DTMC: An Actionable e-Customer Lifetime Value Model Based on Markov Chains and Decision Trees

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ABSTRACT

We describe a model for estimating the customer lifetime value (CLV) of customers in an e-commerce environment. The model is explained and experiments are performed on real-life data from a large Dutch Internet retailer.

Our method results in CLV estimates that have similar accuracy to estimates generated by the commonly used model, while keeping the number of customer segments much lower, and thus more 'actionable'.

Categories and Subject Descriptors

H.4.m [**Information Systems Applications**]: Miscellaneous; I.2.6 [**Artificial Intelligence**]: Learning

Keywords

Customer Lifetime Value, E-Commerce, CART, Marketing, Markov Chains

General Terms

Algorithms, Economics, Management

1. INTRODUCTION

E-commerce sales have exhibited a staggering growth in the last few years. The U.S. Census Bureau [13] estimates that total retail e-commerce sales in the third quarter of 2006 increased 21% with respect to the same period in 2005, whereas total retail sales increased only 5%. The sales volumes for online retail was $27.5 billion in this period, corresponding to 2.8 percent of the total sales volume of $991.7 billion. Other sources show similar growth percentages [12]. It is thus clear that the web is becoming an increasingly important sales channel and companies should strive for a successful web site.

Various metrics for measuring web site success have been proposed in the literature. (See, e.g., [10, 5].) Commonly used metrics for success include traffic count, conversion rate, click through ratio, user satisfaction, frequency of use and likelihood of return. Only recently [4], there has been some interest in the e-Commerce community in the use of *Customer Lifetime Value* (CLV) as a basis for measuring web site success. Briefly, the CLV of a customer is the expected value of the (discounted) profit she will generate now and in the future. Thus, contrary to the previous metrics, CLV takes a 'long term approach'. The sum of the CLV's for all customers is often referred to a *customer equity* (CE). The CLV concept is adopted from traditional marketing literature [3].

There are two motivations for using CLV/CE as a metric for web site success. First the CLV metric can be used to guide marketing efforts and make these more accountable, including data-mining efforts to promote sales, and help in establishing a firm long-term relationship with high-value customers. Second, CE can play a role in establishing the value of an e-commerce firm [11, 8].

Berger and Nasr [1] proposed a series of mathematical models for calculating CLV in different scenarios, whereas much of the earlier literature had been – in their words – 'dedicated to extolling its use as a decision criterion'. These models were subsequently re-formulated and unified by casting them into a Markov chain framework by Pfeifer and Carraway. Unlike cross sectional or basic longitudinal models for predicting CLV, Markov chains can be used to explicitly model the dynamics of how CLV develops over time for a given customer. Details on this latter model will be given in Section 2.

Contrary to many traditional marketing environments, e-commerce environments are typically very data-rich. Unfortunately, the traditional Markov based model from reference [6] is unable to cope with this data-richness, because the existence of large numbers of attributes and with numerical attributes leads to an explosion of the number of states in the Markov chain and/or partitioning problems.

In this paper, we propose a two-stage CLV model, called DTMC, that is based on CART. The addition of the decision tree (CART) step enables us to apply the Markov chain framework to with data-rich environments by grouping the customers into segments of similar value based on

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their attributes, thereby reducing the number of states. Besides enabling the Markov approach for e-commerce data, this has the added advantage that the model becomes more actionable, since it is easier to direct a single effort to a complete segment than varying efforts to each customer. Details on the DTMC model will be given in Section 3.

We evaluated the DTMC model using real world customer purchase data from a large Dutch Internet retailer selling ink cartridges for ink-jet printers. This gives encouraging results. These experiments and results are described in Sections 4 and 5 respectively. Finally, Section 6 gives conclusions, discussion and an outlook.

2. CUSTOMER EQUITY AND CUSTOMER LIFETIME VALUE

This section gives an overview of CLV and the model from [6] to estimate it.

Customers are seen as an asset in customer centric marketing, therefore financial theory is very useful when estimating CLV. The value of an asset is measured by discounting its future value to the present, thus a basic CLV model can be formulated as

$$CLV_i = \sum_{t=1}^{T} \frac{\text{Profit}_{i,t}}{(1+d)^t}, \qquad (1)$$

where CLV_i refers to lifetime value of customer i, T is the time horizon, $Profit_{i,t}$ is the profit gained from customer i at time t and d is the discounting factor. Profit is gained when revenues are larger then associated costs. Consequently, the CLV model can be split into two parts. Mathematically this becomes

$$CLV_i = \sum_{t=1}^{T} \frac{\text{Revenue}_{i,t}}{(1+d)^t} - \sum_{t=1}^{T} \frac{\text{Cost}_{i,t}}{(1+d)^t}. \qquad (2)$$

Because customer relationships are viewed as an asset. The total value of all customer relationships can be seen as an equity to the firm. Thus the sum of all individual CLV's of customers in the industry results in customer equity (CE):

$$CE = \sum_{i=1}^{I} CLV_i. \qquad (3)$$

The CLV model of Equation 2 is all that is needed to calculate CLV. The discount factor can be easily determined from business rules, but estimating future revenues and costs for every customer is were the difficulty lies.

