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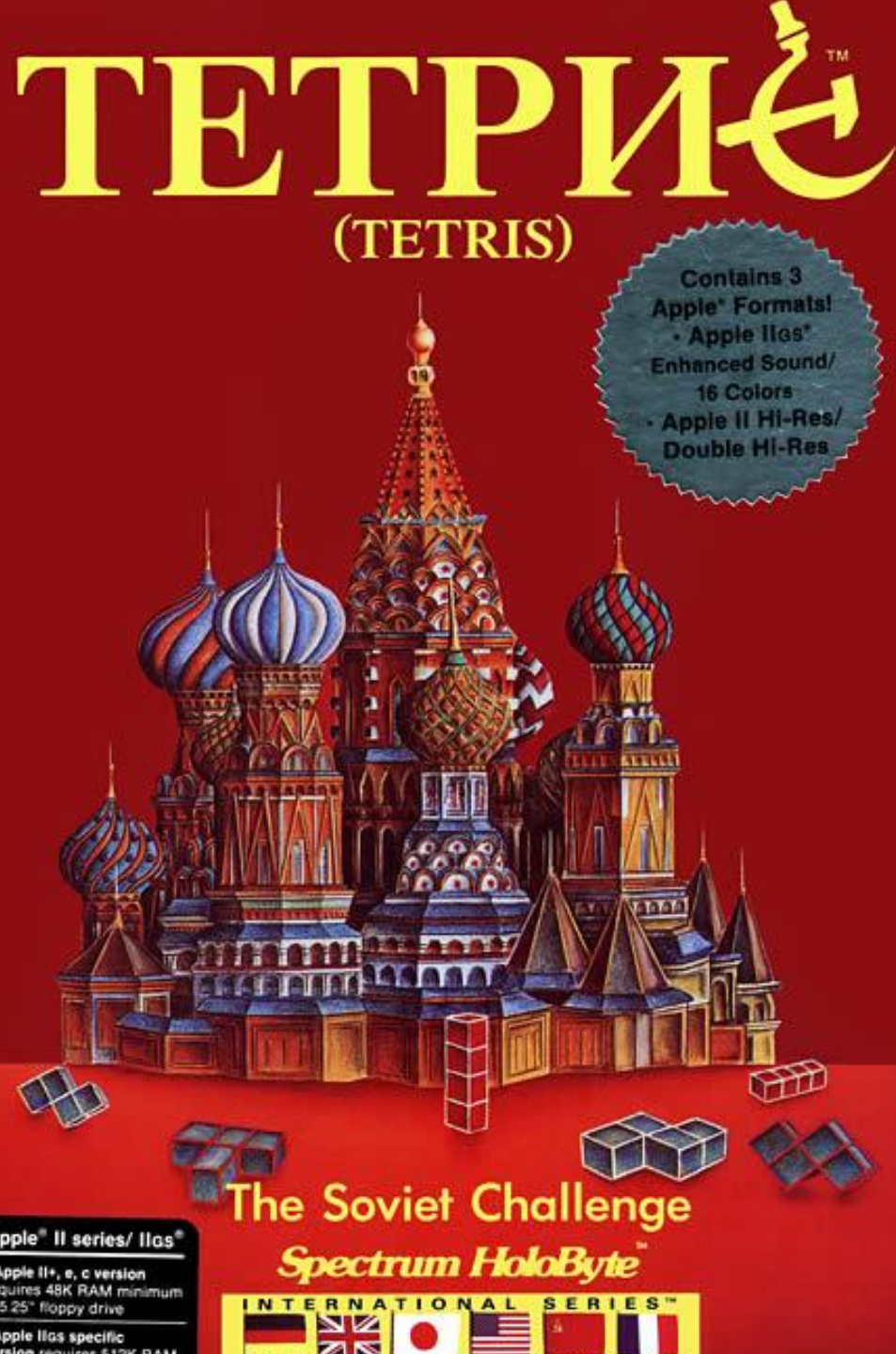
Ron Breukelaar

Hendrik Jan Hoogeboom

Walter Kosters

(LIACS algoritmen)

23 jun 2006





Tetris?

- Tetris is NP complete !!
- what configurations ?
- undecidable Tetris
- the AI of Tetris





How hard is Tetris?

- Basic Rules
- **Offline Tetris**
- Complexity
- Reduction

Breukelaar, Demaine, Hohenberger, Hoogeboom, Kusters, Liben-Nowell.
[Tetris is Hard, Even to Approximate](#). Selected Papers from the Ninth
International Computing and Combinatorics Conference (COCOON 2003).
Int. J. of Computational Geometry and Applications 14 (2004) 41-68.



History

- History
- Basic Rules
- Offline Tetris
- Complexity
- Reduction
- Conclusion

1985

Alexey Pajhitnov (Алексей Пажитнов) invents Tetris inspired by 'pentominoes'.



1989

Nintendo released Tetris on 8-bit console and Game-Boy

Now

Many other versions of Tetris are still sold, played and loved.



