TEGELS

van patronen tot berekeningen

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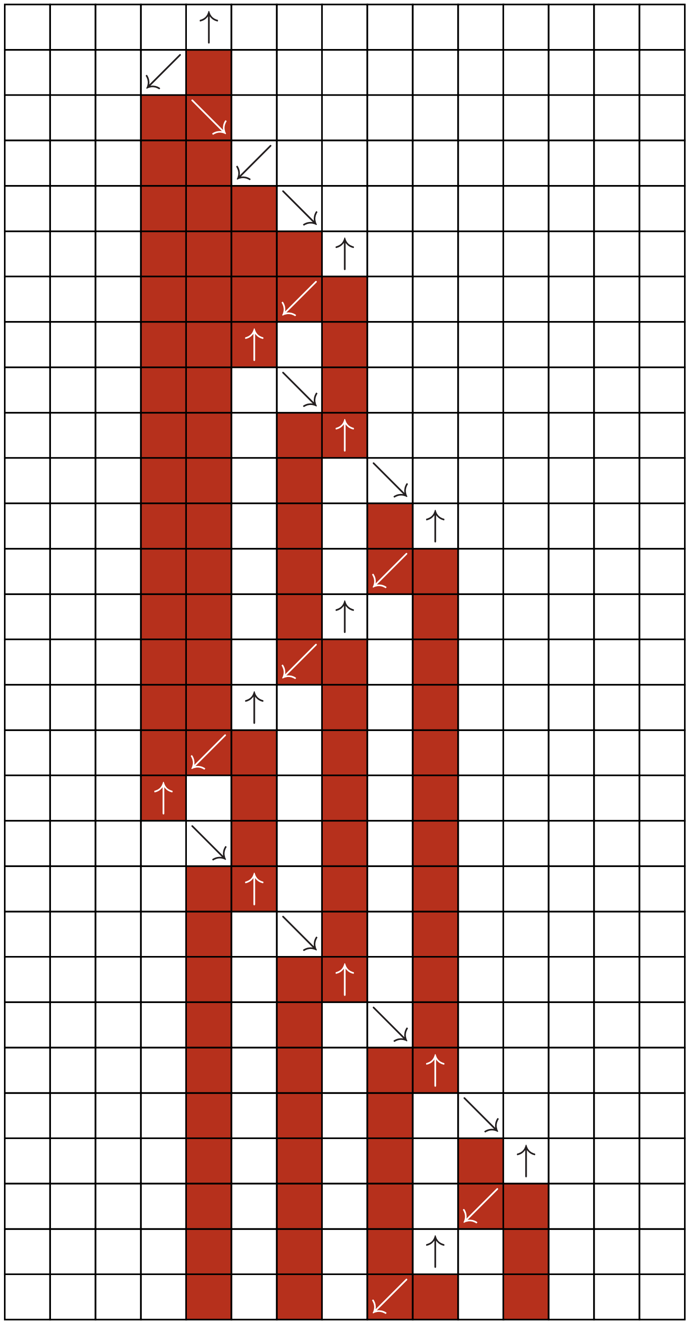
www.liacs.nl/home/hoogeboo/praatjes/tegels/