Different approaches to this problem can be found in literature. For instance, references [9] and [14] develop individual level CLV models based on marketing theory, whereas [7] proposes a segment level CLV model based on pre-determined segments. As stated earlier, in this paper we build on the Markov chain approach from [6].

Pfeifer and Carraway propose the use of RFM variables, i.e. variables capturing the recency (time elapsed since the last purchase), frequency (total number of purchases) and monetary value (total generated income) of a customer. After discretization, these variables are used to define the states in the Markov model. We now give a short example, adopted from [6], to illustrate this idea.

The example uses only recency to model customer behavior, which is recorded months. The different values for recency are then used as states for the Markov chain. In

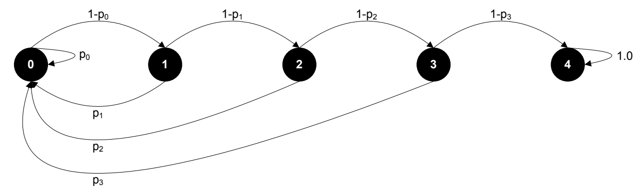

Figure 1: Graphical representation of the switching probabilities in the Markov matrix.

the example 5 states are defined. State 0 corresponds to recency 0, indicating a purchase in the current period. States 1, 2, and 3 respectively correspond to recency 1, 2, and 3 indicating a purchase 1, 2, or 3 periods ago. And state 4 corresponds to recency 4 or more. A customer in state 4 is considered to be a lost customer and state 4 therefore is said to be a 'dead state', since its not possible to return to another state from this state.

These states are then used to construct a Markov matrix $\mathbf{P}$, where element $\mathbf{P}_{ij}$ represents the probability of going from state i to state j. This matrix can either be estimated by experts or by using historical data. Figure 1 gives a graphical representation of such a matrix. (This figure was adopted from [6].) Each node in the figure represents a state and each arrow a possible switch with its associated probability.

Each customer arriving in a state represents a certain value for the company. A reward vector is therefore used to assign a value to each state. This is done by including gains and costs. NC is used to present the net contribution of a state and M indicates the marketing expenses. Gains are only being made if a customer purchases anything and therefore only state 0 has a net contribution. Marketing expenses are made if a customer is thought of as active. Thus there are no marketing expenses for state 4. This results in the following reward vector

$$\mathbf{R} = \begin{bmatrix} NC - M \\ -M \\ -M \\ -M \\ 0 \end{bmatrix}.$$

Now by combining the Markov matrix and reward vector the CLV of a customer in state s with recency r can be valued T periods ahead. Therefore the transition probabilities have to be calculated for every future period t ($t = 1, \ldots, T$). These probabilities are found by multiplying the Markov matrix t times, which is a well known property of Markov chains [6]. Thus for every period t a transition matrix $\mathbf{P}^t$ is found. This matrix has to multiplied by the reward vector for every period, which results in the value derived from a customer in that period. By summing over all periods CLV is found:

$$\mathbf{CLV} = \sum_{t=0}^{T} [(1+d)^{-1}\mathbf{P}]^t \mathbf{R}. \qquad (4)$$

This equation shows how the vector **CLV** is calculated using a Markov chain. This vector contains the expected future value, T periods ahead, of a customer in state s ($s = 1, \ldots, S$) at time $t = 0$. Furthermore d is the discount rate of money, $\mathbf{P}$ is the Markov matrix containing

Customer/Period	Recency	Frequency	Monetary/discretized
1/1	0	1	10/1
1/2	1	1	10/1
1/3	2	1	10/1
1/4	3	1	10/1
1/5	4	1	10/1
1/6	5	1	10/1
1/$\geq$ 7	6	1	10/1
2/1	0	2	40/1
2/2	1	2	40/1
2/3	2	2	40/1
2/4	3	2	40/1
2/5	4	2	40/1
2/6	0	4	140/2
2/7	1	4	140/2
2/8	2	4	140/2
2/9	3	4	140/2
2/10	0	5	190/2
2/11	1	5	190/2
2/12	2	5	190/2

Table 1: Typical discretized data for the e-CLV model. Recency is recorded in months with a maximum of 6. Frequency represents the aggregated number of purchases, and has a maximum of 6, and Monetary value is the aggregated spending for that customer. Monetary is discretized by introducing categories of 100 dollars wide. (Monetary 1 represents spending between \$0 and \$100, 2 represents spending between \$100 and \$200, and so on.)

switching probabilities between states and $\mathbf{R}$ is the reward vector containing the monetary contribution of each state.

Now, by counting the number of customers in each state at $t = 0$ customer equity can be calculated. This is done by multiplying this number by the respective CLV, as shown below.

$$CE = \sum_{s=1}^{S} CLV_s C_s \qquad (5)$$

In above equation CLV_s is the CLV of a customer in state s and C_s is the number of customers in this state at time $t = 0$.

The paper by Pfeifer and Carraway is mostly intended to show the usefulness of Markov chains in CLV modeling, while no empirical use of their model is presented. This gap is filled by the e-CLV model proposed by Etzion et. al. [4], which is based on attributes specific to the e-commerce domain. In their application to online electronic auctions, Etzion et. al. use customer sessions, bids, and purchases to derive various attributes, such as recency and frequency of sessions, bids, and purchases.

