There's just one thing I got to know
Can you tell me please, who won?

– Crosby, Stills and Nash

Lessons in Play

Outcome Classes

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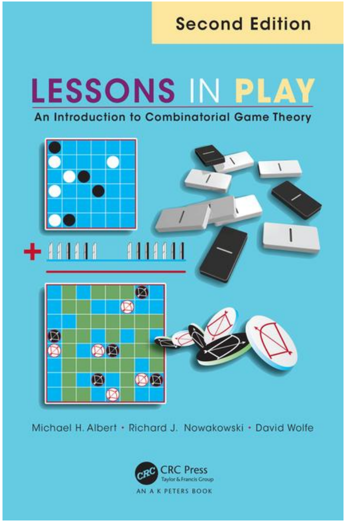


Fundamental theorem of combinatorial games

In a game between Albert and Bertha, with Albert moving first, either Albert can force a win moving first, or Bertha can force a win moving second (not both).

The four outcome classes

Class	Name	Definition
$\mathcal{N}$	Fuzzy	Next player can force a win (First player win)
$\mathcal{P}$	Zero	Previous player can force a win (Second player win)
$\mathcal{L}$	Positive	Left can force a win (regardless of starting player)
$\mathcal{R}$	Negative	Right can force a win (regardless of starting player)

Outcome classes		*When right moves first*	
		Right wins	Left wins
When left moves first	Left wins	$\mathcal{N}$	$\mathcal{L}$
	Right wins	$\mathcal{R}$	$\mathcal{P}$

Outcome Functions (of position G)

$$O_L(G) = \begin{cases} \text{😎}, & \text{if Left can force a win moving first} \\ \text{😵}, & \text{if Left cannot force a win moving first} \end{cases}$$

$$O_R(G) = \begin{cases} \text{😎}, & \text{if Right can force a win moving first} \\ \text{😵}, & \text{if Right cannot force a win moving first} \end{cases}$$

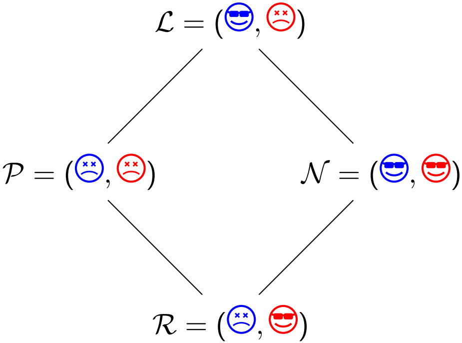
$$O(G) = (O_L(G), O_R(G))$$

Relation to Outcome Classes

Class	Outcome Function
$\mathcal{N}$	$O(G) = ($, $)$
$\mathcal{P}$	$O(G) = ($, $)$
$\mathcal{L}$	$O(G) = ($, $)$
$\mathcal{R}$	$O(G) = ($, $)$

Outcome classes	$O_R(G) =$ 	$O_R(G) =$ 
$O_L(G) =$ 	$\mathcal{N}$	$\mathcal{L}$
$O_L(G) =$ 	$\mathcal{R}$	$\mathcal{P}$

Partial order



Positions and Options

From a game position:
The moves available to Left (Left's options): $\mathcal{G}^L$.
The moves available to Right (Right's options): $\mathcal{G}^R$.

Game position G, consists of its options $\mathcal{G}^L$ and $\mathcal{G}^R$:
$G = \{ \mathcal{G}^L \mid \mathcal{G}^R \}$

Outcome Class from Options

Possible moves	Some $G^R \in \mathcal{R} \cup \mathcal{P}$	All $G^R \in \mathcal{L} \cup \mathcal{N}$
Some $G^L \in \mathcal{L} \cup \mathcal{P}$	$\mathcal{N}$	$\mathcal{L}$
All $G^L \in \mathcal{R} \cup \mathcal{N}$	$\mathcal{R}$	$\mathcal{P}$