Universiteit Leiden

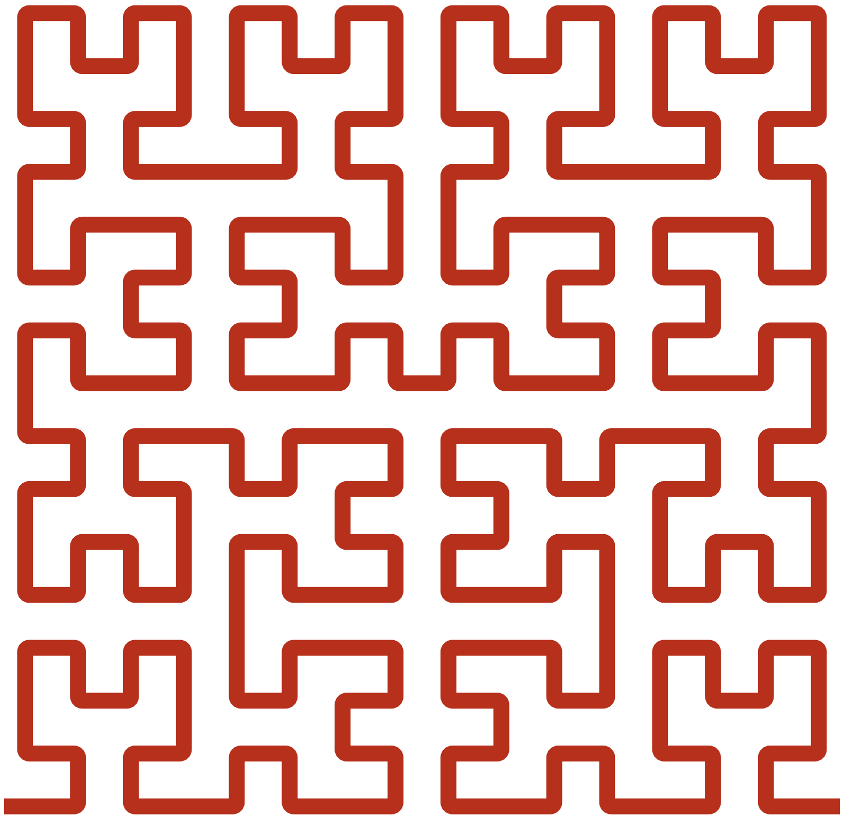


wiskunde: regelmaat, patronen

informatica: berekeningen

modellen:

- Wang tiles
- Cellular automata
- Turing machines



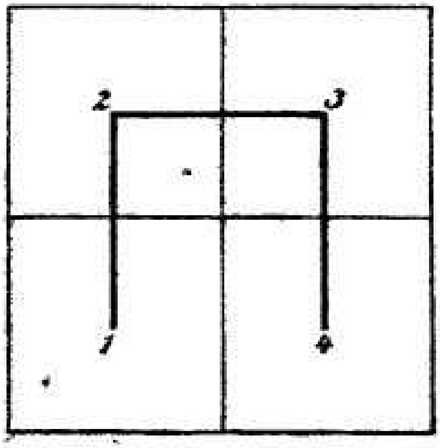


Fig. 1.

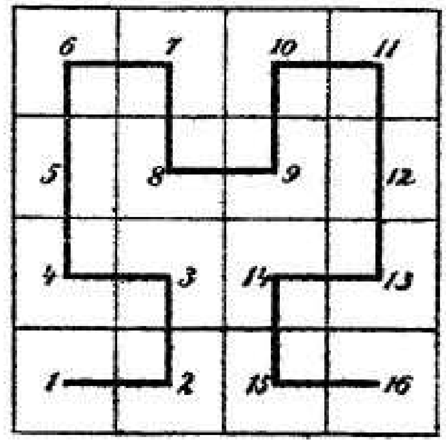


Fig. 2.

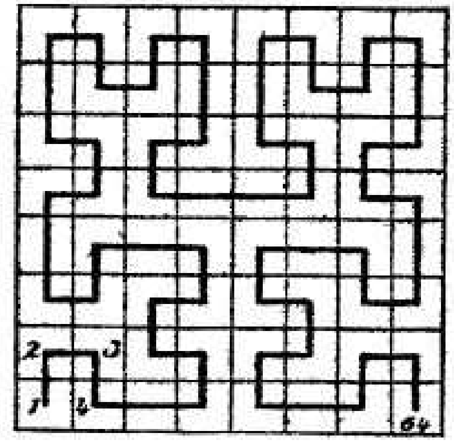
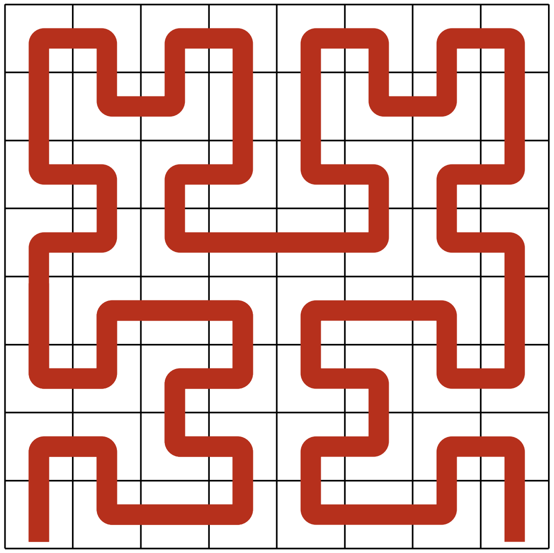
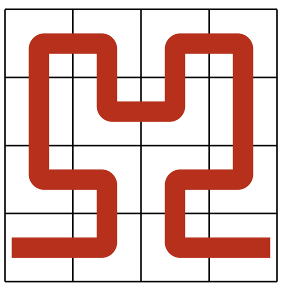


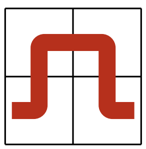
Fig. 3.

Ueber die stetige Abbildung einer Linie auf ein Flächenstück (1891)

David Hilbert in Königsberg i. Pr.