After discretization, these attributes are used to form the states of the Markov matrix, i.e., every possible combination of discretized values for these attributes forms a separate state. An example of such discretized data is shown in Table 1. The Markov matrix $\mathbf{P}$ is then estimated by counting the state transitions customers go through in these data. A major drawback is that the number of states grows exponentially with the number of attributes: The example generates $7.5.2 = 70$ states.

The Markov model based on RFM variables seems a good solution to estimate CLV for Internet retailing businesses, simply because complete customer purchase records to extract RFM variables are always available since purchases have to be billed and/or shipped to customers. Secondly other information, for instance site visiting behavior may be available, which can be used for prediction. The study by [4] already uses such information.

Although conceptually elegant, the model put forward by [6] is susceptible to some criticism. A first point of critique is the so called 'dead state'. If customers are inactive for a certain number of periods they are considered lost for good. And, if they return to the company they are treated as new customers. [8] indicate that this approach systematically underestimates CLV. In [14] this problem is also put forward. With Markov chains this problem is easily solved by letting the possibility exist to return to other states from the 'dead state'.

Secondly, the exponential number of states leads to problems. The first problem is that the reward vector is difficult to estimate since many of the states may lack observations. (Therefore, the authors of [6] suggest it should be estimated by experts, but this may be infeasible or give unreliable estimates.) Moreover, it is important for business managers to keep the states actionable, since marketing strategies will be based on different CLV's. Designing 70 different strategies, as in the example dataset, is unfeasible.

Therefore a segment based approach as in [7] may be more practical. Their approach is inappropriate for our purposes however, since it is tailored to the telecommunications industry, where customers have long term contracts, lending itself to survival modeling. In our case, these long term contracts are lacking, and a Markov model approach seems more appropriate, but states should somehow be grouped into distinctive clusters in order to keep them actionable. The next section will introduce such a model.

3. DTMC MODEL

This section will introduce the Decision Tree Markov Chain (DTMC) model, able to estimate segment level CLV. The next section will illustrate the use of the DTMC model by presenting a real life business case. To estimate segment level CLV a CART decision tree [2] is used to form distinctive groups based on the input variables. The groups formed by the CART tree are subsequently used as states in the Markov chain model.

Data for the DTMC model has to consist of variables describing customers. These variables have to be recorded for a number of periods. The reward generated by a customer also needs to be recorded every period. This typical data layout is shown in Table 2. For every customer (1 to I) for every period (1 to P) descriptive variables $x1, \ldots, xk$ and reward/spending y are recorded.

Various descriptive variables can be used for the segmentation. for instance demographic information, the number of site visits this period, or RFM variables as in [6] and [4]. Traditional RFM variables are aggregated over the complete customer history. Since we want to find segments of customers with similar value *within one period* rather than over their entire lifespan for use in the Markov model, we use *per period* versions of the RFM variables in the experiments below. Recency now indicates how many periods ago a customer bought something relative to this period, frequency is the number of purchases during this period and monetary is the total amount spent during the current period.

The choice for per-period-RFM variables introduces a problem, since CART will place all customers who have been inactive for at least one period in the same state. This way

Customer/Period	$x1$	$x2$		xk	y
$1/1$	$x1_{11}$	$x2_{11}$	$\cdots$	xk_{11}	y_{11}
$1/2$	$x1_{12}$	$x2_{12}$	$\cdots$	xk_{12}	y_{12}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$2/P$	$x1_{1P}$	$x2_{1P}$	$\cdots$	xk_{1P}	y_{1P}
$2/1$	$x1_{21}$	$x2_{21}$	$\cdots$	xk_{21}	y_{21}
$2/2$	$x1_{22}$	$x2_{22}$	$\cdots$	xk_{22}	y_{22}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$2/P$	$x1_{2P}$	$x2_{2P}$	$\cdots$	xk_{2P}	y_{2P}
			$\vdots$		
$I/1$	$x1_{I1}$	$x2_{I1}$	$\cdots$	xk_{I1}	y_{I1}
$I/2$	$x1_{I2}$	$x2_{I2}$	$\cdots$	xk_{I2}	y_{I2}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
I/P	$x1_{IP}$	$x2_{IP}$	$\cdots$	xk_{IP}	y_{IP}

Table 2: Typical data for the DTMC model.

valuable customer discrimination information is lost. This problem is circumvented by deleting the records that contain no actual purchases. Only data containing purchases is then used to fit the CART tree. Such data is illustrated in Table 3.

Record	Customer/Period	Recency	Frequency	Monetary
1	1/1	0	1	10
2	2/1	0	2	40
3	2/6	5	2	100
4	2/10	4	1	50
5	3/6	0	3	75
6	4/3	0	4	125
7	4/11	8	4	150
8	4/14	3	5	250
9	4/16	2	2	75

Table 3: Typical RFM data for the DTMC model, with non-buy periods left out.

Using a CART tree, the monetary amount is estimated using only recency and frequency. This groups the data into certain value segments, through the leafs found at the bottom of the tree. Besides the states defined by the CART tree we introduced an 'end state', which customers enter if they have been inactive for 6 periods. These customers are considered lost, or semi-lost for the company. (We will discuss this shortly.) Figure 2 depicts the process of assigning states to records. States 1 through 4 represent the 'buy' states modeled by the CART tree, state 5 represents the end state. As an example: leaf 2 of the CART tree takes records 3,4 and 9, from Table 3 and thus has an average value of $75.