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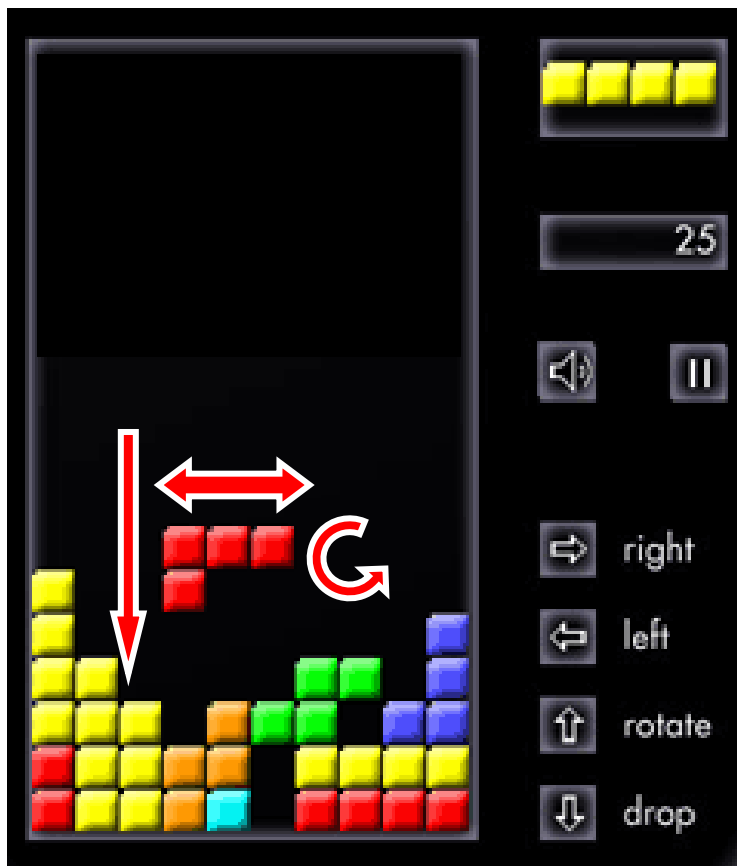
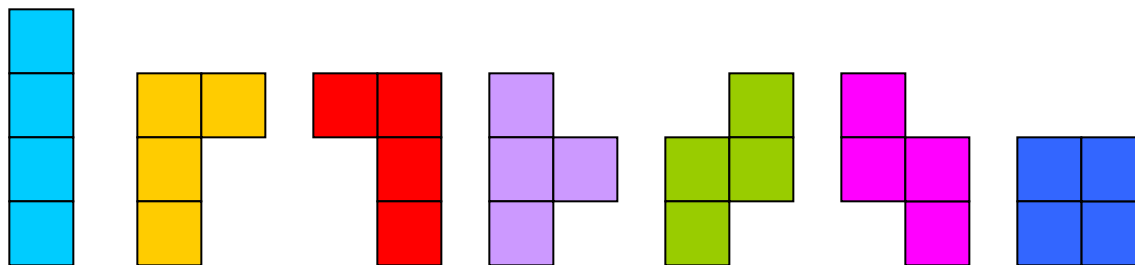
Other versions ...





Basic Rules of Tetris

- History
- Basic Rules
- Offline Tetris
- Complexity
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- Conclusion



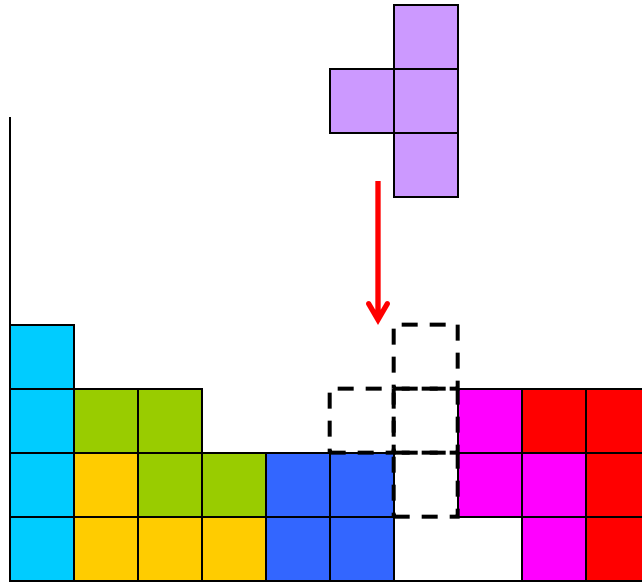
7 different pieces,
4 blocks each

- left / right
- rotate: 90 degrees
- drop
- one block look-ahead



Basic Rules of Tetris

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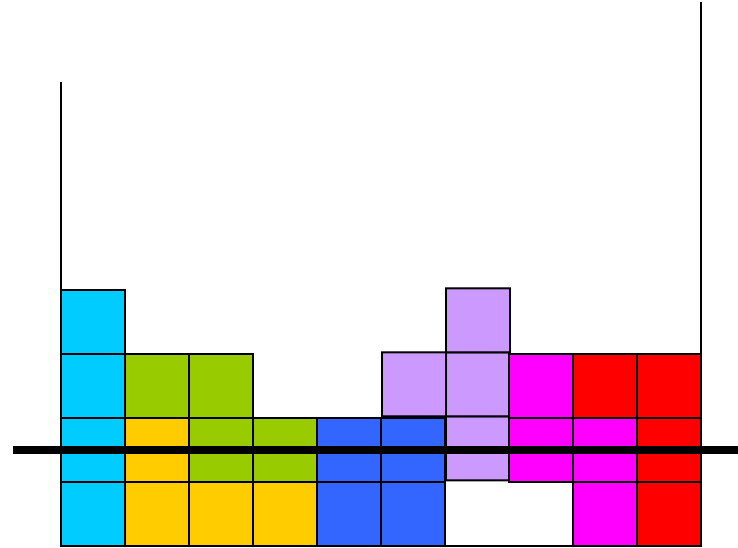


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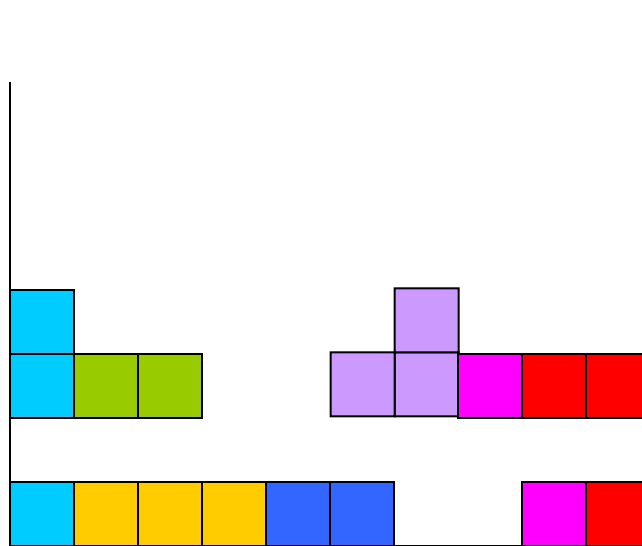


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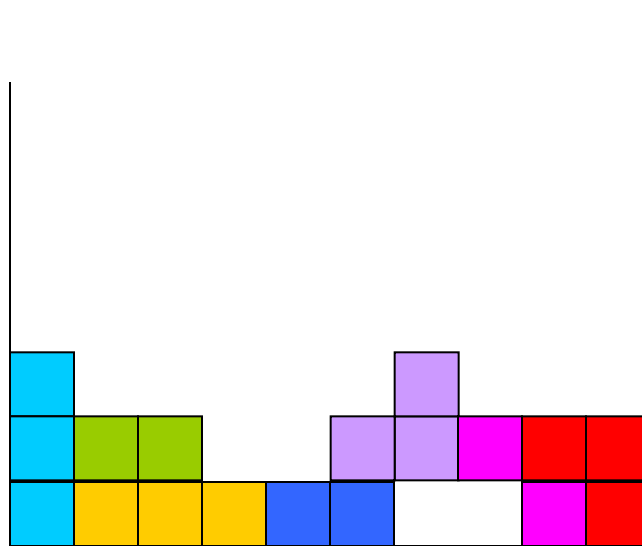
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Basic Rules of Tetris