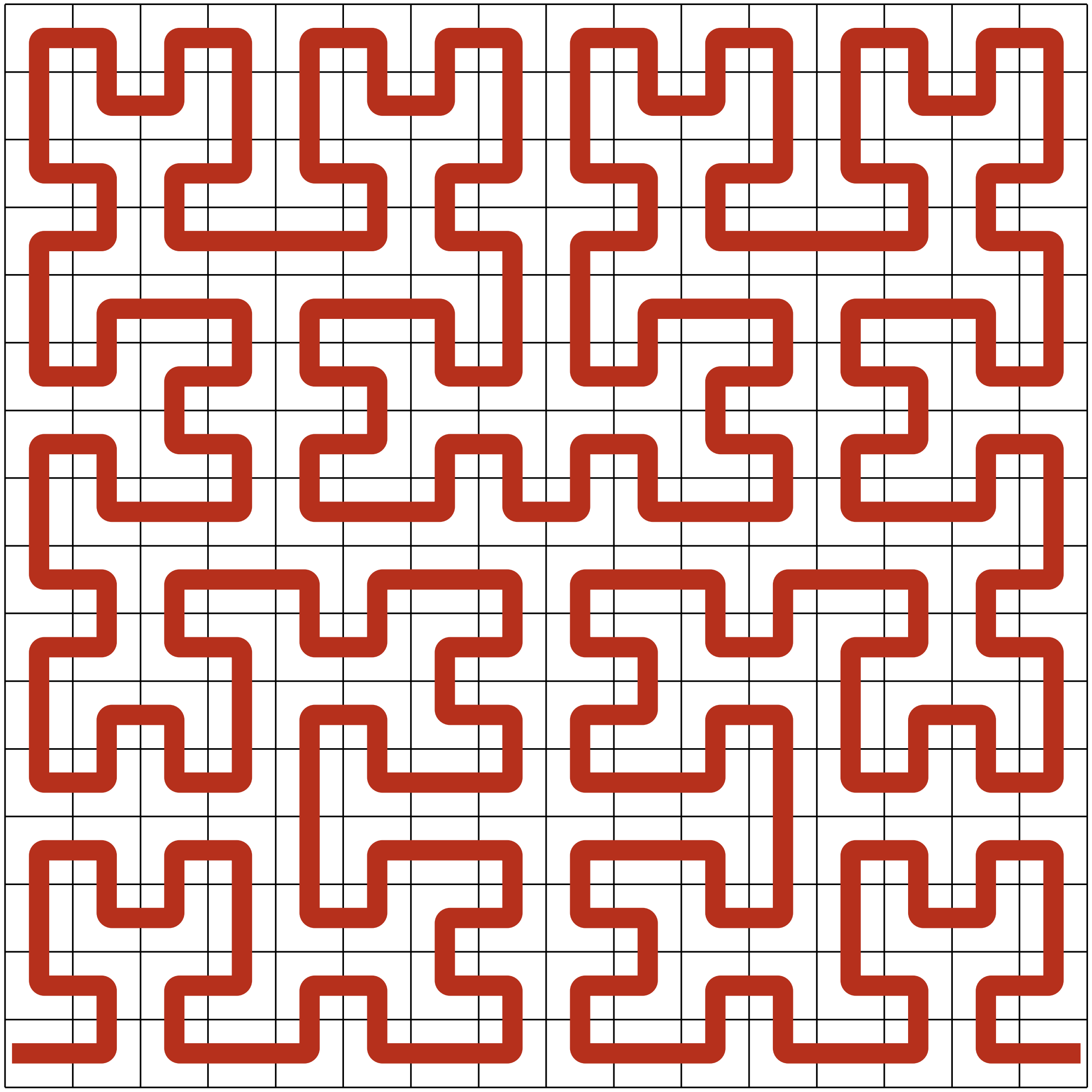


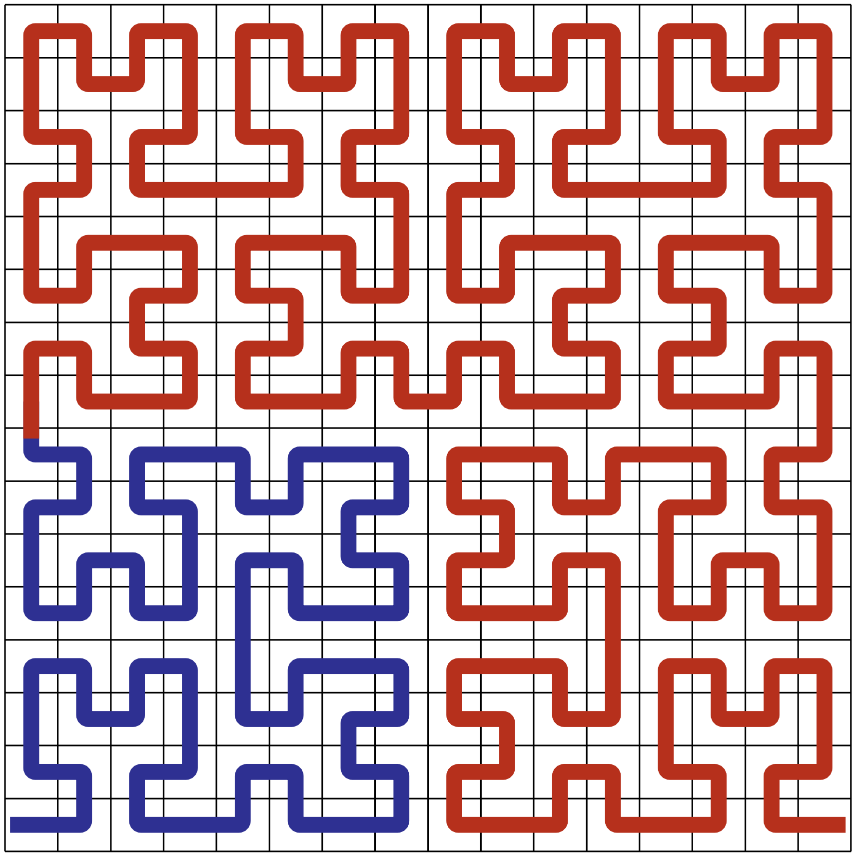




