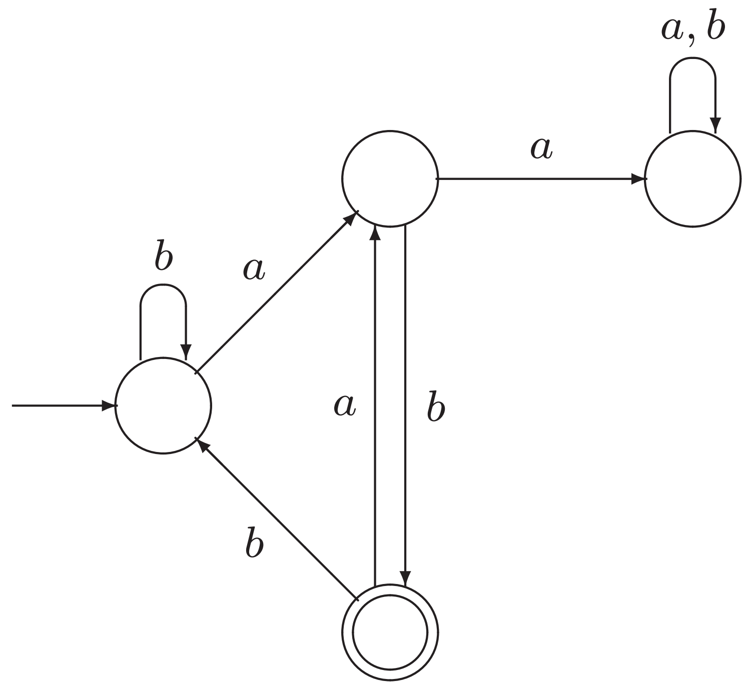
Exercise 2.27.
Describe decision algorithms to answer each of the following questions.

a. Given two FAs M_1 and M_2,
are there any strings that are accepted by neither?

d. Given an FA M accepting a language L, and a string x,
is x a prefix of an element of L?

g. Given two FAs M_1 and M_2,
is $L(M_1) \subseteq L(M_2)$?

Exercise 2.13.
For the FA pictured below, show that there cannot be any other FA with fewer states accepting the same language.



Exercise 2.17.
Let L be the language *AnBn* $= \{a^n b^n \mid n \geq 0\}$.

a. Find two distinct strings x and y in $\{a, b\}^*$ that are not L-distinguishable.

b. Find an infinite set of pairwise L-distinguishable strings.

Exercise 2.15.
Suppose L is a subset of $\{a, b\}^*$.

If $x_0, x_1, \ldots$ is a sequence of distinct strings in $\{a, b\}^*$,
such that for every $n \geq 0$, x_n and x_{n+1} are L-distinguishable,
does it follow that the strings $x_0, x_1, \ldots$ are pairwise L-distinguishable?

Either give a proof that it does follow,
or find an example of a language L and strings $x_0, x_1, \ldots$ that represent a counterexample.

Exercise 2.21. For each of the following languages $L \subseteq \{a, b\}^*$, show that the elements of the infinite set $\{a^n \mid n \geq 0\}$ are pairwise L-distinguishable.

a. $L = \{a^i b a^{2i} \mid i \geq 0\}$

b. $L = \{a^i b^j a^k \mid k > i + j\}$

d. $L = \{a^i b^j \mid j \text{ is a multiple of } i\}$

e. $L = \{x \in \{a, b\}^* \mid n_a(x) < 2n_b(x)\}$

f. $L = \{x \in \{a, b\}^* \mid \text{ no prefix of } x \text{ has more } b\text{'s than } a\text{'s}\}$

h. $L = \{ww \mid w \in \{a, b\}^*\}$