System-level Scheduling of Real-time Streaming Applications using a Semi-partitioned Approach

Emanuele Cannella, Mohamed A. Bamakhrama, and Todor Stefanov
Leiden Institute of Advanced Computer Science
Leiden University, Leiden, The Netherlands
Email: {e.cannella, m.a.m.bamakhrama, t.p.stefanov}@liacs.leidenuniv.nl

***Abstract*—Modern multiprocessor streaming systems have hard real-time constraints that must be always met to ensure correct functionality. At the same time, these streaming systems must be designed to use the minimum required amount of resources (such as processors and memory). In order to meet such constraints, using scheduling algorithms from the classical real-time scheduling theory represents an attractive solution approach. These algorithms enable: (1) providing timing guarantees to the applications running on the system, and (2) deriving analytically the minimum number of processors required to schedule the applications. So far, designers in the embedded systems community have focused on global and partitioned scheduling algorithms. However, recently, a new *hybrid* class of scheduling algorithms has been proposed. In this work, we investigate the applicability of a sub-class of these hybrid algorithms, called semi-partitioned algorithms, to applications modeled as Cyclo-Static Dataflow (CSDF) graphs. The contribution of this paper is two fold. First, we devise an approach that enables semi-partitioned scheduling algorithms, even soft real-time ones, to be applied to CSDF graphs while providing hard real-time guarantees at the input/output interfaces with the external environment. Second, we focus on an existing soft real-time semi-partitioned approach, for which we propose an allocation heuristic, called FFD-SP. The proposed heuristic reduces the minimum number of processors required to schedule the applications compared to a pure partitioned scheduling algorithm, while trying to minimize the buffer size and latency increases incurred by the soft real-time approach.**

I. Introduction

The design of modern embedded systems is a difficult task due to the complex functionalities that have to be implemented and the strict constraints that must be met. In order to implement these complex functionalities, Multiprocessor Systems-on-Chip (MPSoCs) are nowadays the standard choice for system designers. Realizing such complex functionalities on MPSoCs entails two challenges. The first challenge is expressing the parallelism in the applications in order to efficiently exploit the multiple processors found in MPSoCs. The common practice for expressing the parallelism in an application is to use Models of Computation (MoCs) [1]. Several MoCs have been proposed such as Synchronous Dataflow (SDF) [2] and its generalization, Cyclo-static Dataflow (CSDF) [3].

The second challenge is how to allocate and schedule the applications' tasks on the system in such a way that all the timing constraints are guaranteed to be met. Many algorithms have been proposed in the classical real-time scheduling theory to deal with the problem of scheduling applications on multiprocessor systems [4]. These algorithms assume that the application tasks satisfy a certain *real-time task model* which allows them to analytically reason about the timing behavior of the tasks. Several real-time task models have been proposed such as the periodic task model and the sporadic task model. Recent works (such as [5], [6]) have shown that applications modeled as CSDF graphs can be scheduled as a set of real-time periodic tasks. This provides two main benefits. The first benefit is that the designer can apply hard real-time (HRT) scheduling algorithms [4] to streaming applications in order to guarantee timing constraints and temporal isolation among different applications. The second benefit is that applications can be loaded at run-time on the system, provided that the appropriate schedulability tests are satisfied. The benefits mentioned above are advantages of [5], [6] over conventional static scheduling.

So far, the approaches proposed in [5], [6] consider only hard real-time *global* or *partitioned* scheduling algorithms. Under *global* scheduling algorithms, all the tasks can migrate among all the processors. Such algorithms can be *optimal* for multiprocessor systems, which means that they can fully exploit the available computational resources (see for instance [7]). However, this comes at the cost of high scheduling overheads due to excessive task preemptions and migrations. Moreover, modern MPSoC systems that ensure predictability of execution are commonly implemented as distributed memory systems in order to avoid the unpredictability of accessing shared resources. Therefore, implementing a global scheduler on such distributed memory systems implies that the code of each task has to be replicated[1] on all the processors, incurring a large memory overhead. *Partitioned* scheduling algorithms, in contrast, incur neither migration overhead nor memory overhead because each task is statically allocated to a single processor. However, these algorithms are affected by bin-packing issues [9]. If no limit on the maximum task utilization of a task set is imposed, partitioned algorithms may require almost twice as many processors compared to an optimal scheduler [10].

Recently, a third class of algorithms, called *hybrid* scheduling algorithms, has been proposed. Among the hybrid scheduling algorithms, *semi-partitioned algorithms* (e.g., [11]) seem to be a good match for distributed memory systems. Under semi-partitioned algorithms, most of the tasks are statically allocated to processors, and only a subset of the tasks is allowed to migrate among different processors. Migrating tasks follow a migration pattern derived at design-time. Thus, semi-partitioned approaches represent a "middle ground" between partitioned and global scheduling algorithms. In fact, semi-partitioned scheduling algorithms can overcome (or mitigate) the bin-packing effects that affect partitioned approaches. As a result, semi-partitioned algorithms require less processors than partitioned algorithms to schedule certain task sets. At the same time, these algorithms do not incur large memory overheads and task migration/preemption overheads like global algorithms.

Several semi-partitioned scheduling algorithms have been proposed [11]–[15]. These algorithms can be classified based on *when* migrations are allowed to occur. In *restricted-migration* approaches [11]–[13] migrations can happen at job boundaries only. In *unrestricted-migration* (or *portioned*) approaches, migrations can happen at any time during a job execution. We argue that the restricted-migration class of semi-partitioned schedulers is the most suitable for distributed memory MPSoCs. This is because migrating at job boundaries reduces the amount of data (state) to be transferred from one processor to the next. Moreover, if the task

[1]We assume task migration using code replication, as in [8], because of its low overhead in distributed memory MPSoC systems.