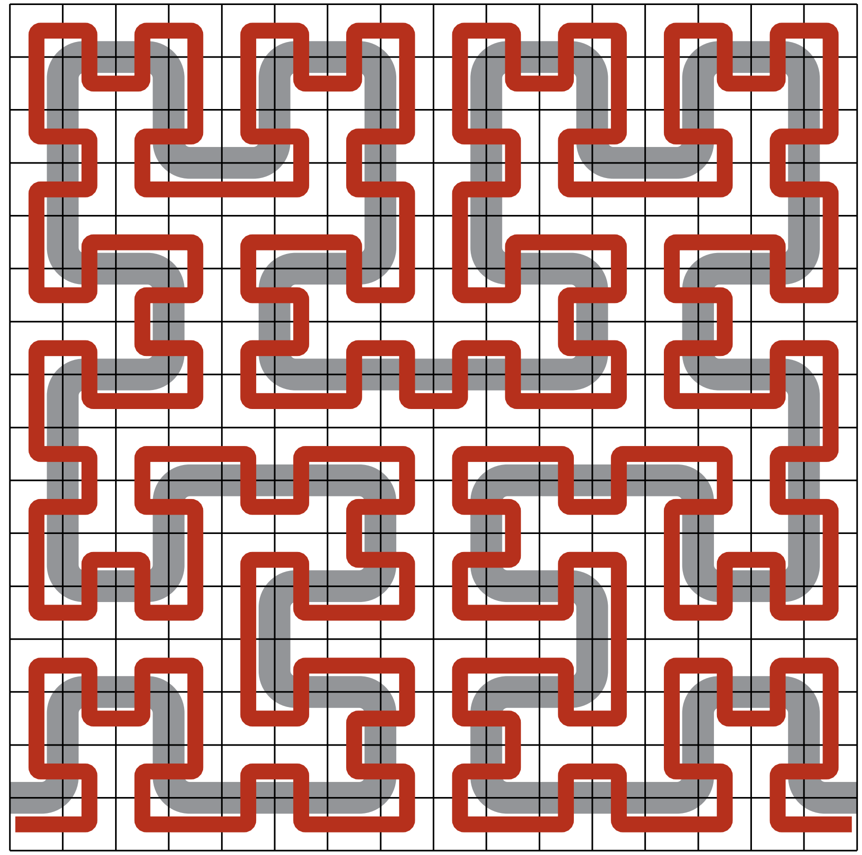
zes ‘tegels’:

twee recht, vier bochten

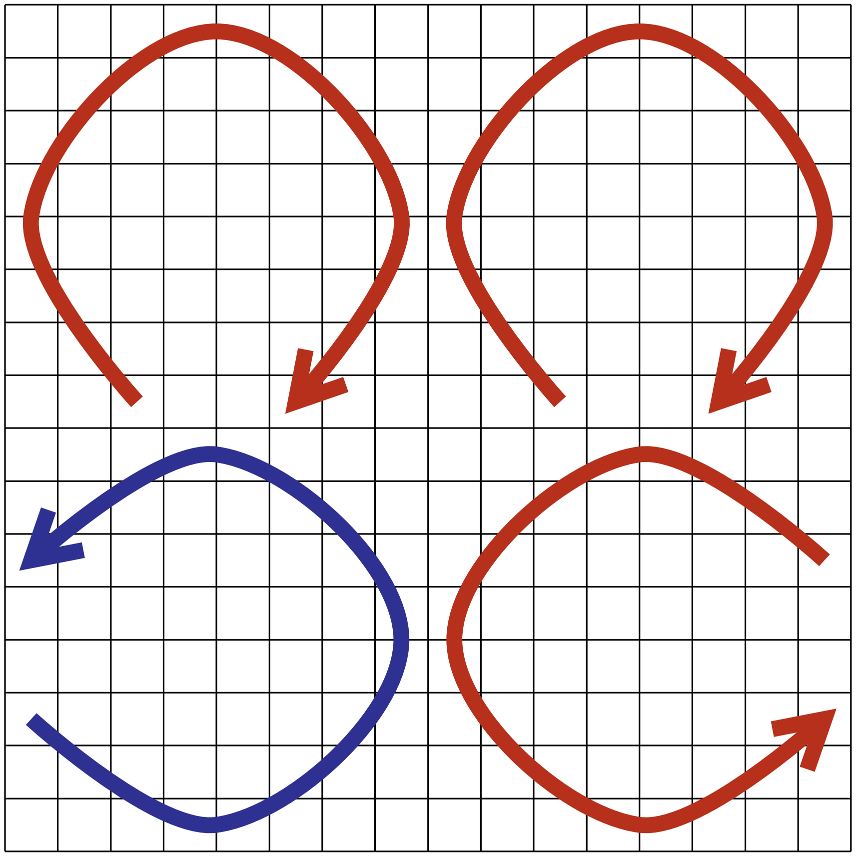




vier kopieën



verfijning



wikipedia

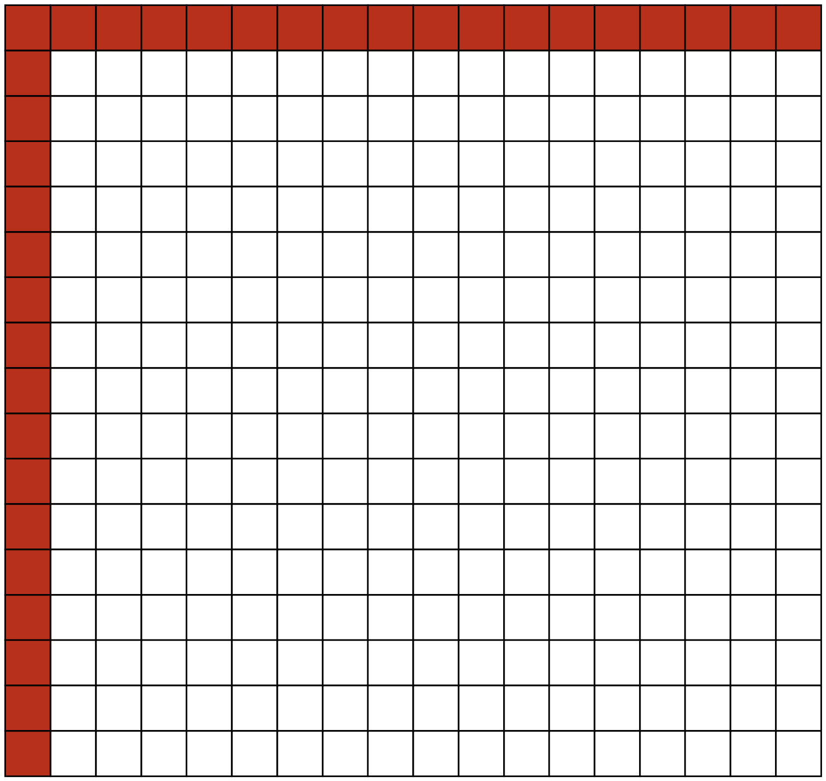
```
to starthilbert :size :level
  hilbert (:size / power 2 (:level-1)) :level 1
end

to hilbert :size :level :parity
  if  :level = 0 [stop]
  right 90 * :parity
  hilbert :size (:level-1) (:parity * -1)
  forward :size
  right -90 * :parity
  hilbert :size (:level-1) :parity
  forward :size
  hilbert :size (:level-1) :parity
  right -90 * :parity
  forward :size
  hilbert :size (:level-1) (:parity * -1)
  right 90 * :parity
end
```

