

Chapter 6: Structure

Games born by Day 2

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What to answer? (Chapter 6)

Classifying games: informative/interesting manner

Prove theorems that say something about games born on, or before a given day.

Birthday

- Definition 4.1 (page 76):

The birthday of a game $G = \{g^L \mid g^R\}$ is defined recursively as 1 plus the maximum birthday of any game in: $\mathcal{G}^L \cup \mathcal{G}^R$

For the base case, if $\mathcal{G}^L = \mathcal{G}^R = \emptyset$, then the birthday of G is 0.

Birthday

- Definition (of 6.1):

$\mathcal{G}_n =$ the set of games born by day n .

$g_n = |\mathcal{G}_n| =$ the number of games born by day n .

$\mathcal{G}_0 = \{0\}$ $g_1 = 1$ Left and Right set?

Birthday

$$\mathcal{G}_1 = \{1, \star, 0, -1\}$$

$$1 = \{0 \mid \}$$

$$\star = \{0 \mid 0\}$$

$$0 = \{ \mid \}$$

$$-1 = \{ \mid 0\}$$

\mathcal{G}_n games are subsets of \mathcal{G}_{n-1}

Observation 6.1

$$g_n \leq 2^{g_{n-1}} * 2^{g_{n-1}} = 2^{2g_{n-1}}$$

Loose upper limit for number of games born by day n.

$$n=2: g_n \leq 2^8 = 256$$

Comparable options means Dominating options

Antichain: set of incomparable elements

All antichain games available on day 1

$\{1\}, \{0, *\}, \{0\}, \{*\}, \{-1\}, \{\}$

$$g_2 = 6 * 6 = 36$$

Antichains are arranged here:

		Right					
		-1	0, *	0	*	1	\emptyset
Left	1	\mathcal{N}			\mathcal{L}		
	0, *						
	0	\mathcal{R}			\mathcal{P}		
	*						
	-1						
	\emptyset						

All antichain games available on day 1

$$\{1|-1\} = \pm 1, \quad \{*\|*\} = 0, \quad \{0\|*\} = \uparrow$$

22 unique outcomes, 22 games born day 2

		Right					
		-1	0,*	0	*	1	\emptyset
Left	1	± 1	1 0,*	1 0	1 *	1*	2
	0,*	0,* -1	*2	$\uparrow*$	\uparrow	$\frac{1}{2}$	1
	0	0 -1	$\downarrow*$	*			
	*	* -1	\downarrow		0		
	-1	-1*	$-\frac{1}{2}$				
	\emptyset	-2	-1				

Exercise 6.2:

Values of (non-canonical form) games:

$$\{0, * \mid *\} = \uparrow$$

$$\{0, * \mid 1\} = \frac{1}{2}$$

$$\{0, * \mid \ } = 1$$

Exercise 6.2:

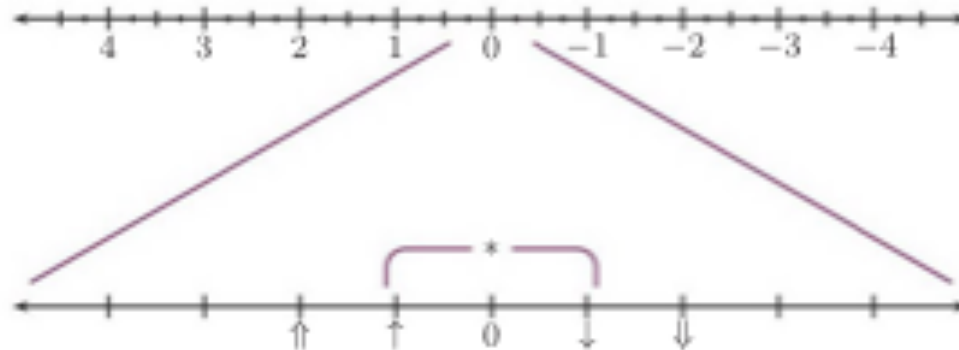
Value of:

0 is incomparable to \star , but the left option to \star , reverses out through 0

$$\{0, \star \mid \star\} = \{0 \mid \star\}$$

- **Definition 5.41: (page 119)**

$$\{0 \mid \star\} = \uparrow$$



Exercise 6.2:

Value of:

Same dominating option here:

$$\{0, \star \mid 1\} = \{0 \mid 1\} = \frac{1}{2}$$

Exercise 6.2:

Value of:

Same dominating option here:

page 106:

$$\{0, \star \mid \} = \{0 \mid \} = 1$$

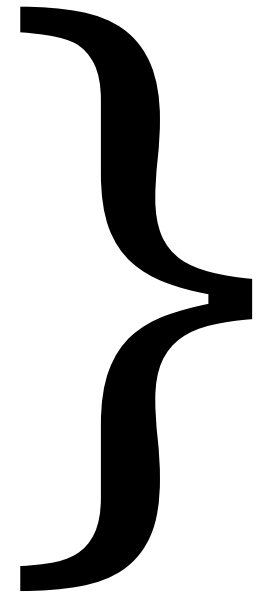
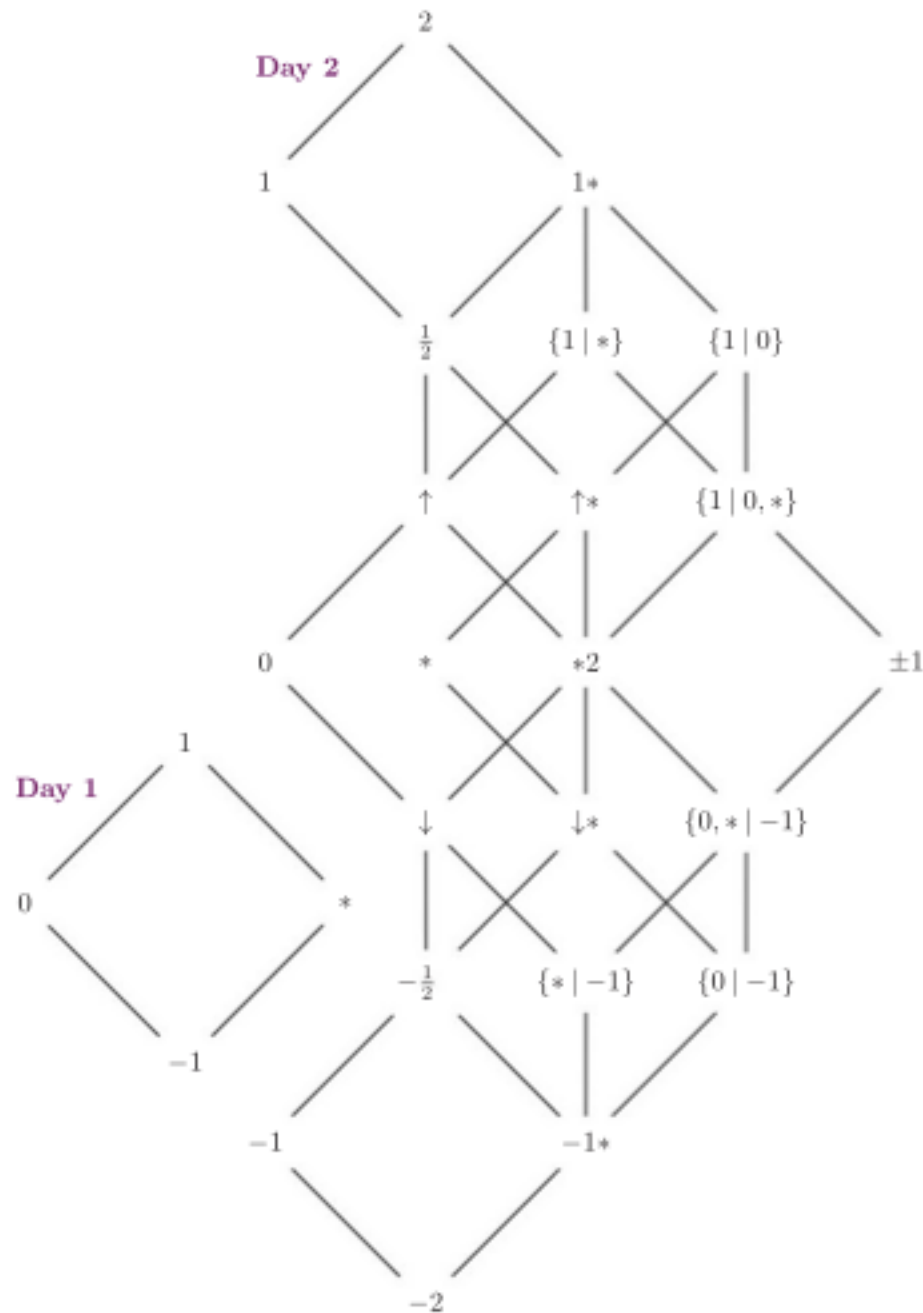
Hasse Diagram