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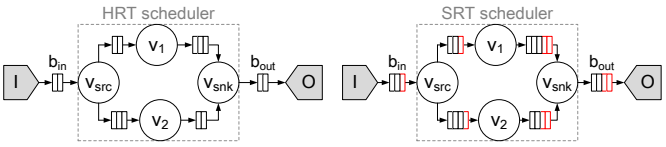


(a) Analysis under HRT schedulers (e.g., [5])

(b) Analysis under SRT schedulers (this paper)

Fig. 1. Scheduling framework under both HRT (a) and SRT (b) schedulers.

does not keep an internal state between two successive jobs[2], no state migration is needed. Within the class of restricted-migration semi-partitioned approaches, EDF-fm [11] is particularly suited to distributed memory systems because in that approach a migrating task is allowed to migrate only between two processors (contrary to [12], [13], in which migrating tasks may span among all the processors). This property reduces substantially the overhead caused by replicating the task code. In addition, EDF-fm uses a fast utilization-based schedulability test (contrary to [12], [13]), that can be easily executed at run-time for incoming applications. For the reasons explained above we employ EDF-fm in our work.

Although EDF-fm can have great benefits for distributed memory MPSoCs, it provides hard real-time guarantees only for migrating tasks and soft real-time (SRT) guarantees for fixed tasks. This means that fixed tasks can miss their deadlines by a bounded value called *tardiness*. As a consequence, the proposed scheduling frameworks in [5], [6] can not be used directly with such an algorithm, since these frameworks assume that all task deadlines are met. Therefore, in this paper, we extend the framework proposed in [5] to support soft real-time scheduling algorithms while providing hard real-time guarantees on the input/output interfaces of the application with the external environment. This is illustrated in Fig. 1 which shows the difference between using HRT and SRT schedulers. Given an application, we want to provide hard real-time guarantees to I and O, which denote the external provider/consumer of the data streams. I and O are assumed to execute periodically. I delivers the data into an input buffer b_{in}, from which the application reads the data and processes it. After that, the application writes the processed data into an output buffer b_{out} which is read by the external data consumer O. The framework in [5] shows that the application can be scheduled under a partitioned HRT scheduling algorithm as a set of real-time periodic tasks by computing sufficient minimum buffer sizes between application tasks. In this paper, we show that the same can be done under semi-partitioned SRT schedulers, achieving the same throughput and using less processors, albeit requiring larger buffers and increased application latency. This increase in the buffering requirements is visualized in Fig. 1(b) using red color. Therefore, under both schemes (i.e., Fig. 1(a) and 1(b)), the external data consumer O will always find enough data in b_{out} (and I enough space in b_{in}), regardless of the fact that v_{snk} (and v_{src}) might miss deadlines under the SRT scheduler.

A. Paper Contributions

Given an implicit-deadline periodic task set representation of a CSDF graph and any soft real-time scheduling algorithm that ensures a bounded tardiness to each task, we derive the earliest task start times and minimum buffer sizes that guarantee the existence of a valid schedule of the given application. Valid schedule means that, even in presence of task tardiness, tasks can be released periodically and neither buffer underflow nor overflow can occur. We call this approach *tardiness-aware periodic scheduling*. Moreover, we guarantee that the interfaces of the environment with the application (see I and O in Fig. 1(b)) can execute in a strictly periodic way with neither underflow nor overflow on input and output buffers (see b_{in} and b_{out} in Fig. 1(b)), i.e., we provide HRT guarantees for the input/output interfaces.

Then, using the above result, we focus on a specific restricted-migration semi-partitioned scheduling algorithm, namely EDF-fm [11], with the goal of reducing the number of required processors compared to partitioned approaches. We propose a novel heuristic, called FFD-SP, to assign tasks to processors while taking data dependencies into account. This heuristic replaces the ones proposed in [11] that are intended for independent task sets. Our proposed heuristic is aimed at reducing the number of required processors while keeping a low buffer size and latency overhead when the EDF-fm algorithm is used.

Finally, we show on a set of real-life benchmarks that our approach can lead to significant benefits by reducing the number of processors required to schedule a given application, compared to a partitioned approach, while maintaining the same throughput. Among the used benchmarks, we show that this is true for all the applications that suffer from the bin-packing effects of partitioned algorithms. Moreover, our experiments show that the increase in memory requirements and application latency introduced by our approach is acceptable (on average +24% and +29%, respectively), especially for systems in which the throughput constraint is more important than memory or latency constraints.

II. Related Work

To the best of our knowledge, real-time semi-partitioned scheduling algorithms have never been studied when mapping streaming applications with inter-task data dependencies to MPSoCs. In fact, existing semi-partitioned solutions [11]–[15] only consider sets of independent tasks. In the real-time community, however, techniques different from pure partitioning to assign data-dependent application tasks to a multiprocessor platform have already been devised. Existing approaches which are close to our work are [16] and [17] by Liu and Anderson. These approaches use a global scheduler which, similar to our case, satisfies soft real-time requirements. In particular, [16] describes a way to guarantee bounded tardiness of an application specified as a pipeline of tasks under a SRT global scheduler. A strong limitation in [16] is that only simple pipeline application topologies are handled, contrary to our approach that can handle more complex topologies like CSDF graphs. In [17], they extend their analysis to guarantee bounded task tardiness in more complex application graph topologies, such as Processing Graph Method (PGM) graphs. However, PGM graphs are still less expressive than CSDF graphs, which are supported in our approach. Moreover, the work in [17] does not address the calculation of minimum buffer sizes, which is an important metric to evaluate the practicability of the approach. In contrast, the calculation of buffer sizes is supported by our approach.