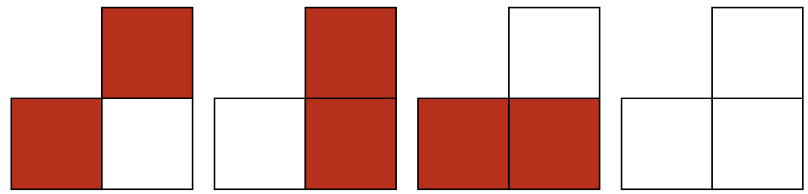
aanroep

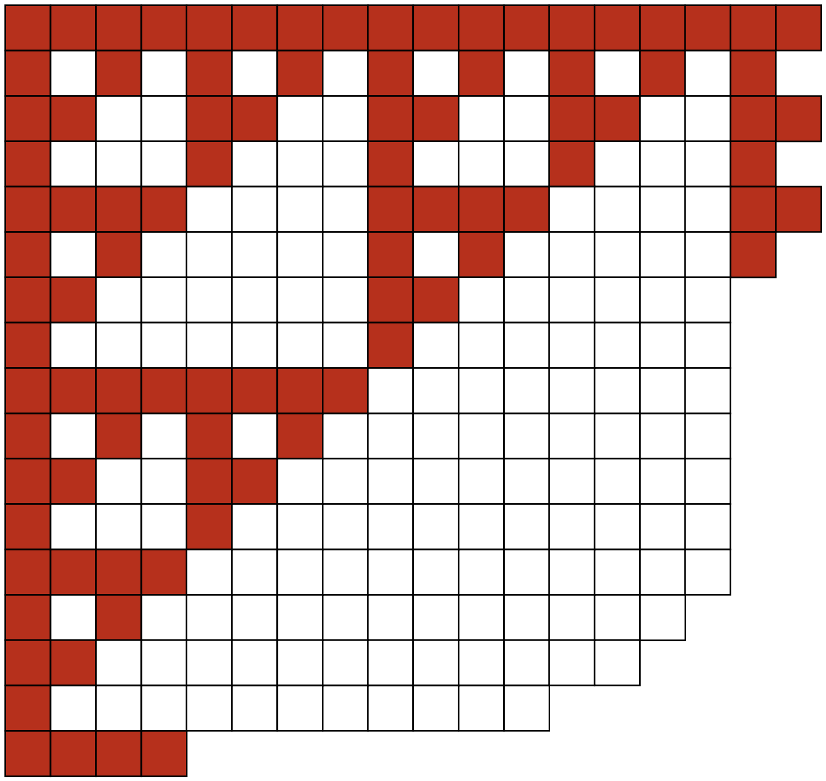
```
starthilbert 200 5
```

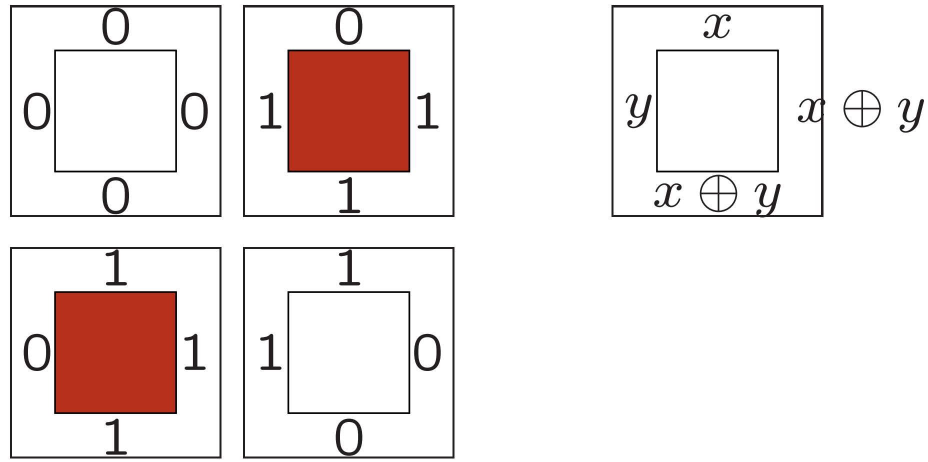
■ Zelf proberen