Investigate the partial orders of games born by day 1 and day 2

Two games $G > H$ are connected if no $G > J > H$ exists

Greater game is above the lesser one

From Left's perspective:



Right

	-1	0,*	0	*	1	∅
1	±1	1 0,*	1 0	1 *	1*	2
0,*	0,* -1	*2	↑*	↑	1/2	1
0	0 -1	↓*	*			
*	* -1	↓	0			
-1	-1*	-1/2				
∅	-2	-1				

Left

Konane positions: Exercise 1 page 151

REMOVE PHASE - Black removes one black stone from the board center (I suppose in even boards, that means the 2x2 square center) or from one of the edges. White then removes a white stone adjacent to the removed black stone. (irrelevant, as we only have 4 stones left)

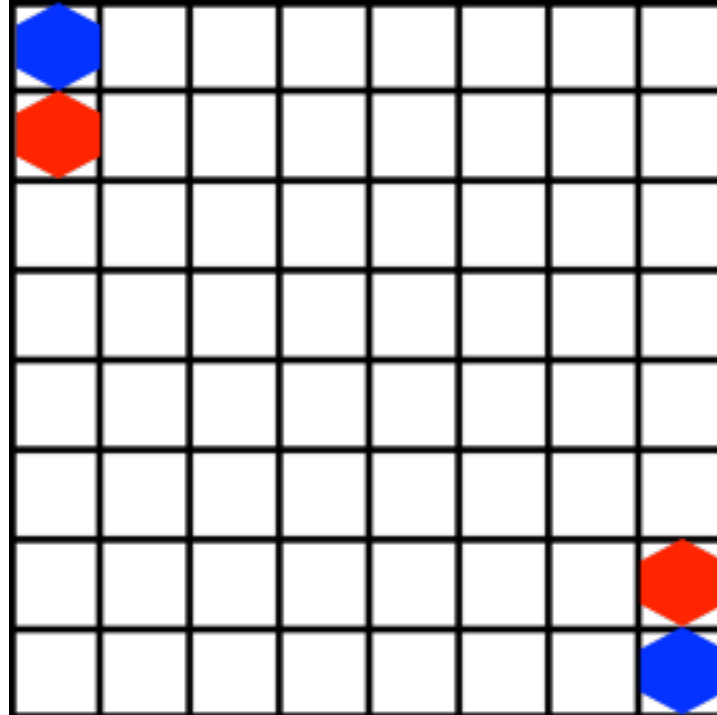
JUMP PHASE - Each player, in turns, pick a friendly stone and jump an enemy stone (on the same rank or column), capturing it.

Left = Blue

Right = Right

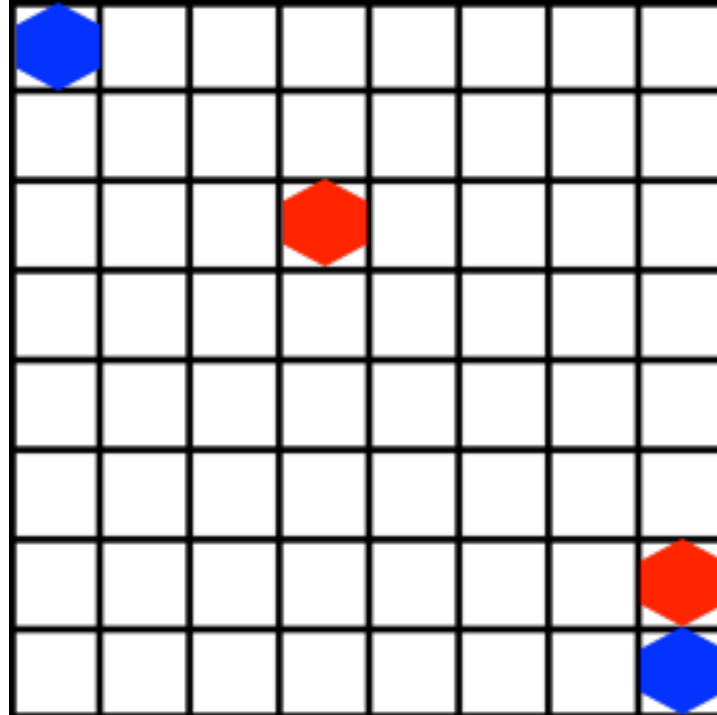
Exercise 1 page 151:

$$2 = \{1 \mid \}$$



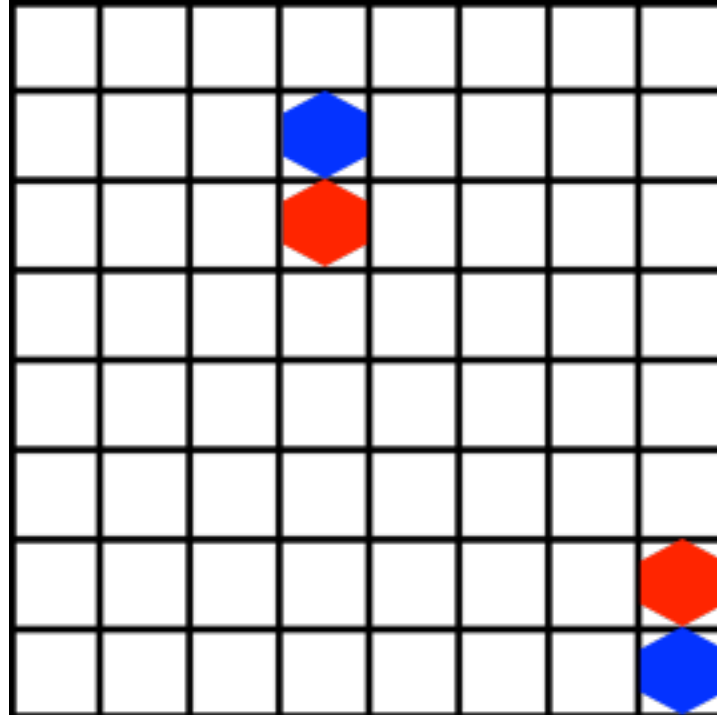
Exercise 1 page 151:

$$1 = \{0 \mid \}$$



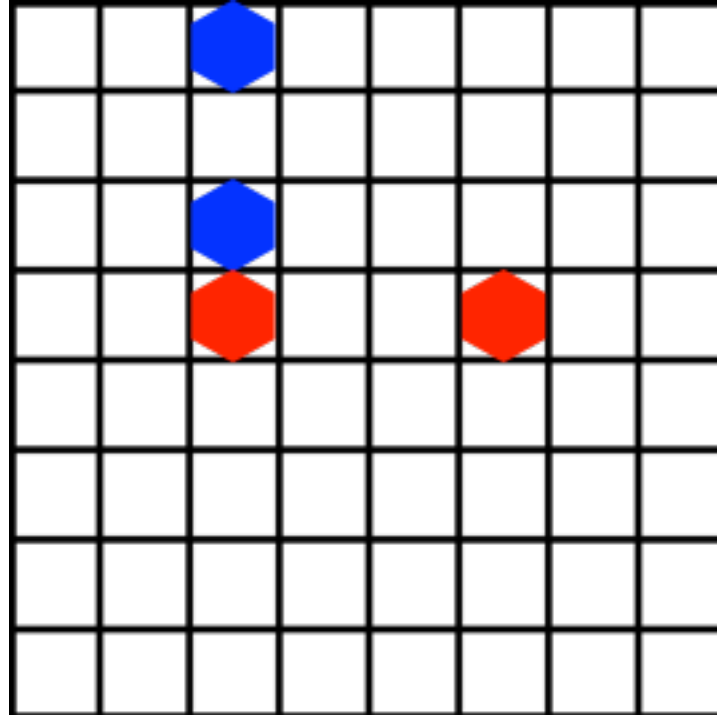
Exercise 1 page 151:

$$1\star = \{1 \mid 1\}$$



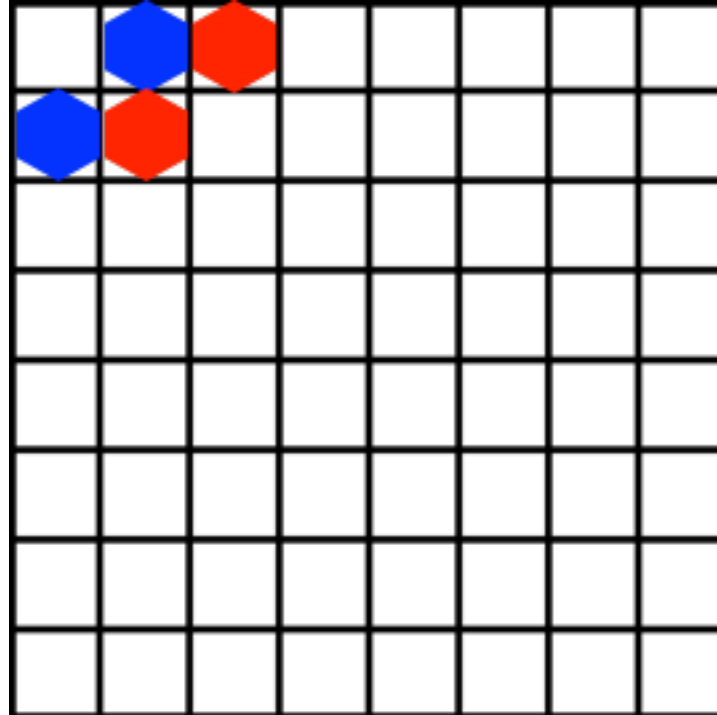
Exercise 1 page 151:

$$\frac{1}{2} = \{0 \mid 1\}$$



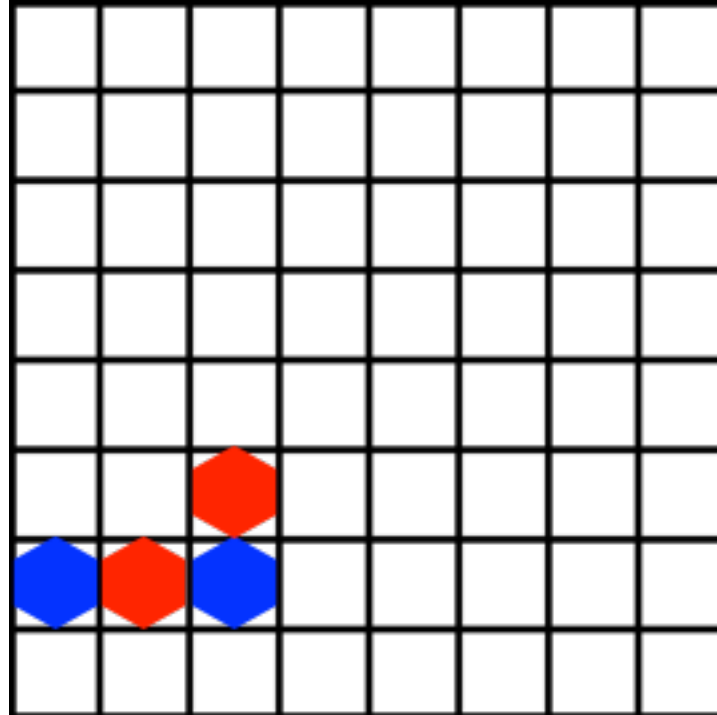
Exercise 1 page 151:

$\{1 \mid \star\}$



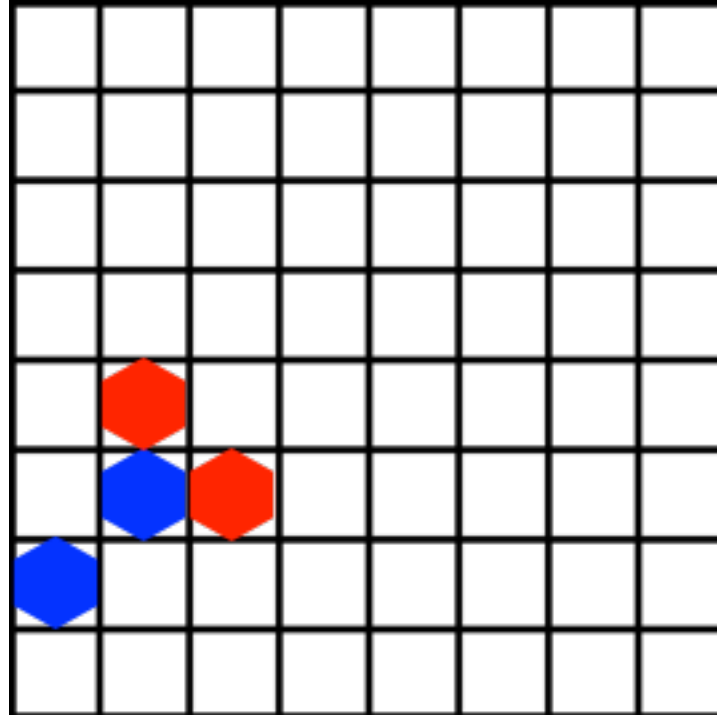
Exercise 1 page 151:

$\{1 \mid 0\}$



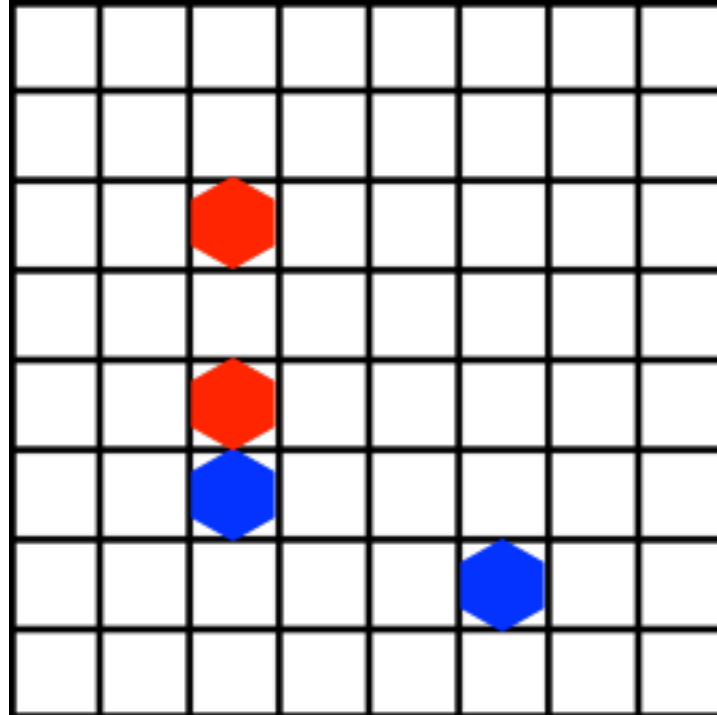
Exercise 1 page 151:

$$\uparrow = \{0 \mid \star\}$$



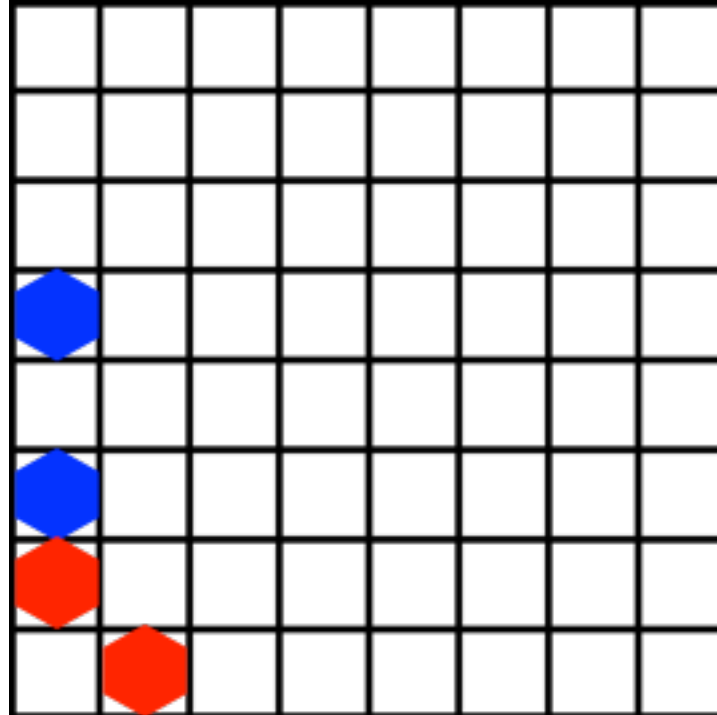
Exercise 1 page 151:

$$\uparrow * = \{0, * \mid 0\}$$



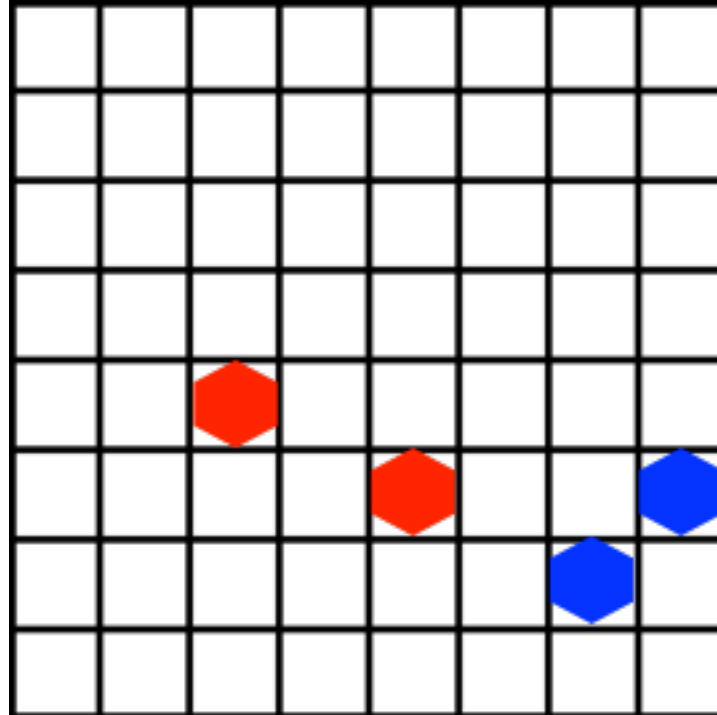
Exercise 1 page 151:

$\{1 \mid 0, \star\}$



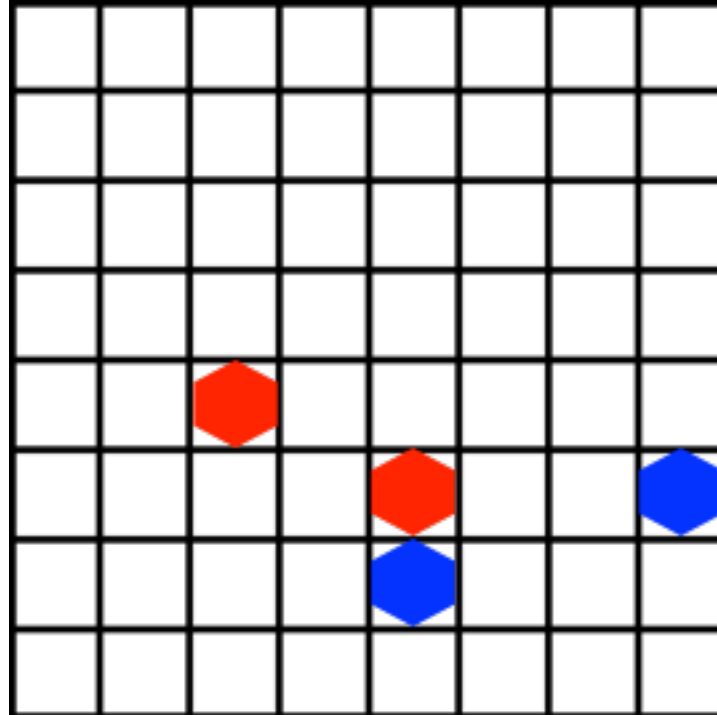
Exercise 1 page 151:

0



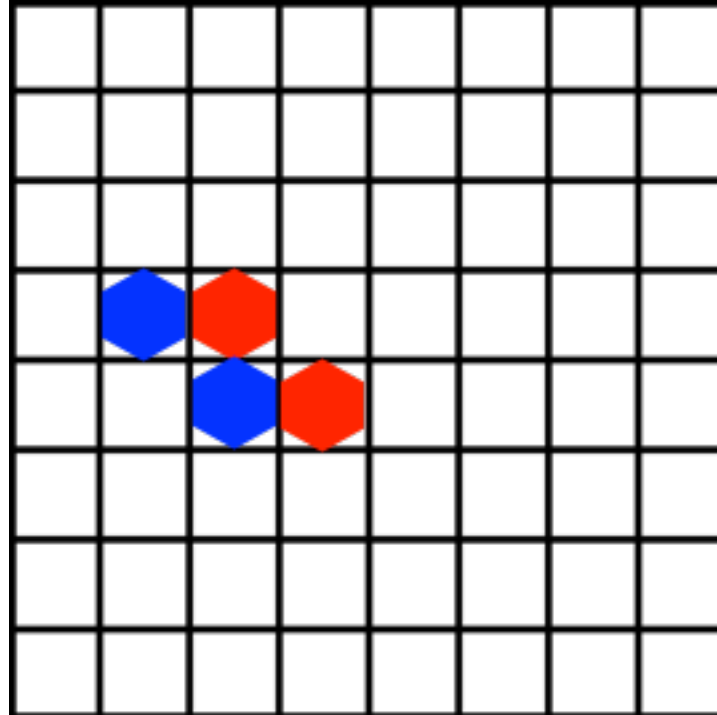
Exercise 1 page 151:

★



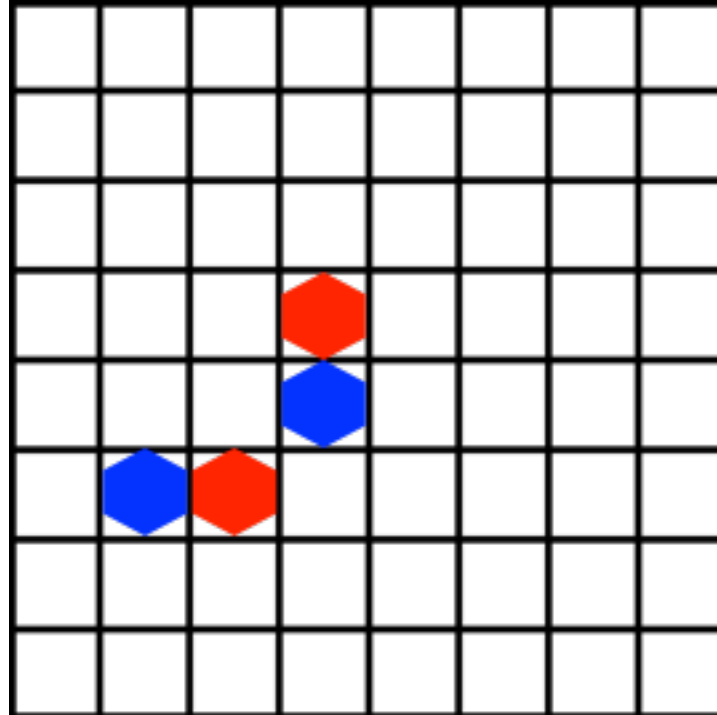
Exercise 1 page 151:

$$\star 2 = \{0, \star \mid 0, \star\}$$



Exercise 1 page 151:

$$\pm 1 = \{1 \mid -1\}$$



Summary

- Classify games by their birthday recursively
- \mathcal{G}_n and \mathfrak{G}_n definitions
- Hasse diagram of day 1 and day 2 games and their relations
- Konane for specific day 1 and day 2 game values

“Noem me Walter” - Kusters



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