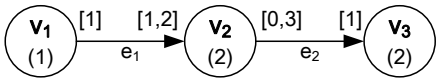
III. Background

In Sec. III-A we introduce the system model considered in our work. Then, we summarize two techniques instrumental to our approach: strictly periodic scheduling of acyclic CSDF graphs (Sec. III-B) and the EDF-fm semi-partitioned scheduling algorithm (Sec. III-C).

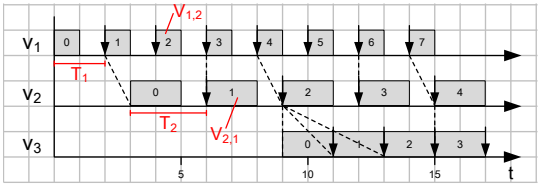
A. System Model

We consider a system composed of a set $\Pi = \{\pi_1, \pi_2, \cdots, \pi_m\}$ of m homogeneous processors. The processors execute a task set $\Gamma = \{\tau_1, \tau_2, \cdots, \tau_n\}$ of n periodic tasks, which can be preempted at any time. A periodic task $\tau_i \in \Gamma$ is defined by a 4-tuple $\tau_i = (C_i, T_i, S_i, \Delta_i)$, where C_i is the worst-case execution time of the task, T_i is the task period, S_i is the start time of the task, and Δ_i represents the task tardiness bound, as defined in Definition 1 below. In this paper we consider only *implicit-deadline* tasks, which have relative deadline D_i equal to their period T_i. The utilization of a task τ_i is given by $u(\tau_i) = C_i/T_i$ (also denoted

[2]Tasks that possess this property are called *stateless* and are common in streaming applications.



(a) Simple example of a CSDF graph. Actor execution times are indicated between parentheses.



(b) Derived periodic task set and minimum start times. Down arrows represent task deadlines, dotted lines represent data dependencies.

Fig. 2. Example of the approach described in [5].

as u_i). Given a set γ of tasks mapped onto processor π_k, the total assigned utilization on π_k is denoted with $U(\pi_k) = \sum_{\tau_i \in \gamma} u(\tau_i)$.

The kth job of task τ_i is denoted with $\tau_{i,k}$. Job $\tau_{i,k}$ of τ_i, for all $k \in \mathbb{N}_0$, arrives in the system at the time instant $r_{i,k} = S_i + kT_i$. The absolute deadline of job $\tau_{i,k}$ is $d_{i,k} = S_i + (k+1)T_i$, which is coincident with the arrival of job $\tau_{i,k+1}$. We denote the actual completion time of $\tau_{i,k}$ with $f_{i,k}$. In SRT systems, tasks are allowed to miss their deadline by a certain bounded value, as defined below.

Definition 1 (Tardiness bound)**.** A task τ_i is said to have a *tardiness bound* Δ_i if $f_{i,k} \leq (d_{i,k} + \Delta_i), \forall k \in \mathbb{N}_0$.

B. Strictly Periodic Scheduling of CSDF Graphs

A CSDF graph [3] is composed of a set of actors V and a set of edges E, through which actors communicate. The authors in [5] show that the actors in an acyclic CSDF graph can be scheduled as a set of real-time periodic tasks. An example of this is given in Fig. 2. Based on the properties of the graph, they derive the minimum period of each actor using the following expression:

$$T_i = \frac{Q}{q_i} \left\lceil \frac{\eta}{Q} \right\rceil \tag{1}$$

where q_i is the number of repetitions of actor v_i per graph iteration, C_i is the worst-case execution time of v_i, $\eta = \max_{v_i \in V}\{C_i\, q_i\}$ and $Q = \mathsf{lcm}\{q_1, q_2, \cdots, q_n\}$. Consider the CSDF graph shown in Fig. 2(a). Its repetition vector is $\vec{q} = [3, 2, 3]$. Its worst-case execution time vector is $\vec{C} = [1, 2, 2]$. Then, it follows that $\eta = 6$ and $\vec{T} = [2, 3, 2]$. In general, the derived period vector $\vec{T}$ satisfies the condition:

$$q_1 T_1 = q_2 T_2 = \cdots = q_n T_n = \alpha \tag{2}$$

where α is defined as *iteration period*, and represents the duration needed by the graph to complete one iteration.

Then, the authors in [5] define the following functions:

Definition 2 (Cumulative Production Function)**.** The cumulative production function of actor v_i producing into channel e_u during a time interval $[t_s, t_e)$, denoted by $\mathsf{prd}_{[t_s,t_e)}(v_i, e_u)$, is the sum of the number of tokens produced by v_i into e_u during the interval $[t_s, t_e)$.

Definition 3 (Cumulative Consumption Function)**.** The cumulative consumption function of actor v_j consuming from channel e_u over a time interval $[t_s, t_e)$, denoted by $\mathsf{cns}_{[t_s,t_e)}(v_j, e_u)$, is the sum of the number of tokens consumed by v_j from e_u during the interval $[t_s, t_e)$.

Note that the time interval in Definitions 2 and 3 can be either open or closed from the right. Using the functions defined in Definitions 2 and 3, the authors derive the earliest start times of actors and minimum buffer sizes of the channels, as described in the following.