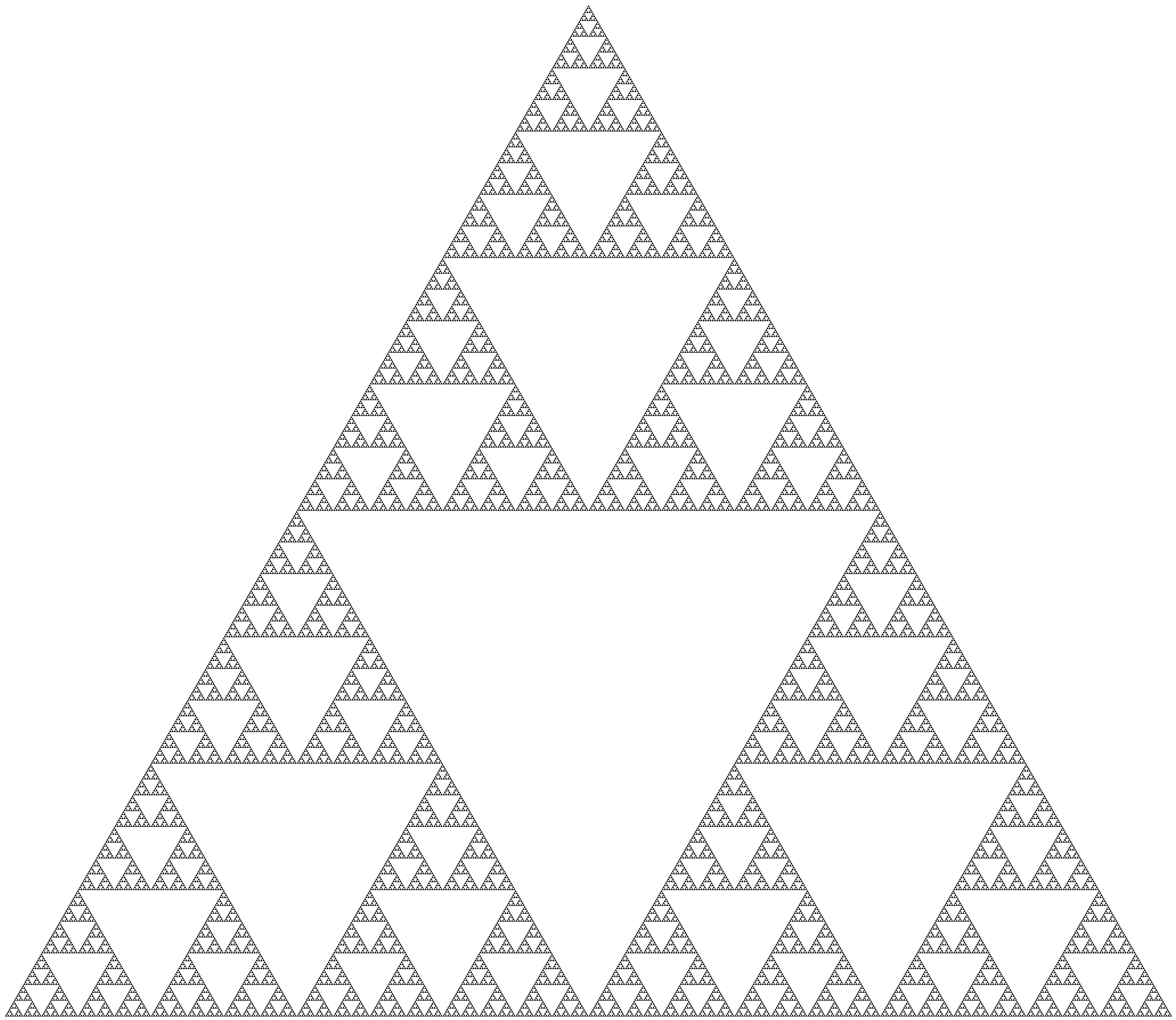
de buren bepalen de kleur







x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0



Qef's Website
wikipedia