One might wonder why we did not use the actual value segments as states for the Markov model, i.e., create a segmentation based on the Monetary attribute and discard the Recency and Frequency attributes. However, there is a large random component in short term customer buying behavior which would lead to quite random transition behavior across states, causing an explosion of uncertainty with even a low number of iterations.

Expected value is a better reflection of the potential value of a customer than actual value, and will lead to more stable state switching behavior. Accordingly, our approach segments customers based on a limited set of non monetary attributes that are assumed to have a relation to future buying behavior. In this study we used the classical attributes recency and frequency, but in principle any combination of suitable attributes can be used.

	begin	**1**	**2**	**3**	**4**	**5**
begin	0	2	0	2	0	0
1	0	0	1	0	0	1
2	0	0	1	0	0	1
3	0	0	0	0	0	2
4	0	0	1	0	1	0
5	0	0	0	0	1	0

Figure 3: The number of state transitions counted from Table 3. The row represents the old state, the column the new state.

After the CART step, the states of the Markov matrix are known. The states for the example are summarized in Table 4. The average contribution of each state is also shown in this table. Together these contributions form the reward vector needed to calculate CLV. Table 4 also introduces a begin state, which captures *potential* customers. Transition probabilities from this state to the other states indicate how likely it is that a new customer will enter in a certain state.

Frequency	Recency	State	Contribution
-	-	**begin**	0
≤ 2	0	**1**	25
≤ 2	≥ 1	**2**	75
> 2	0	**3**	100
> 2	≥ 1	**4**	200
0	6	**5**	0

Table 4: State definitions for the Markov matrix.

There are two interpretations we can give to state 5, the end state. Customers arriving in state 5 can be considered lost for good, which makes returning to one of the other states impossible. This approach is followed by [6] and [4]. But, customers can also be considered to be semi-lost. This way it is possible to return to the other states. Note the introduction of this semi-lost concept is also a contribution of this paper

Both scenarios will be tested with our empirical application presented in the remainder.

The state sequence of every customer is used to calculate the switching probabilities between states. This is done by a similar procedure as in [4]. First the Markov matrix is initialized to zero. Then the state transitions are counted. The results of this counting procedure applied to the data of Table 3 is shown in Figure 3. (The transitions to state 5 are not shown in Table 3 but are inferred from the original data.) We then transform this matrix into a Markov matrix by normalizing each row (except the row representing the end state) to 1. If we want to use the 'lost-for-good' scenario we first set all counts from the 'semi-lost-state' (state 5) to other than the end states to 0, thereby ignoring such transitions had they occurred. (In the example such a transition occurs from state 5 to state 4.) Depending which procedure we apply we end up with one of the matrices in Figure 4.

The last modeling step is the estimation of average CLV per state T periods ahead. In principle, this is done using Equation 4, but since we discarded observations where no purchase was made we must make a few adjustments. First, we compute the average between-purchase-period by dividing the length of the total lifespan for each customer by the number of occasions in which she made a purchase and de-

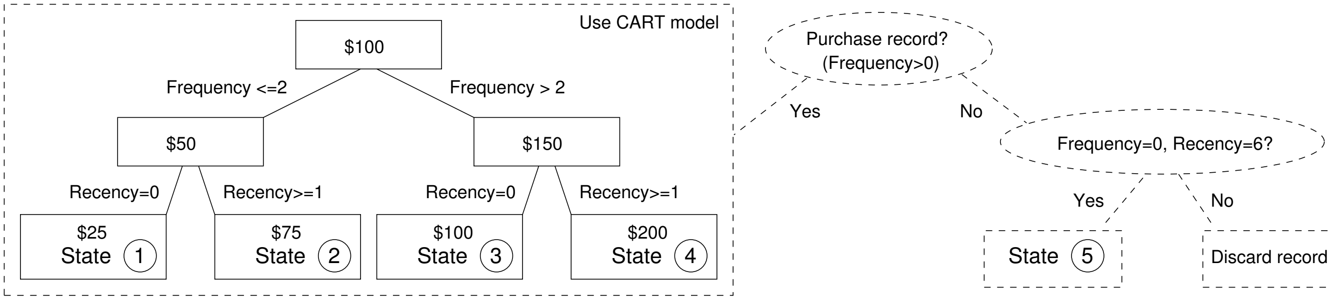

Figure 2: Assigning states to records. The encircled numbers represent the states.

	begin	**1**	**2**	**3**	**4**	**5**
begin	0	0.5	0	0.5	0	0
1	0	0	0.5	0	0	0.5
2	0	0	0.5	0	0	0.5
3	0	0	0	0	0	1
4	0	0	0.5	0	0.5	0
5	0	0	0	0 0.25	0.75	

	begin	**1**	**2**	**3**	**4**	**5**
begin	0	0.5	0	0.5	0	0
1	0	0	0.5	0	0	0.5
2	0	0	0.5	0	0	0.5
3	0	0	0	0	0	1
4	0	0	0.5	0	0.5	0
5	0	0	0	0	0	1

Figure 4: Markov matrices for semi-lost scenario (top) and lost-for-good scenario (bottom).

note the result by N. (Thus, $N = 2.7$ means that on average there is a 2.7 month period between purchases.) Next, we redefine the total number of periods T to reflect the longer period length $T_{new} = \lceil T/N \rceil$, and we accordingly adjust the discount rate $d_{new} = (1 + d)^{T/N} - 1$.