1) Earliest start times: The earliest start time of actor v_j, denoted with S_j, is derived using the following expression (according to Lemma 3 in [5] and assuming no task tardiness):

$$S_j = \begin{cases} 0 & \text{if } \mathsf{prec}(v_j) = \emptyset \\ \max_{v_i \in \mathsf{prec}(v_j)} \left(S_{i \to j}\right) & \text{if } \mathsf{prec}(v_j) \neq \emptyset \end{cases} \tag{3}$$

where

$$S_{i \to j} = \min_{t \in [0, S_i + \alpha]} \Big\{ t : \mathop{\mathsf{prd}}_{[S_i, \max(S_i,t)+k)}(v_i, e_u) \geq \mathop{\mathsf{cns}}_{[t, \max(S_i,t)+k]}(v_j, e_u) \quad \forall k = 0, 1, \cdots, \alpha \Big\} \tag{4}$$

where α is the iteration period as defined by Equation (2), and S_i is the start time of the predecessor actor v_i. Equation (4) considers the dependency between predecessor actor v_i and successor actor v_j, over channel e_u. It computes the *earliest* start time $S_{i \to j}$ such that v_j, in its periodic execution, is never blocked on read. This is ensured by checking that at each time instant the cumulative number of tokens produced by v_i over e_u is greater than or equal to the number of tokens consumed by v_j from the same channel. For instance, actor v_2 in Fig. 2(b) must start no earlier than time $t = 3$ to satisfy its data dependency from actor v_1.

Start times $S_{i \to j}$ are computed for each actor in the predecessor set of v_j (denoted with $\mathsf{prec}(v_j)$). Then, when actor v_j has several predecessors, the start time S_j has to be set to the maximum of start times $S_{i \to j}$ considering each predecessor in isolation, as captured by Equation (3).

2) Minimum buffer sizes: Once the start time of actors have been calculated, the authors derive the following expression for the *minimum* size b_u of communication channel e_u connecting actors v_i and v_j (see Lemma 4 in [5]):

$$b_u(v_i, v_j) = \max_{k \in [0,1,\cdots,\alpha]} \Big\{ \mathop{\mathsf{prd}}_{[S_i, \max(S_i,S_j)+k)}(v_i, e_u) - \mathop{\mathsf{cns}}_{[S_j, \max(S_i,S_j)+k)}(v_j, e_u) \Big\} \tag{5}$$

In other terms, Equation (5) evaluates the maximum number of unconsumed tokens in e_u during one iteration of v_i and v_j.

C. EDF-fm Semi-partitioned Scheduling

Anderson et al. in [11] describe the EDF-fm semi-partitioned scheduling algorithm for sporadic tasks sets in the context of soft real-time multiprocessor systems. Under EDF-fm tasks can be either *fixed* or *migrating*. Migrating tasks are allowed to migrate between only two processors, with the restriction that migration can only happen at job boundaries.

The EDF-fm approach consists of two phases: the *assignment phase* and the *execution phase*, which are summarized in the following sections.

1) Assignment phase: Consider the following definitions:

Definition 4 (Task share)**.** A task τ_i is said to have a share $s_{i,k}$ on π_k when a fraction $s_{i,k}$ of π_k's available utilization is allocated to τ_i.

Definition 5 (Task fraction)**.** Given $s_{i,k}$, π_k executes a fraction $\varphi_{i,k} = \frac{s_{i,k}}{u_i}$ of τ_i's total execution requirement.

In the assignment phase each task is assigned to either one processor (fixed task) or two processors (migrating task). In particular, the assignment phase assigns tasks in sequence to processors. Tasks are assigned to a processor π_k until its capacity is exhausted. If a task τ_i cannot entirely fit on processor π_k, then a share $s_{i,k} = 1 - U(\pi_k)$ of its utilization is assigned to π_k and the remaining utilization $s_{i,k+1} = (u_i - s_{i,k})$ is assigned to the next processor, π_{k+1}. The assignment phase of EDF-fm ensures that at most two migrating tasks are assigned to any processor.

Example 1. Given the task set $\{\tau_1 = (C_1 = 3, T_1 = 10), \tau_2 = (2, 5), \tau_3 = (2, 5), \tau_4 = (1, 2), \tau_5 = (1, 2), \tau_6 = (2, 5), \tau_7 = (1, 2)\}$, the EDF-fm algorithm derives the task assignment shown in Fig. 3. For instance, task τ_3 in Fig. 3 cannot entirely fit onto

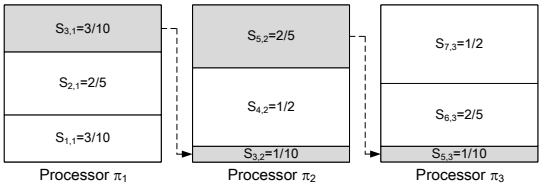


Fig. 3. EDF-fm assignment of the task set considered in Example 1.

π_1, thus its utilization is split between π_1 and π_2 with shares $s_{3,1} = 3/10$ and $s_{3,2} = 1/10$, respectively.

2) Execution phase: The execution phase employs a simple online scheduling algorithm that is derived from EDF and ensures bounded tardiness with a minimal overhead compared to a canonical EDF scheduler. Jobs belonging to a task τ_i, which migrates between processors π_k and π_{k+1}, are assigned such that in the long run the fraction of τ_i's workload executed on π_k is close to $\varphi_{i,k}$. For instance, in the scenario depicted in Fig. 3, on average 1 out of 4 jobs of τ_3 are assigned to π_2 and the remaining 3 jobs are assigned to π_1. When two migrating tasks, τ_i and τ_j, are assigned to π_k, the tardiness bound for a fixed task τ_u assigned to the same processor is given by:

$$\Delta(\tau_u) = \frac{C_i \cdot (\varphi_{i,k} + 1) + C_j \cdot (\varphi_{j,k} + 1) - T_u \cdot (1 - U(\pi_k))}{1 - s_{i,k} - s_{j,k}} \tag{6}$$

where C_i and C_j are the worst-case execution times of τ_i and τ_j (as defined in Sec. III-A), T_u is the period of task τ_u, and $U(\pi_k)$ is the sum of fixed tasks' utilization and migrating tasks' shares allocated to π_k. By contrast, migrating tasks do not miss any deadline, therefore their tardiness bound is zero.