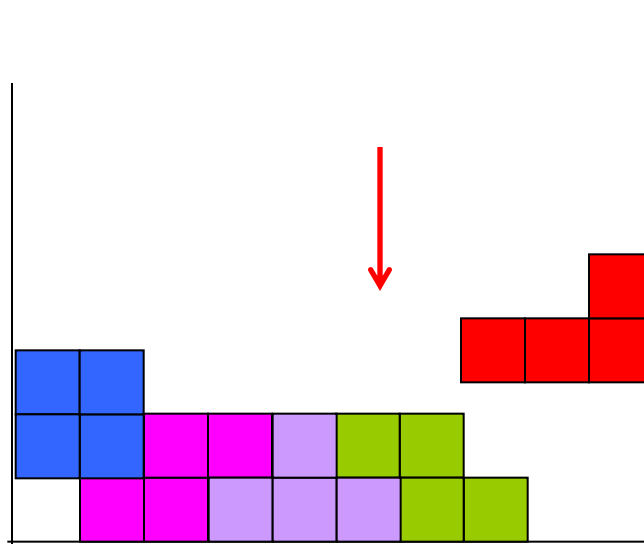


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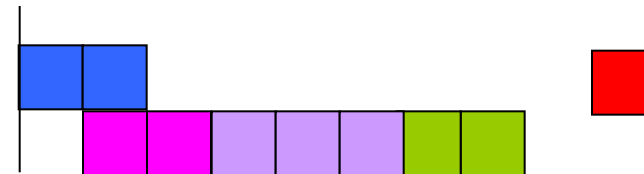


Basic Rules of Tetris

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Full lines are deleted
... and may leave 'floating' blocks





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'Offline' Tetris

- Partially filled board.
- All pieces are known at the beginning.

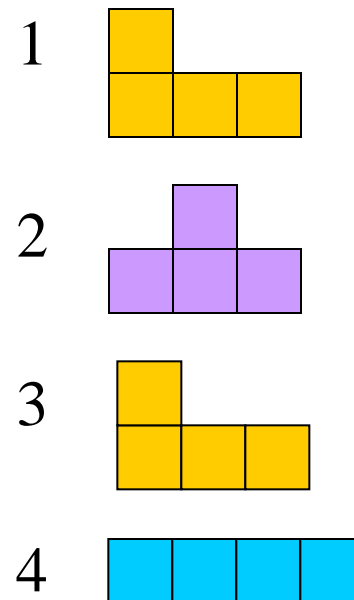
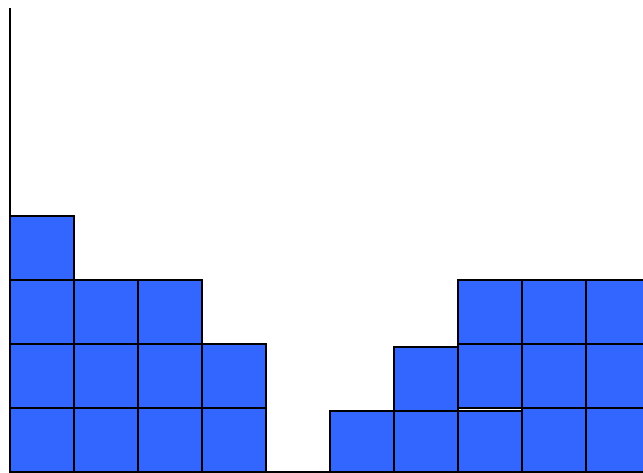
*"Given an **initial game board** and a **sequence of pieces**, can the board be cleared?"*



'Offline' Tetris

- History
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- Complexity
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"Given an initial game board and a sequence of pieces, can the board be cleared?"

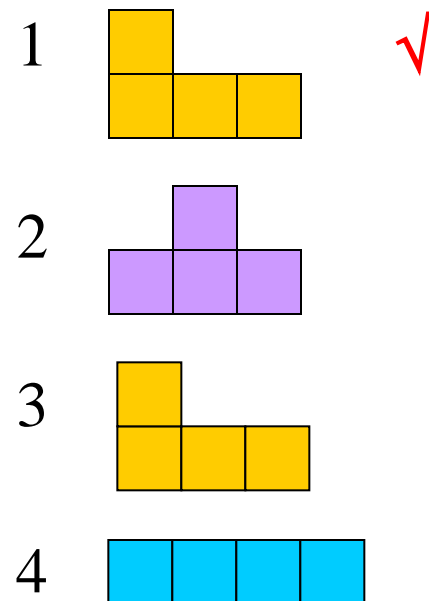
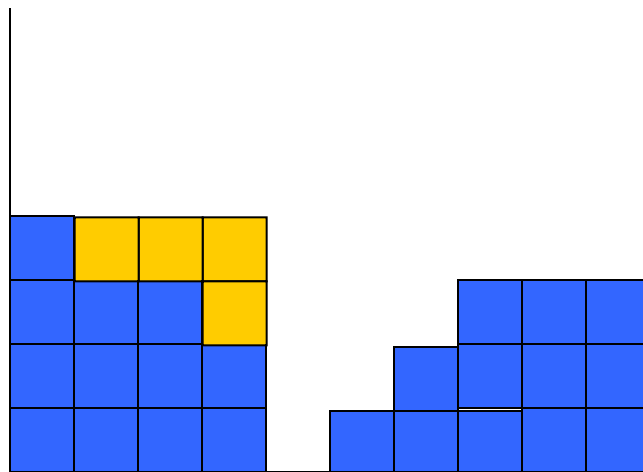