This results in the following CLV equation:

$$\mathbf{CLV} = \sum_{t=1}^{T_{new}} ((1 + d_{new})^{-1}\mathbf{M})^t \mathbf{R}, \tag{6}$$

where $\mathbf{CLV}$ is the $S \times 1$ vector containing the average CLV for every state s ($s = 1, \dots, S$), $\mathbf{M}$ is the Markov transition matrix containing the switching probabilities, d_{new} is the discounting factor and $\mathbf{R}$ is the reward vector containing the estimated expenditure of a customer when he arrives in a certain state. In practice the final term of this sum may not represent a full N-month period and should be weighted accordingly.

Table 5 shows the resulting CLV's for the lost for good and semi-lost scenarios. For each state, the number of customers and the average CLV is recorded. Thus customers in state 4 at $t = 0$ have a CLV of 260 for the lost for good scenario and 275 for the semi-lost scenario. The obsoleteness of absorption state 5 for the lost for good scenario also becomes clear from this table because no value is derived from customers who arrived in this state for this scenario.

The customer equity (CE), as described earlier, can now be calculated. For current customers (states 1 though 5) the CLV's have to be multiplied by the number of customers in each state at $t = 0$ of the testing period. In our example data 1 customer is found in state 2 and 3 customers in state 5 at $t = 0$. Mathematically the CE calculation for current

State	**#Customers at $t = 0$**	**CLV Lost for good**	**CLV Semi-lost**
begin	0	85	123
1	0	61	114
2	1	61	114
3	0	0	79
4	0	260	275
5	3	0	100

Table 5: Number of customers and average CLV's per state for the lost for good scenario and the semi-lost scenario.

customers becomes

$$CE_{current} = \sum_{s=1}^{S} CLV_s \times C_s, \tag{7}$$

where S denotes the number of states, CLV_s is the estimated average CLV for customers in state s and C_s is the number of customers currently in state s. By summing over all states customer equity for current customers is calculated.

Next, we estimate the number of new customers per period C_{new} based on historical data, and compute the CE value for these new customers:

$$CE_{new} = CLV_{\mathbf{begin}} \times C_{new}, \tag{8}$$

where $CLV_{\mathbf{begin}}$ denotes the CLV of customers in the begin state. Combining both equities results in total customer equity

$$CE = CE_{current} + CE_{new}. \tag{9}$$

After explaining the model, we describe its application to a real-world data set.

4. EXPERIMENTAL SETUP

Customer purchase information was made available to us by a large Dutch Internet retailer. This data is used to test and validate our model. This section will describe the data, data transformations and conducted experiments as well as model validation. Two benchmark models are introduced for validation: a naive model and the e-CLV model.

4.1 Data

The Internet retailer (whose name we are not allowed to disclose) sells all kinds of products on his site. We only have customer purchase information for one category: ink cartridges for ink-jet printers. Customers normally buy something in this category once in about every three months. Retaining customers thus is very important for this particular product category.

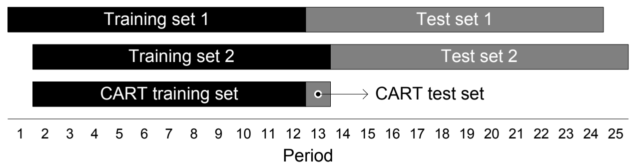

Figure 5: Graphical representation of training and test sets.

Customer purchase information from May 2004 up to and including May 2006 was available. Recency, frequency, and monetary (RFM) variables for the DTMC and e-CLV model were calculated for every month. This resulted in 25 periods with RFM variables for every customer.

As was explained earlier, these RFM variables are defined differently for the DTMC model and the e-CLV model. e-CLV uses aggregated variables, whereas DTMC uses per-period variables. Moreover, the DTMC models only uses records with actual purchases and records in which customers enter the end state. Customers enter this state after 6 periods of inactivity.

4.2 Model building

Model training consists of two separate steps. First the decision tree has to be trained. Data from periods 2 through 12 is used to train this tree and period 13 is used to test it. The decision tree is then used to define the states for the Markov matrix.

The second step calculates the reward vector and the Markov matrix, enabling the calculation of CLV. To compare CLV predictions across time two training and test sets are used for this second step. The states for the Markov matrix are kept identical for both sets in order to enable comparison of CLV's per state across time. The reward vector is calculated from the training data and state definitions. A graphical representation of the used training and test sets is given in Figure 5. Periods 1 to 12 will be training set 1, periods 2 to 13 training set 2, periods 13 to 24 test set 1, and periods 14 to 25 test set 2. Furthermore, both the 'lost-for-good' en the 'semi-lost' scenarios, which were introduced earlier, are modeled, resulting in slightly different Markov matrices. All our modeling is done using the free statistical software package R, available from `http://cran.r-project.org/`.

Before training the decision tree, outliers were deleted from the dataset. We chose to delete all rows that had a monetary value bigger or smaller than three standard deviations from the mean in the training period. This resulted in removing 176 records with values larger than 435 on a total of 24482 records.