On a processor running two migrating tasks, τ_i and τ_j, EDF-fm requires that the sum of their utilization does not exceed one:

$$U_{mig}(\pi_k) = u_i + u_j \leq 1 \tag{7}$$

This condition is automatically satisfied if the maximum utilization of any task is limited to $1/2$, given the fact that at most two migrating tasks can be assigned to a single processor. However, tasks that exceed this utilization limit can still be considered, provided that condition (7) is valid for all the processors.

IV. Tardiness-Aware Periodic Scheduling

In this section, we present the first main contribution of this paper. We show how a CSDF graph, represented by a set of periodic tasks derived using the approach in [5] (see Sec. III-B), can be scheduled using soft real-time schedulers, i.e., schedulers that may introduce a bounded tardiness on the completion of tasks. The SRT scheduler considered in this paper is the EDF-fm algorithm, whose per-task tardiness bound is given by Equation (6). Note, however, that the results obtained in this section are valid for any SRT scheduler which provides bounded task tardiness. Our solution extends the framework in [5] by deriving new earliest start times for each task (see Sec. IV-A) and minimum buffer sizes (see Sec. IV-B) that can handle task tardiness and still allow a periodic release of each task.

A. Earliest Start Times in Presence of Tardiness

Based on Definitions 1, 2, and 3 introduced in Sec. III, we give the following Lemma:

Lemma 1. *In presence of task tardiness, bounded by Δ_i for source actor v_i and by Δ_j for destination actor v_j, the earliest start time $S_{i\to j}$ of actor v_j due to its dependency from v_i, under a valid schedule, is given by:*

$$S_{i\to j} = \min_{t\in[0,S_i+\Delta_i+\alpha]} \Big\{ t : \underset{[S_i+\Delta_i,\max(S_i+\Delta_i,t)+k)}{\mathsf{prd}}(v_i,e_u) \geq \underset{[t,\max(S_i+\Delta_i,t)+k]}{\mathsf{cns}}(v_j,e_u) \quad \forall k = 0,1,\cdots,\alpha \Big\} \tag{8}$$

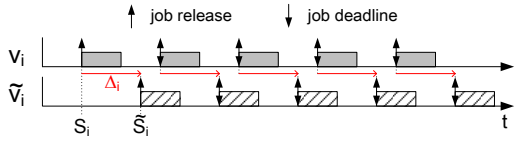


Fig. 4. Worst-case scheduling of source actor v_i when deriving $S_{i\to j}$ in presence of tardiness.

Proof: If actors v_i and v_j may be affected by tardiness, Equation (4) can not be applied in its original form. For instance, if job $v_{1,2}$ in Fig. 2(b) completes later than its deadline, job $v_{2,1}$ (that depends on the completion of $v_{1,2}$) cannot be released at time $t = 6$. It follows that the start time of actor v_2 has to be changed.

We can define a worst-case scenario of the execution of v_i and v_j to derive the earliest start time $S_{i\to j}$ in case of tardiness. This worst-case scenario to determine task start times occurs when: (i) all the invocations of the source actor v_i are completed with the maximum tardiness; (ii) none of the invocations of the destination actor v_j are affected by the tardiness (i.e., v_j meets all its deadlines). In the worst-case execution of v_i, the completion of *all* its invocations are delayed by the maximum considered tardiness Δ_i, such that $f_{i,k} = d_{i,k} + \Delta_i$, for all k. We can associate these worst-case delayed invocations to a fictitious task $\tilde{v}_i$. Task $\tilde{v}_i$ has start time $\tilde{S}_i = S_i + \Delta_i$, constant period $\tilde{T}_i = T_i$, and no tardiness. Notice that invocations $\tilde{v}_{i,k}$ of $\tilde{v}_i$ are strictly periodic. Fig. 4 shows the schedule of actor v_i and its worst-case execution scenario, represented by $\tilde{v}_i$.

By contrast, in the worst-case scenario to determine $S_{i\to j}$, v_j is executed as early as possible, so we assume that all invocations $v_{j,k}$ of v_j are not affected by tardiness. Then, the earliest start time that guarantees the absence of blocking of v_j in its execution, even for the worst-case production and consumption patterns of v_i and v_j, is found by evaluating Equation (4) with $\tilde{v}_i$ as source actor and v_j as destination actor. This scenario is captured by Equation (8). Note that any completion of an invocation $v_{i,k}$ of v_i earlier than its corresponding worst-case $\tilde{v}_{i,k}$ results in an earlier production of tokens, such that the inequality in Equation (8) still holds for all $k \in [0,1,\cdots,\alpha]$. Similarly, if any of the invocations of v_j is affected by tardiness, the token consumption is executed later and Equation (8) guarantees that enough tokens will be available to be read. ■

Note also that the start time $S_{i\to j}$ of actor v_j due to its dependency from v_i is only affected by the tardiness bound Δ_i of the source actor. In addition, when actor v_j has several predecessors, the start time S_j has to be set to the maximum of the start times $S_{i\to j}$ given by Equation (8) considering each predecessor in isolation, as captured by Equation (3).