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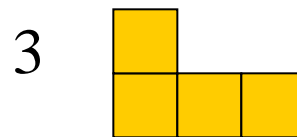
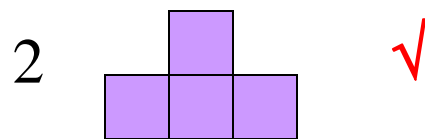
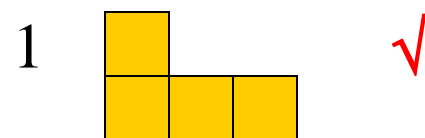
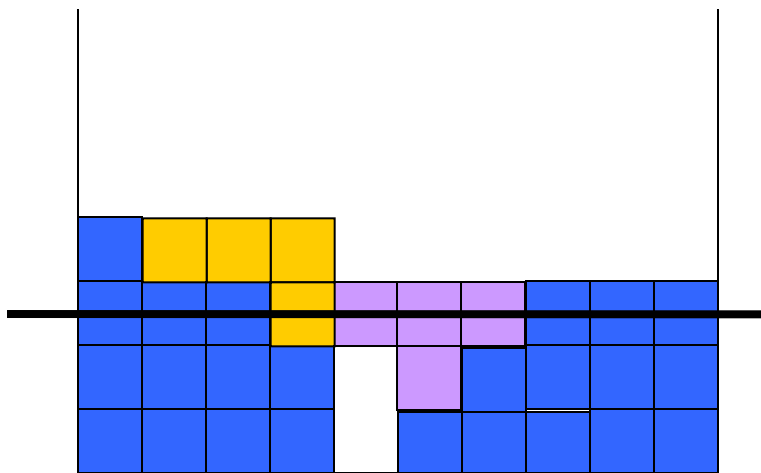




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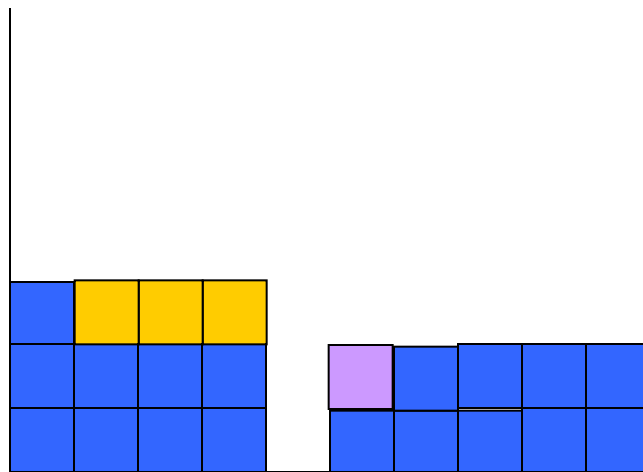


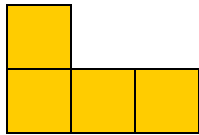
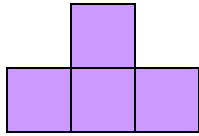
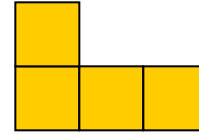



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"Given an *initial game board* and a *sequence of pieces*, can the board be cleared?"



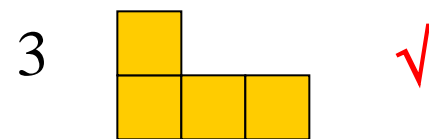
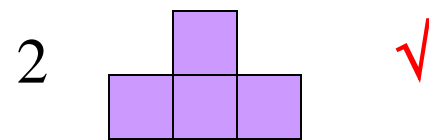
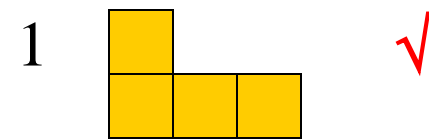
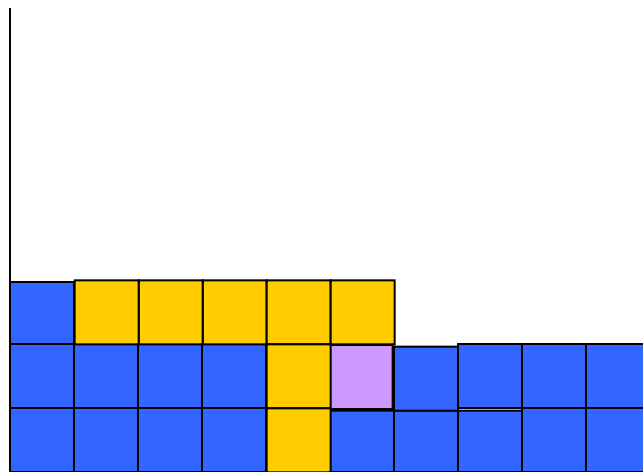
- 1  ✓
- 2  ✓
- 3 
- 4 



'Offline' Tetris

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"Given an *initial game board* and a *sequence of pieces*, can the board be cleared?"

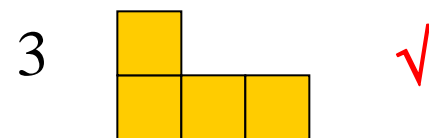
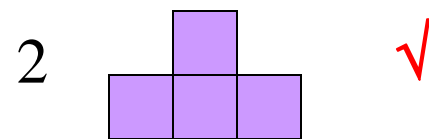
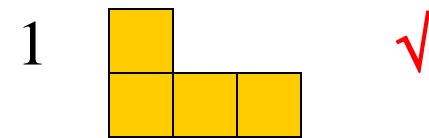
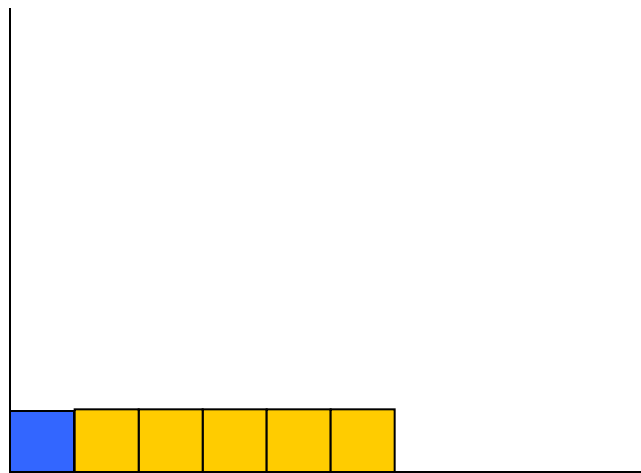




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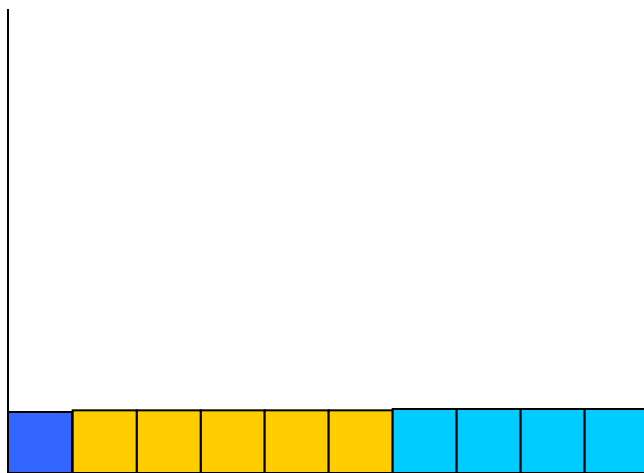


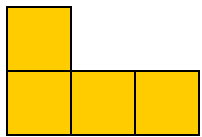
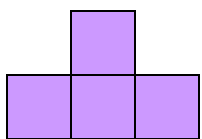
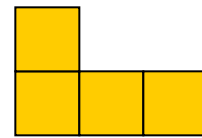



'Offline' Tetris

- History
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"Given an *initial game board* and a *sequence of pieces*, can the board be cleared?"