Next two CART parameters were chosen appropriately, the complexity parameter and the minimum number of records per leaf. The CART tree should have a high accuracy, with the restriction of keeping the tree size down. We chose 250 records as a minimum per leaf, being approximately 1% of the data. Subsequently, the complexity parameter was varied and 0.0001, the value giving tree with the highest accuracy, was chosen.

Now that the decision tree is found the reward vector for training set 1 and 2, as described above and displayed in Figure 5, has to be calculated. Each leaf in the decision tree can be seen as a state. For each state the associated monetary value is calculated by averaging over all records in this state except outliers. Thus for training set 1 all records from period 1 to 12 are used and for training set 2 all records from period 2 to 13 are used. The next step is to calculate Markov matrices for both data sets.

For both training sets the Markov matrices are calculated. First the state sequence of every customer is determined. The filtered records are also included again, since no outlier filtering is necessary when calculating the Markov matrices. The state sequence then is determined by assigning a state number to every period for every customer using the state definitions as described above. The Markov matrices for both data sets are then determined by counting the number of transitions, as described above.

Using the constructed DTMC models average CLV's of current and prospective customers can be predicted. By using the number of current customers in every state, customer equity for this group can be calculated. When the number of new customers is estimated customer equity can also be calculated for this group. To calculate these values we use a discount factor of 1% per month.

4.3 Evaluation metric

The results of our experiments will be evaluated by comparing actual net present values with our predicted CLV's.

Net present value. Predicted CLV is compared to the realized value by a net present value (NV) calculation over the test period. NV can be seen as the 'target value' for CLV – it represents the discounted *actual* aggregated spending by a customer:

$$NV_i = \sum_{t=1}^{T} \frac{A_{it}}{(1+d)^t}, \tag{10}$$

where NV_i is the net present value of customer i ($i = 1, \ldots, I$), A_{it} is the amount spend by customer i in period t and d is the discount factor.

Now that NV of every customer is known NV per state and total NV can be calculated. The relative difference between CLV and NV values will be used to measure the accuracy of the tested models. The difference is thus calculated as

$$E = |1 - CLV/NV|, \tag{11}$$

and should be as small as possible.

4.4 Benchmark models

Our model performance will be tested against two benchmark models, a naive model and the e-CLV model.

Naive model. Our naive model assumes customer behavior remains unchanged in every period. Thus, the T-period ahead customer equity is estimated simply by summing the current sales T times and discounting each term appropriately

$$CE = \sum_{t=1}^{T} \frac{Sales_0}{(1+d)^t}, \tag{12}$$

where $Sales_0$ are the sales in the baseline period.

e-CLV model. As explained earlier, the e-CLV model uses aggregated RFM variables. These variables have to be discretized in order to be applicable. Recency is discretized into 7 categories. Frequency was given an imposed maximum bound of 6, with 6 indicating 6 or higher. And the monetary amount was discretized into 6 categories, category

State	Recency	Frequency	Monetary 1	Monetary 2
begin	-	-	0	0
1	≥ 1	$= 2$	85.9	85.8
2	$= 0$	$= 2$	104.5	103.2
3	-	≥ 3	142.6	141.8
4	$= 0$	$= 1$	59.2	58.1
5	$= 1$	$= 1$	47.5	47.5
6	≥ 2	$= 1$	43.4	43.4
7	$= 0$	$= 6$	0	0

Table 6: Recency, frequency and average monetary values per state for training sets 1 and 2, found using the CART decision tree.

1 running from 0 to 100, 2 from 100 to 200, 3 from 200 to 300, 4 from 300 to 400, 5 from 400 to 500, and 6 500 and above.

With these RFM variables the e-CLV model is trained. The discretized variables give rise to $7 \cdot 6 \cdot 6 = 252$ states, plus a begin state. The state sequence in the training data sets are counted to calculate the Markov matrix. Using the counts, Markov matrices for the 'lost-for-good' and 'semi-lost' scenarios are calculated as in the DTMC model. ([4] only uses the 'lost-for-good' scenario.)

Average monetary values for the reward vector are calculated the same way as with the DTMC model. The added monetary amount is averaged over all customers in every state. Thus only states with a recency of 0 may have a value higher than 0 associated with it, since these are the only states in which actual purchases took place. For states without observations we estimated the reward to be the average reward over all observations with recency 0. Note that Pfeifer and Carraway tackle this problem by using expert knowledge to construct the reward vector [6]. The model in [4] is not clear about the net contributions of the reward vector, but this is probably also the average of the training set due to the large state space.

5. RESULTS

This section presents the results of the experiments described in the previous section.

Decision tree step. The resulting decision tree is shown in Figure 6. Six value segments were identified by the tree. These groups are numbered 1 through 6. Group 7 represents the end state. All state definitions as well as the average monetary values for both training sets can be found in Table 6. The monetary values show a logical pattern, states with higher frequencies have higher values. Some variation is captured through recency as well, however. States 1 and 2 differ only in recency for instance. State 7, which functions as the end state, logically has a value of 0 associated with it.

Markov chain step. After generating the sequence data for both training sets and counting the transitions the uncorrected Markov matrices were found. After correction the Markov matrix for the semi-lost scenario for both training set 1 and 2 are shown in Figure 7. For the lost for good scenario the probability of going to state 7 from state 7 is 1 instead of 0.95 for both matrices.