B. Minimum Buffer Sizes in Presence of Tardiness

Based on Definitions 1, 2, and 3 introduced in Sec. III, and given the actor start times calculated leveraging Lemma 1, the following lemma provides a way to derive minimum buffer sizes in case of task tardiness:

Lemma 2. *In presence of task tardiness, bounded by Δ_i for source actor v_i and by Δ_j for destination actor v_j, the minimum buffer size b_u of a communication channel e_u connecting v_i and v_j, under a valid schedule, is given by:*

$$b_u(v_i,v_j) = \max_{k\in[0,1,\cdots,\alpha]} \Big\{ \underset{[S_i,\max(S_i,S_j+\Delta_j)+k)}{\mathsf{prd}}(v_i,e_u) - \underset{[S_j+\Delta_j,\max(S_i,S_j+\Delta_j)+k)}{\mathsf{cns}}(v_j,e_u) \Big\} \tag{9}$$

Proof: To get the minimum buffer size in presence of task tardiness, we consider the worst-case scenario that would result in the maximum buffer requirement for channel e_u. This worst-case scenario occurs when: (i) all the invocations $v_{j,k}$ of the destination actor v_j complete with the maximum tardiness (such that $f_{j,k} =$

Algorithm 1: FFD-SP task assignment heuristic.

Input: The number of processors m, a task set $\Gamma = \{\tau_1, \tau_2, \cdots, \tau_n\}$ of n periodic tasks.
Result: *True* and an m-partition describing the task assignment onto m processors if Γ is schedulable, *False* otherwise.

1 Find $\Gamma_s = \{\tau : \tau \in \Gamma \wedge \tau$ is stateful$\}$;
2 Assign tasks in Γ_s using FFD;
3 **if** Γ_s *cannot be assigned* **then**
4 **return** False;
5 **for** $\tau \in (\Gamma - \Gamma_s$, *sorted in decreasing utilization*$)$ **do**
6 Try to assign task τ using First-Fit heuristic;
7 **if** *First-Fit is successful* **then**
8 **continue**;
9 **for** $\pi' \in (\Pi$ *sorted in decr. available utilization*$)$ **do**
10 $s_1(\tau) = 1 - U(\pi')$;
11 Assigned = False;
12 **if** *sp_assign*$(s_1(\tau), \pi')$==*True* **then**
13 $s_2(\tau) = u(\tau) - s_1(\tau)$;
14 **for** $\pi'' \in (\Pi$ *sorted in incr. available utilization*$)$ **do**
15 **if** *sp_assign*$(s_2(\tau), \pi'')$==*True* **then**
16 Assigned = True;
17 **break**;
18 **if** *Assigned*==*False* **then**
19 Revert assignment of $s_1(\tau)$ to π';
20 **else**
21 **break**;
22 **if** *Assigned*==*False* **then**
23 **return** False;
24 Optimize the obtained partition;
25 **return** True;

Algorithm 2: *sp_assign* function.

Input: The share s of task τ to be assigned, a processor π.
Result: *True* if s can be assigned to π, *False* otherwise.

1 **if** $(U(\pi) + s \leq 1)$ *and* $(U_{mig}^{current}(\pi) + u(\tau) \leq 1)$ **then**
2 Assign s to π;
3 **return** True;
4 **else**
5 **return** False;

$d_{j,k} + \Delta_j$, for all k); (ii) none of the invocations of the source actor are affected by tardiness.

We can then prove Lemma 2 with a procedure similar to the one used in the proof of Lemma 1. We associate the worst-case completion of all the invocations $v_{j,k}$ to a fictitious actor $\tilde{v}_j$. Actor $\tilde{v}_j$ is strictly periodic, with no tardiness, constant period $\tilde{T}_j = T_j$ and start time $\tilde{S}_j = S_j + \Delta_j$. Then, the minimum buffer requirement of the communication channel e_u is found by evaluating Equation (5) with v_i as source actor and $\tilde{v}_j$ as destination actor. This scenario is captured by Equation (9).

Note that any earlier completion of any of the iterations of v_j would not increase the buffer size requirement. This is because an earlier completion of v_j would results in an earlier consumption of tokens from channel e_u. Similarly, any delayed completion of an iteration of v_i would result in a delayed production of tokens to the considered channel. Thus, the derived value of b_u is sufficient and minimum. ■

Note that Equations (8) and (9) can also be used to analyze the interfaces between the external data provider and consumer (I and O in Fig. 1(b)) and the source and sink nodes of the application (v_{src}, v_{snk}). Compared to the HRT approach shown in Fig. 1(a), in the SRT approach of Fig. 1(b) v_{src} and/or v_{snk} may experience tardiness. In this case, Equation (8) and (9) derive delayed start time of the external consumer O and larger buffer sizes of b_{in} and b_{out} such that both I and O can execute strictly periodically with neither buffer overflow nor underflow occurring on b_{in} and b_{out}.

V. FFD-SP Task Assignment Heuristic

In this section, we present the second main contribution of this paper. Considering the results (Lemma 1 and Lemma 2) shown in Sec. IV, we propose a task assignment heuristic that does not follow the sequential approach common to all the heuristics proposed in [11]. In fact, as explained in Sec. III-C, the heuristics in [11] assign tasks to processors in a sequential way, which means that in most cases processors are assigned migrating tasks. In turn, this makes most tasks in the system affected by tardiness. Actor tardiness imposes larger buffer sizes (according to Lemma 2) and postponed start times of successor actors (according to Lemma 1). Overall, this leads to larger memory requirements and increased application latency.