- 1  ✓
- 2  ✓
- 3  ✓
- 4  ✓

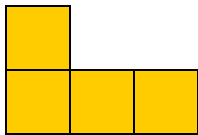
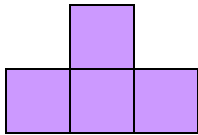
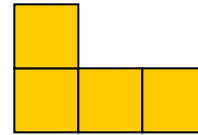



- History
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'Offline' Tetris

"Given an *initial game board* and a *sequence of pieces*, can the board be cleared?"



- 1  ✓
- 2  ✓
- 3  ✓
- 4  ✓



Complexity

- History
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Problem groups:

NP

solution *checkable* within 'reasonable' time

P

problem *solvable* within 'reasonable' time

NP-complete

problem is NP and
algorithm for this problem can be 'translated' to
any other NP problem

big question: **P = NP?** -- \$1.000.000

http://www.claymath.org/millennium/P_vs_NP/

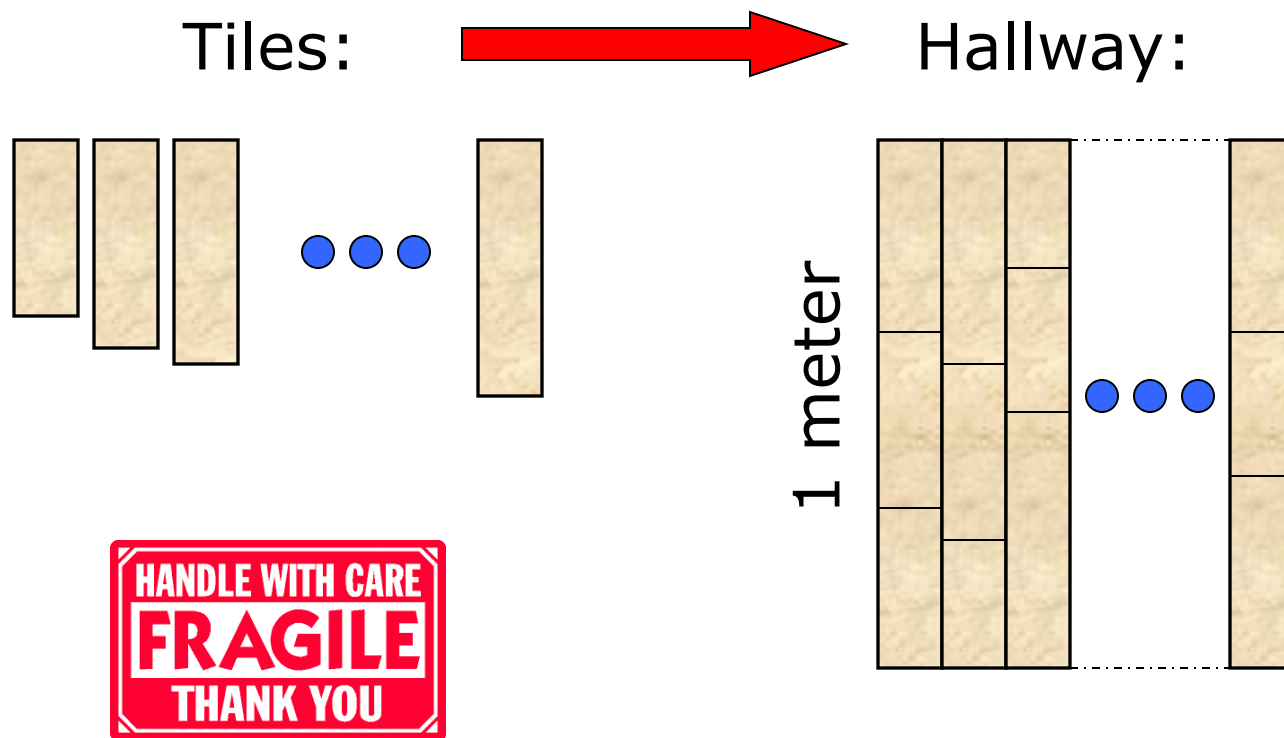


Complexity

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Floortje has bought a new floor, the salesman told no sawing was required: "every row can be filled with three tiles from the pack".

Question: Was salesman telling the truth?





Complexity

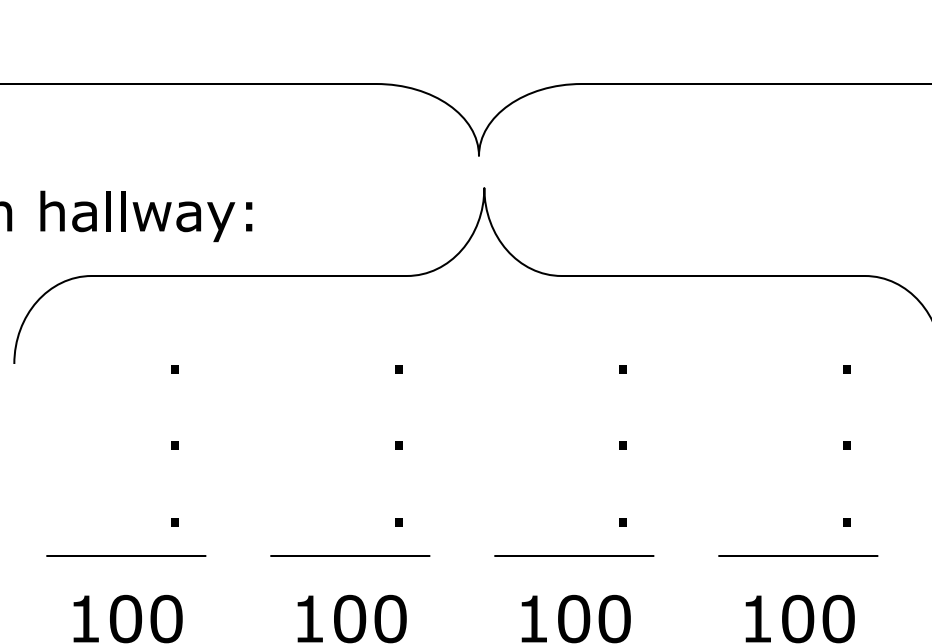
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Floor tile example:

12 floor tiles (in centimeters):

26, 26, 28, 30, 31, 32, 33, 34, 36, 36, 40, 48

4 rows in hallway:



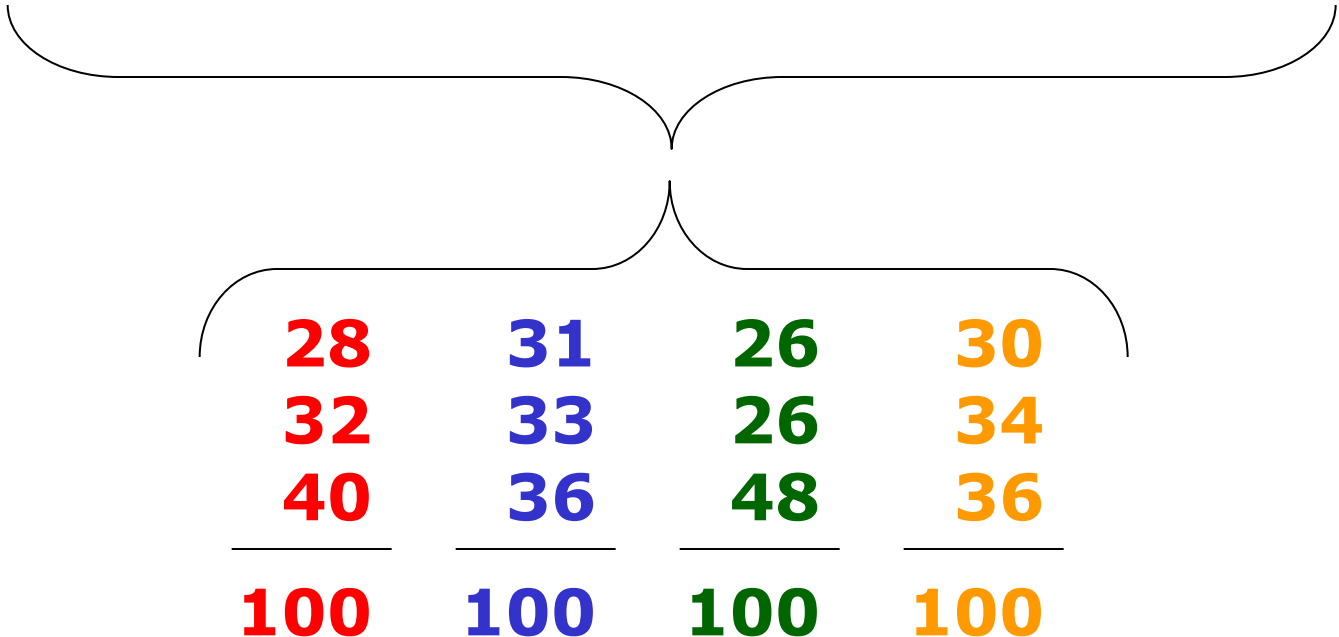


Complexity

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Floor tile example:

26, 26, 28, 30, 31, 32, 33, 34, 36, 36, 40, 48





Complexity

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Number of possible configurations:

9 tiles	→	1.680
12 tiles	→	369.600
15 tiles	→	168.168.000

Grows exponentially: NP ... intuitively ...

3-partitioning problem

proven to be NP-complete

so ... any NP problem can be solved using the algorithm for 3-partitioning



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- History
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Reduction

now:

translate the floor tiling problem into a Tetris problem

if we can solve Tetris
then we can solve floor tiling
then we can solve every NP-problem