When looking at the Markov matrices the large transition probabilities to state 7 are especially interesting. These values indicate that customers are not very loyal to this company. Furthermore state 6 seems a popular state, because switching probabilities to this state from the others

	b	**1**	**2**	**3**	**4**	**5**	**6**	**7**
begin	0	0	0.16	0.04	0.80	0	0	0
1	0	0.15	0	0.02	0	0.09	0.24	0.50
2	0	0.05	0	0.01	0	0.04	0.11	0.79
3	0	0.08	0	0.02	0	0.05	0.10	0.76
4	0	0.03	0	0.01	0	0.04	0.10	0.82
5	0	0.07	0	0.02	0	0.09	0.23	0.59
6	0	0.11	0	0.01	0	0.14	0.37	0.38
7	0	0.01	0	0.00	0	0	0.04	0.95

	b	**1**	**2**	**3**	**4**	**5**	**6**	**7**
begin	0	0	0.15	0.04	0.81	0	0	0
1	0	0.13	0	0.02	0	0.08	0.26	0.51
2	0	0.05	0	0.01	0	0.04	0.11	0.80
3	0	0.08	0	0.01	0	0.04	0.10	0.76
4	0	0.03	0	0	0	0.04	0.10	0.83
5	0	0.06	0	0.01	0	0.09	0.24	0.60
6	0	0.09	0	0.01	0	0.12	0.34	0.44
7	0	0.01	0	0.00	0	0	0.04	0.95

Figure 7: Transitions probabilities for the semi-lost scenario for training set 1 (top) and 2 (bottom).

state	**count**	**avg NV**	**avg CLV (semi)**	**avg CLV (lost)**
begin	0	43	72.2	72.2
1	450	34.4	37.9	37.1
2	1135	10.6	16.1	14.8
3	293	18.7	20.2	19.0
4	7579	9.1	13.7	12.4
5	294	30.9	29.4	28.5
6	1589	30.1	42.9	42.3
7	11226	5.0	3.4	0
total	22566	215132	260369	208799

Table 7: Results for test set 1. For each state, the number of customers, average NV, average CLV for the semi-lost scenario and average CLV for the lost for good scenario are shown. Aggregate results are shown in the last row.

are relatively high. For instance 0.24 from state 1 to state 6. Differences between both matrices are rather small and their effect on CLV's is therefore hard to predict. Take for example the switching probabilities from the begin state to the others. The probability of going to state 2 decreases from 0.16 to 0.15 and going to state 4 it increases from 0.80 to 0.81. Overall this analysis shows that customer behavior is complex, as migration is not limited to segments with similar monetary value.

CLV results. Combining the results of the decision tree step and the Markov chain step resulted in the CLV's shown in Tables 7 and 8. A quick look learns that the NV values are more or less matched by the CLV's. See for instance state 1 in Table 7 where average NV is 34.4, average CLV is 37.9 for the semi-lost scenario and average CLV is 37.1 for the lost for good scenario.

Another interesting thing to note is the number of customers per state. State 7 is the biggest group for both test sets with respectively 11226 and 12912 customers. The large increase in this number indicates that a lot of customers are becoming inactive. Especially because the other states do not have an increasing number of customers except state 6, this indicates a retention problem for the company. Total CLV's are shown in the bottom row of Tables 7 and 8. The quality of these estimates will be further discussed below, where our model is compared to the benchmark results.

Net present value comparison. The DTMC models is compared to the Naive and e-CLV models using the evalua-

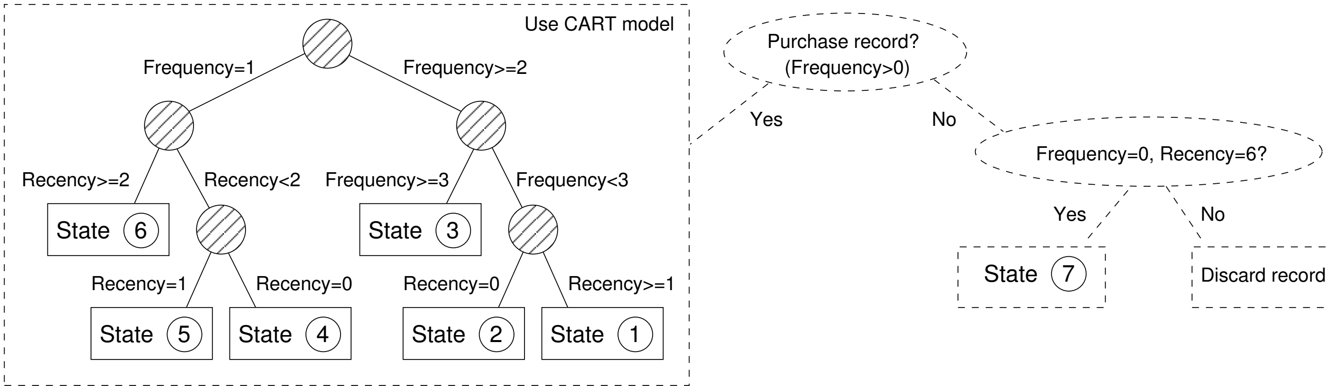

Figure 6: Decision tree resulting from the training data of period 2 to 12. The encircled numbers indicate the states for the Markov step.