In contrast to the heuristics in [11], our proposed heuristic starts to consider semi-partitioning only when the *First-fit Decreasing* (FFD) heuristic [9] fails to assign a certain task in the system. The proposed allocation/assignment heuristic, called FFD-SP, is described in Algorithm 1. The algorithm accepts as input the number of processors m onto which the task set Γ has to be assigned. At the first execution of Algorithm 1, m is set to m_{OPT}, the number of processors required by an optimal scheduler. If the task set cannot be assigned to m processors, m is increased by one and Algorithm 1 is executed again until a successful assignment is found.

At first, Algorithm 1 builds Γ_s, the set of stateful actors, which are then assigned using the FFD heuristic (lines 1-4). Stateful actors are considered first because this way they are fixed to a processor and there is no need to migrate their state. Then, considering task $\tau \in (\Gamma - \Gamma_s)$, the algorithm tries to assign task τ to one of the processors using FFD (lines 6-8). If FFD does not succeed, the algorithm tries to divide the utilization of task τ in two shares, $s_1(\tau)$ and $s_2(\tau)$. Traversing the processor list in decreasing order of available utilization, a share $s_1(\tau) = 1 - U(\pi')$ is tried to be mapped on processor π' (lines 9-12). If the assignment of $s_1(\tau)$ is successful, the algorithm attempts to map a share $s_2(\tau) = u(\tau) - s_1(\tau)$ by traversing the list of processors in increasing order of available utilization (lines 13-17).

The rationale behind this assignment heuristic is two-fold: (1) tasks are semi-partitioned only when the FFD assignment does not succeed; this way the number of processors with migrating tasks (and thus with tardiness) are likely to be less; (2) when a task has to be semi-partitioned, the algorithm tries to allocate the largest share possible $s_1(\tau)$ from the remaining utilization on processors; then, it tries to find the *best fit* for the remaining share $s_2(\tau)$, in order to leave larger "chunks" of processor available utilizations to remaining (unallocated) tasks.

The algorithm makes use of the *sp_assign* function to try and assign task shares. As shown in Algorithm 2, this function checks two conditions. First, there must be enough available utilization on the processor to accommodate the share. Second, in case another migrating task has already been mapped on processor π, condition (7) in Sec. III-C must be satisfied.

When an m-partition has been successfully found, the heuristic tries to *optimize* it (line 24 in Algorithm 1). The optimization consists in re-assigning the migrating task shares, whenever possible, to processors to which less fixed tasks are assigned. This way, less actors are affected by tardiness, leading to lower application latency and buffer size requirements. Note that in Algorithm 1 the first share of a migrating task $s_1(\tau)$ is set to the largest possible value, given the current available utilization of processors. This in turns makes the second share of each migrating task $s_2(\tau)$ as small as possible, making the process of optimization of the partition more effective.

VI. Evaluation

We evaluate our approach using the StreamIt benchmarks considered in [18], for which we employ the unfolding technique

TABLE I
COMPARISON OF DIFFERENT ALLOCATION/SCHEDULING APPROACHES.

Benchmark	OPT	Partitioned (FFD)				Semi-partitioned (FFD-SP)				Semi-partitioned (fm-LUF)		
	m_{OPT}	m_{FFD}	$\frac{m_{FFD}}{m_{OPT}}$	M_{FFD}[B]	L_{FFD}[c.c.]	m_{SP}	$\frac{m_{SP}}{m_{OPT}}$	$\frac{M_{SP}}{M_{FFD}}$	$\frac{L_{SP}}{L_{FFD}}$	m_{LUF}	$\frac{M_{LUF}}{M_{FFD}}$	$\frac{L_{LUF}}{L_{FFD}}$
FFT	24	30	1.25	144680	192512	26	1.083	1.413	1.483	26	1.485	1.676
Beamformer	26	28	1.077	14492	60912	26	1.0	1.145	1.474	26	1.229	1.606
TDE	20	25	1.25	516282	1127175	20	1.0	1.560	1.396	21	1.722	1.860
DES	26	33	1.269	3381	33088	27	1.038	1.138	1.218	28	1.684	1.862
MPEG2	8	9	1.125	61909	138240	8	1.0	1.290	1.217	9	3.014	3.432
Bitonic	11	13	1.182	2374	2275	11	1.0	1.139	1.185	11	1.413	1.395
Serpent	39	42	1.077	59815	370296	40	1.026	1.012	1.074	39	1.068	1.479
average	-	-	1.176	-	-	-	1.021	1.243	1.292	-	1.659	1.902

described in [18] to derive larger CSDF graphs with improved throughput. Among these benchmarks, seven applications require, under the partitioned FFD allocation scheme, more processors than an optimal scheduler. This set of applications is listed in Table I. In this section we compare the system metrics obtained with three different allocation/scheduling approaches: (i) Partitioned EDF with FFD heuristic; (ii) Semi-partitioned EDF-fm, with our proposed FFD-SP heuristic; (iii) Semi-partitioned EDF-fm, with the LUF heuristic proposed in [11]. These approaches are denoted in Table I with *FFD*, *FFD-SP*, and *fm-LUF*, respectively.

Note that *all* the approaches in Table I lead to the same application throughput. This is because the throughput of an application depends on the period of its sink actor, which is unchanged in our analysis even in presence of task tardiness. In addition, we choose to compare the results of the LUF heuristic with our FFD-SP heuristic because, among the heuristics proposed in [11], LUF achieves the smallest number of processors.

The m_{OPT} column in Table I lists the number of processors required by an optimal scheduler (for instance [7]) to execute the considered applications.