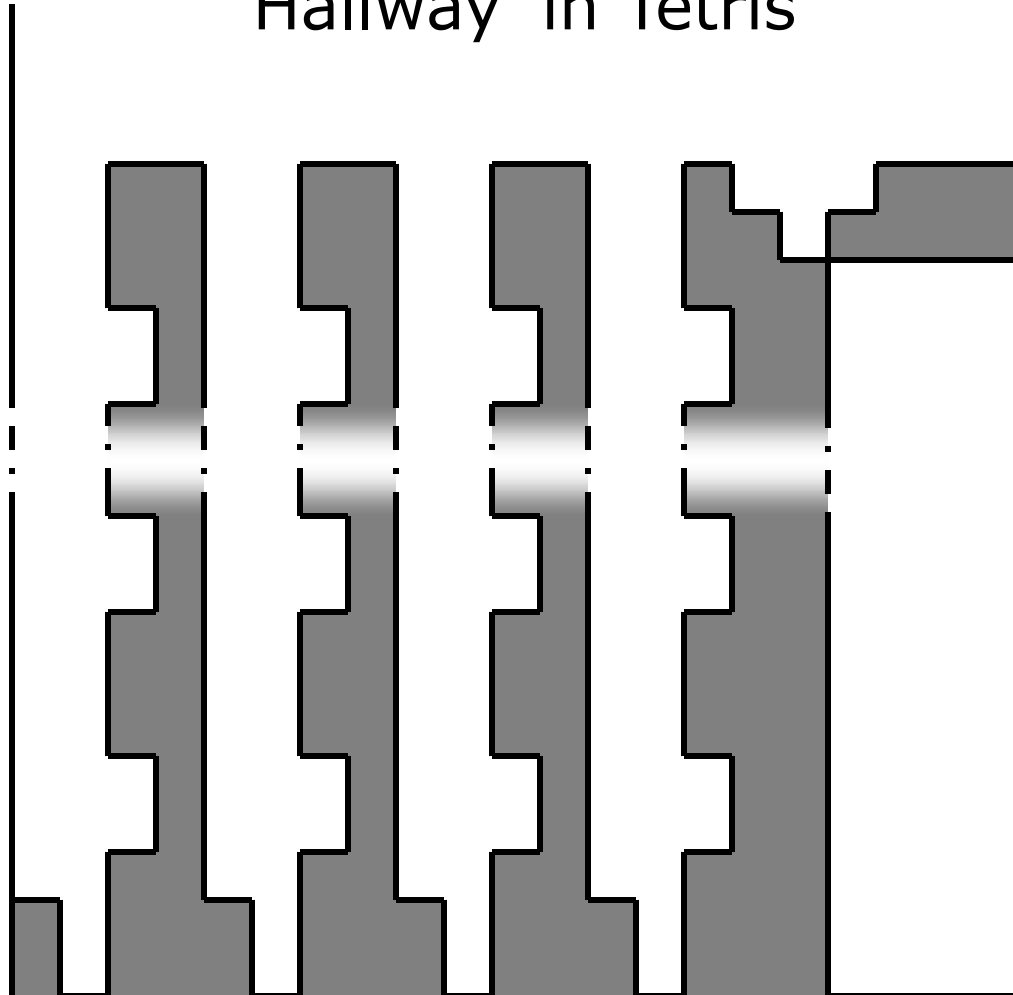
Tetris itself is NP complete



Reduction

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'Hallway' in Tetris



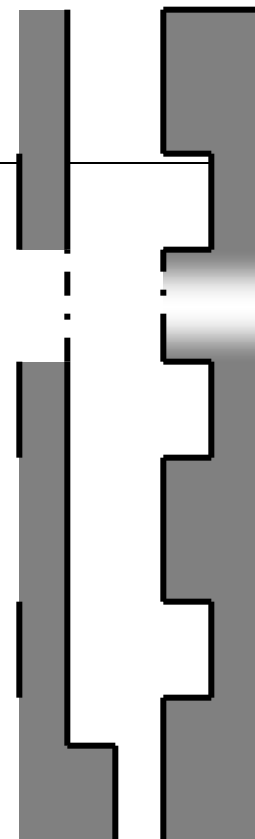
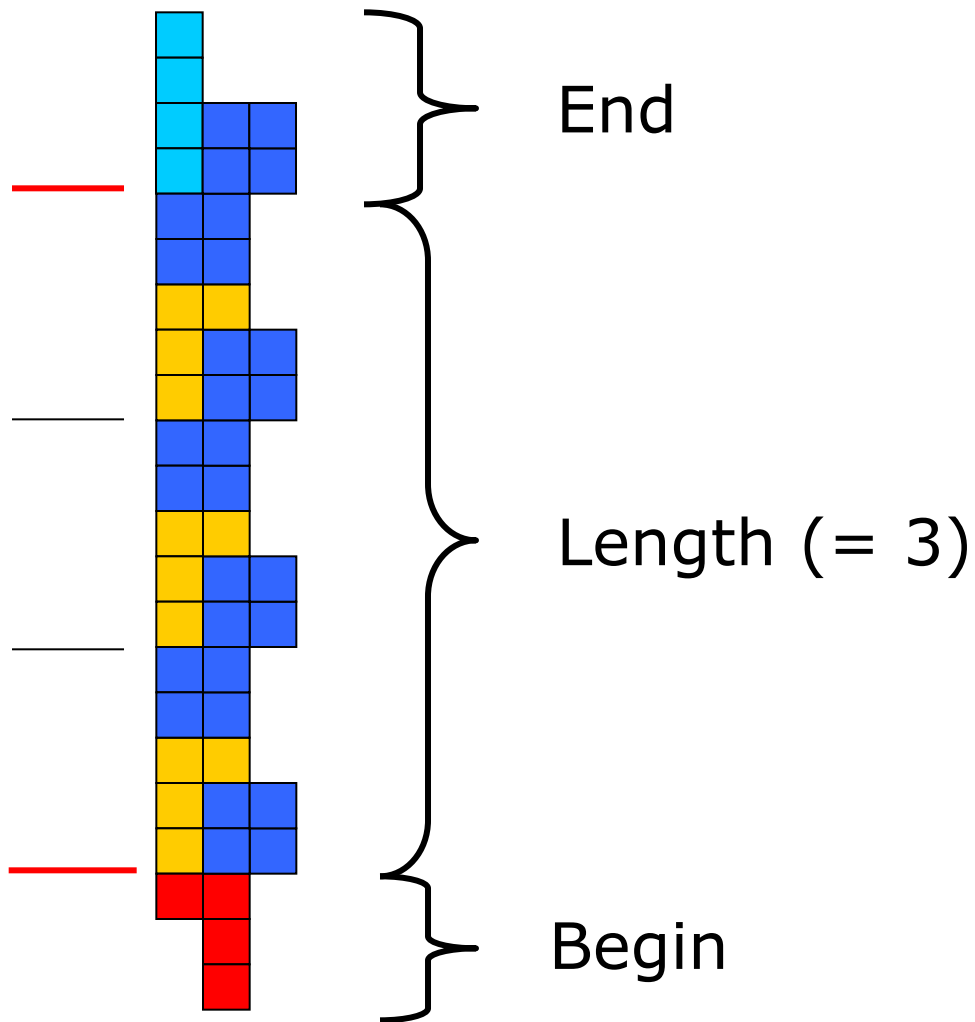


Reduction

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Floor tile in Tetris:



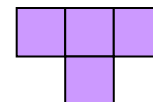


Reduction

... some details on the formalities ...

check: filling the game board is equivalent to filling a hallway:

- floor tiles only fit in one row each.
- the lines can not be cleared before all the floor tiles have been laid.



... “yes” in floor tile problem \Leftrightarrow “yes” in Tetris.

→ Tetris is NP-complete



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Conclusion

Even if there is a finite number of pieces and their order is known, it is very hard (NP-complete) to compute whether a given initial game board can be cleared.

In other words:

If you find an algorithm that plays Tetris optimal within reasonable time, you have proven that **P = NP** and you become famous, ... and **rich**.



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more Tetris?

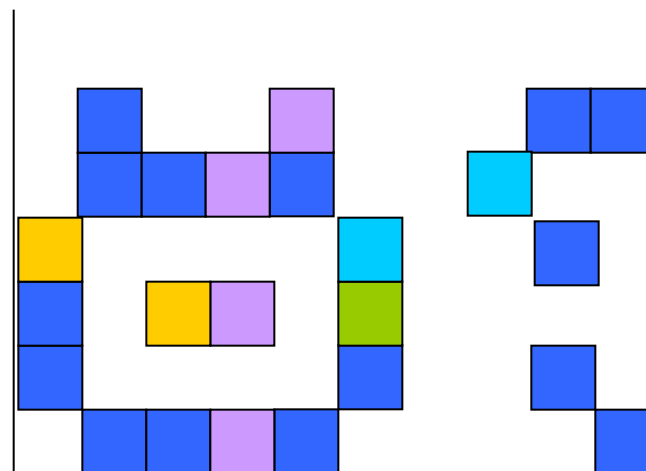
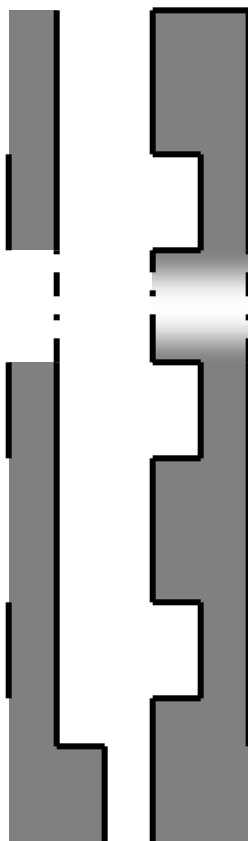