state	count	avg NV	avg CLV (semi)	avg CLV (lost)
begin	0	43.7	69.6	69.6
1	394	31.5	34.9	34.1
2	906	10.6	15.2	14.0
3	217	18.3	19.2	18.1
4	7127	9.2	12.6	11.3
5	286	29.2	27.4	26.5
6	1725	30.7	36.7	36.0
7	12912	5.0	3.4	0
total	23567	217158	236204	180584

Table 8: Results for test set 2.

	set 1	E	set 2	E	avg E
NV	215132	0.0%	217158	0.0%	0.0%
DTMC (semi)	260369	21.0%	236204	8.8%	14.9%
DTMC (lost)	208799	2.9%	180584	16.8%	9.9%
Naive	384935	78.9%	207893	4.3%	41.6%
e-CLV (semi)	231091	7.4%	211213	2.7%	5.1%
e-CLV (lost)	127398	40.8%	105303	51.5%	46.1%

Table 9: NV, CLV and relative errors (see Eqn. (11)) for all tested models for both test sets.

tion metric described in the previous section. Table 9 shows the results on individual test sets and the average error over these sets. On the first test set the DTMC model with the lost for good scenario performs best with an error of 2.9%, the semi-lost scenario e-CLV model is best on the second test set with 2.7% error. On average this model also performs best, with an error of 5.1%, followed by the DTMC lost for good model with 9.9% error and the DTMC semi-lost model with 14.9%. The Naive model and e-CLV lost for good model perform substantially worse with respective average errors of 41.6% and 46.1%.

Discussion of the results. From our analysis a mixed conclusion can be drawn. First the Naive model seems to make a more or less random guess. For the first test set CLV is highly overestimated by this method and the estimate for the second test set is close by. Another shortcoming is the lack of discrimination between customers, because the other models have such a possibility. Overall the Naive model gives a quick, but dirty, estimate of total CLV.

The other models do a better job. The semi-lost scenario e-CLV model performs best overall in terms of accuracy. The e-CLV lost for good model performs way worse, with a very high average error. The difference between both DTMC model scenarios is much smaller. The DTMC lost for good model has the lowest average error and the DTMC semi-lost scenario shows the best trend prediction. Both models perform reasonably well compared to the semi-lost e-CLV model.

The main advantage of the DTMC models as opposed to the e-CLV models is the much smaller number of groups. Interpreting the customer groups is easier for the DTMC model which aids practical actionability. Take for instance the conditions of the variables given in Table 6 and the average CLV's of Tables 7 and 8. Given these values it can be concluded that customers who less recently bought something are more valuable, because the recency of states 1, 5, and 6 are at least 1 and their CLV's are all higher than states 2,3, and 4 which can have recency 0. Put differently, customers who bought something in at least two period are worth more, because customers can not return to states 2 or 4, due to the 0 recency. It is much more difficult to draw such a conclusion for the e-CLV model.

6. SUMMARY, CONCLUSIONS AND DISCUSSION

This paper discussed a customer lifetime value (CLV) model that can be used in e-commerce environments to monitor web site success, provide marketing diagnostics and generate insight. Moreover, a CLV model makes general marketing efforts more accountable – efforts that increase the customer equity (total CLV) are justifiable, others are not.

Because Internet retailing generates a lot of customer data through the extensive use of technology, it should be possible to measure CLV by historical data. In the literature recency, frequency and monetary (RFM) variables are combined with Markov models to calculate CLV [6]. The inclusion of more RFM variables specific to e-commerce into these Markov CLV models was studied by [4], which proved to perform well.

A shortcoming of these studies is the exponential increase of states with the number of variables. The interpretation of the states becomes quite cumbersome this way. A segmentation of the customer records of every period could prevent this. For this reason we developed the DTMC model, based on decision trees and Markov chains.

Using only a few variables concerning customer behavior reasonable CLV estimates can be made. This is true for both the e-CLV model and the DTMC model. These estimates can be used to monitor the overall business effectiveness over time.

One of the shortcomings of the DTMC model is the, rather inelegant, deletion of records. Records without a reward are deleted from the data. This gives some complications with regards to the formulation of the CLV equation. Using different input variables may prevent this problem.

A general point of discussion about our research is the number of variables used. Only RFM variables are used to test a model on applicability in an e-commerce domain. Use of more variables specific to e-commerce and data mining could improve the model. Different CLV drivers may exist in different industries, and this can easily be incorporated in our model.

Using decision trees for grouping the RFM variables was a deliberate choice. Their interpretability greatly aids in explaining the results to decision makers. Especially when incorporating more data this may be very useful for the manager's understanding. When for instance demographic data are also included, it could be the case that there are certain segments of only males, who are very profitable, which would be very useful information for managers. Moreover, the decision tree approach leads to a low number of segments, which can be individually targeted by marketing actions. We encountered business users who appreciate this actionability.

Another future research direction is the formulation of other segment level CLV models. An example could be to replace the decision tree by using a different clustering procedure. (Although general clustering procedures lack the interpretability of CART.) Furthermore the clustering step and Markov chain step could be done simultaneously to get an optimal partitioning according to CLV's. Finally, it could be worthwhile to adapt the CART algorithm such that it finds customer segments that give the highest possible accuracy when combined with a Markov model for predicting CLV. We are currently exploring this option.

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