Let us focus on the comparison between the partitioned approach (FFD) and our proposed semi-partitioned approach (FFD-SP). We note that the FFD approach results in a number of processors (m_{FFD}) which is on average 17.6% greater than the number required by an optimal scheduler (see column $\frac{m_{FFD}}{m_{OPT}}$). In contrast, our FFD-SP algorithm requires on average only 2.1% more processors (see column $\frac{m_{SP}}{m_{OPT}}$), while maintaining the same throughput. This means that our proposed approach can exploit the available processors more efficiently, getting significantly closer to the results obtained by optimal schedulers (see columns m_{SP} and m_{OPT}). However, this comes at two costs. (1) The first cost is the increase of memory requirements. For each benchmark, column M_{FFD} reports the memory required by the partitioned approach, expressed in bytes. Compared to FFD, in FFD-SP the memory requirements increase due to both the size of buffers, that have to be enlarged to handle task tardiness, and the code size overhead of task replicas, which are necessary in case of migrating tasks. In Table I this increase in memory requirements is expressed by the ratio $\frac{M_{SP}}{M_{FFD}}$. On average, our proposed approach requires 24.3% more memory compared to FFD. We argue that this increase in memory requirements is acceptable, given the fact that even the most memory-demanding application requires 516 KB, which is way less than the memory available in modern MPSoC systems. (2) The second cost is the increase in applications' latency, due to the postponement of task start times needed to handle task tardiness. Column L_{FFD} shows the applications' latency, expressed in clock cycles, under FFD. The latency increase of our FFD-SP over FFD is on average 29.2% (see column $\frac{L_{SP}}{L_{FFD}}$). We consider these memory and latency overheads imposed by our approach reasonable, especially when the throughput constraint of an application is more important than memory or latency constraints.

Finally, to evaluate the effectiveness of our proposed FFD-SP heuristic, we compare its results with LUF. We can see from the last two columns of Table I that over the considered benchmarks the EDF-fm approach with LUF heuristic incurs a much larger memory overhead (on average +65.9%, see column $\frac{M_{LUF}}{M_{FFD}}$) and latency increase (on average +90.2%, see column $\frac{L_{LUF}}{L_{FFD}}$) compared to FFD. Moreover, we note that for most applications the number of required processors is equal or greater when using LUF (m_{LUF}) compared to our FFD-SP (m_{SP}), with the exception of the *Serpent* application. This means that for most of the benchmarks our FFD-SP heuristic is equally or more efficient than LUF in exploiting the available processing resources.

VII. CONCLUSIONS

In this paper we prove that streaming applications modeled as acyclic CSDF graphs can be scheduled using a semi-partitioned soft real-time scheduler, providing hard real-time guarantees on the input/output interfaces between the application and the environment. We propose a novel heuristic that is aimed at reducing the number of required processors while keeping a low buffer size and latency overhead when this semi-partitioned approach is used. Finally, we show on a set of real-life benchmarks that our approach can reduce the number of processors required to schedule those applications which incur bin-packing limitations of partitioned approaches, guaranteeing the same throughput and with a reasonable overhead in terms of memory requirements and application latency.

REFERENCES

[1] E. Lee and A. Sangiovanni-Vincentelli, "A framework for comparing models of computation," *IEEE TCAD*, 1998.
[2] E. Lee and D. Messerschmitt, "Synchronous data flow," *P. IEEE*, 1987.
[3] G. Bilsen *et al.*, "Cyclo-static dataflow," *IEEE TSP*, 1996.
[4] R. Davis and A. Burns, "A survey of hard real-time scheduling for multiprocessor systems," *ACM Comput. Surv.*, 2011.
[5] M. Bamakhrama and T. Stefanov, "On the hard-real-time scheduling of embedded streaming applications," *DAES*, 2012.
[6] A. Bouakaz *et al.*, "Affine data-flow graphs for the synthesis of hard real-time applications," in *ACSD*, 2012.
[7] S. Baruah *et al.*, "Proportionate progress: A notion of fairness in resource allocation," *Algorithmica*, 1996.
[8] E. Cannella, O. Derin, P. Meloni, G. Tuveri, and T. Stefanov, "Adaptivity support for MPSoCs based on process migration in polyhedral process networks," *VLSI Design*, 2012.
[9] D. S. Johnson, "Near-optimal bin packing algorithms," Ph.D. dissertation, MIT, 1973.
[10] J. M. López *et al.*, "Utilization Bounds for EDF Scheduling on Real-Time Multiprocessor Systems," *Real-T. Syst.*, 2004.
[11] J. Anderson *et al.*, "An EDF-based restricted-migration scheduling algorithm for multiprocessor soft real-time systems," *Real-T. Syst.*, 2008.
[12] F. Dorin *et al.*, "Semi-partitioned hard real-time scheduling with restricted migrations upon identical multiprocessor platforms," 2010.
[13] L. George *et al.*, "Job vs. portioned partitioning for the earliest deadline first semi-partitioned scheduling," *J. Syst. Architect.*, 2011.
[14] S. Kato and N. Yamasaki, "Portioned EDF-based scheduling on multiprocessors," in *EMSOFT*, 2008.
[15] B. Andersson and E. Tovar, "Multiprocessor scheduling with few preemptions," in *RTCSA*, 2006.
[16] C. Liu and J. Anderson, "Supporting pipelines in soft real-time multiprocessor systems," in *ECRTS*, 2009.
[17] ——, "Supporting Soft Real-Time DAG-Based Systems on Multiprocessors with No Utilization Loss," in *RTSS*, 2010.
[18] J. Zhai *et al.*, "Exploiting just-enough parallelism when mapping streaming applications in hard real-time systems," in *DAC*, 2013.