- Tetris is NP complete !!
- what configurations ?
- undecidable Tetris
- the AI of Tetris



Configurations

- NP complete
- Configurations
- Undecidable
- AI of Tetris



10

restrictions:

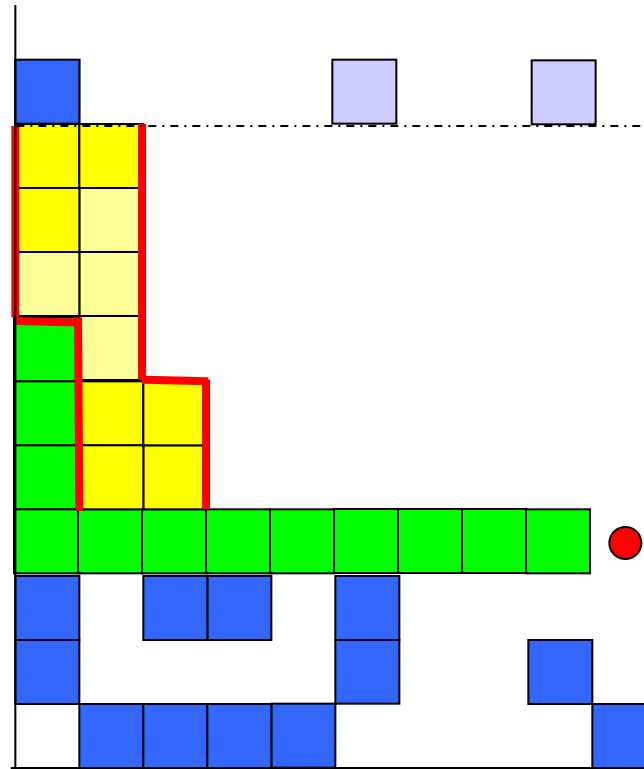
- even number blocks
add 4 blocks
delete 10 blocks
- empty & full rows



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Configurations

- NP complete
- Configurations
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nieuwe rij

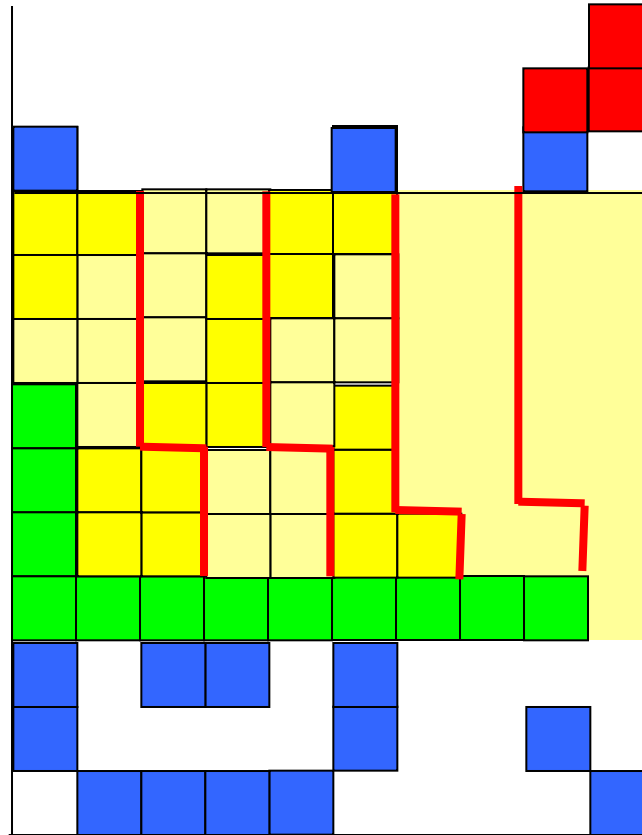
platform



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Configurations

- NP complete
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- AI of Tetris



overflow
nieuwe rij

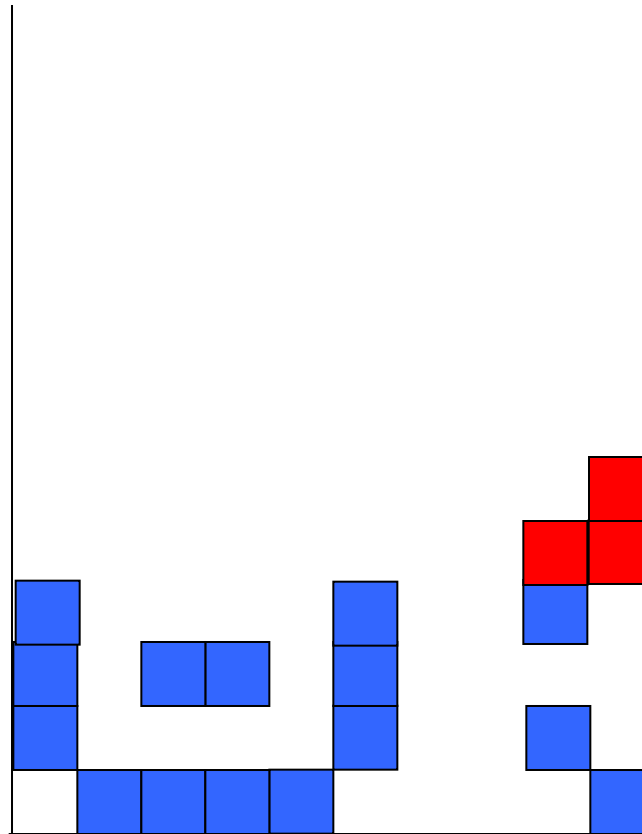
platform



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Configurations

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overflow
nieuwe rij

platform



AI of Tetris

- NP complete
- Configurations
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Mathematical proof:
no optimal stacking of 'S' and 'Z'



What is the best move ?
(using single block look-ahead)





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thank you...





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Questions

- NP complete
- Configurations
- Undecidable
- AI of Tetris