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x	y	$x \oplus y$
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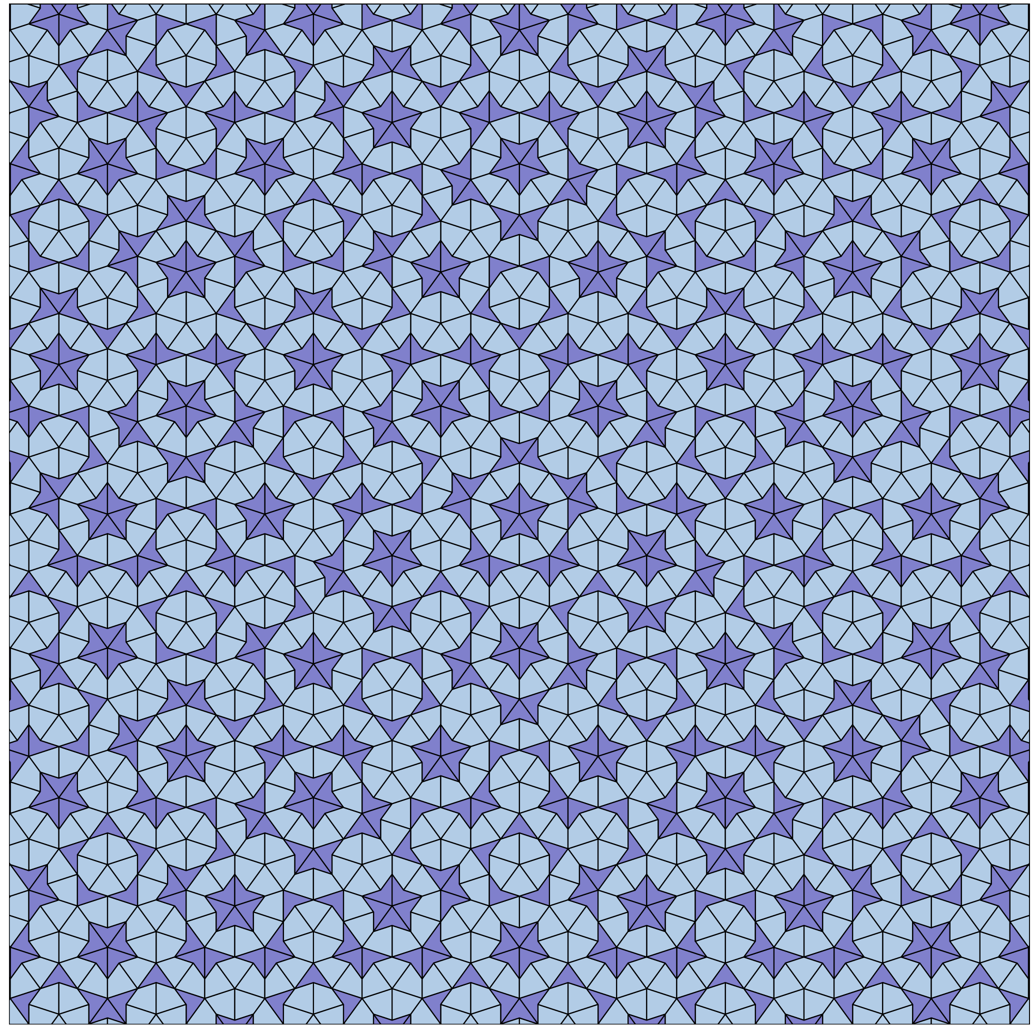
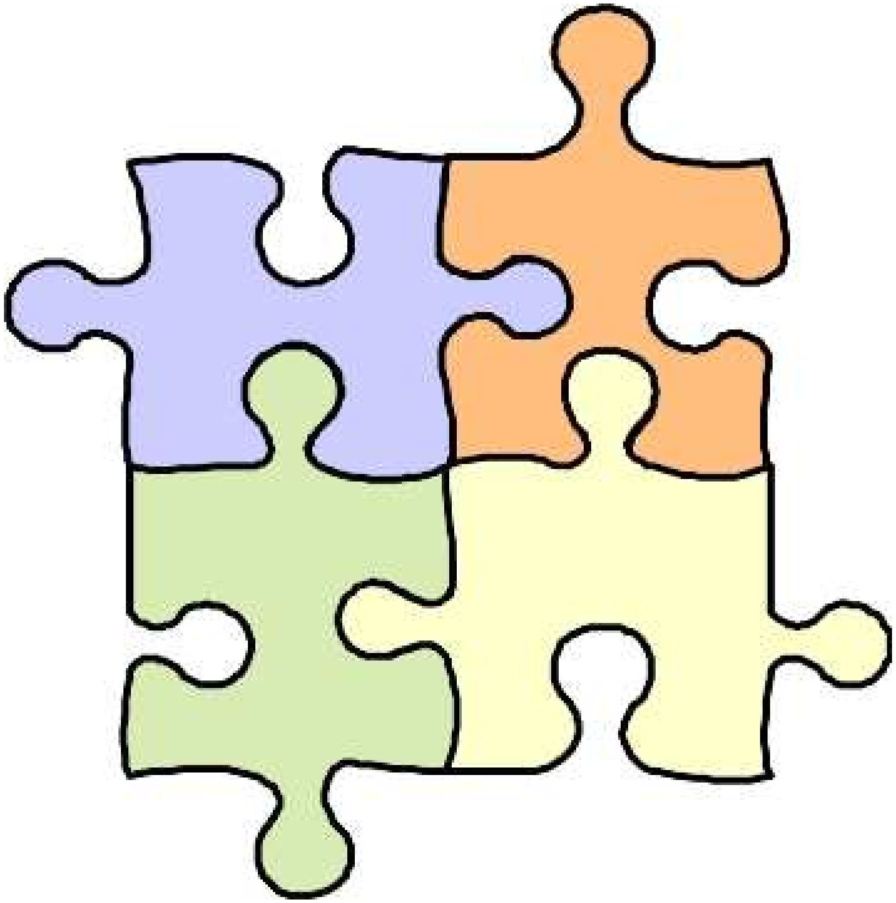
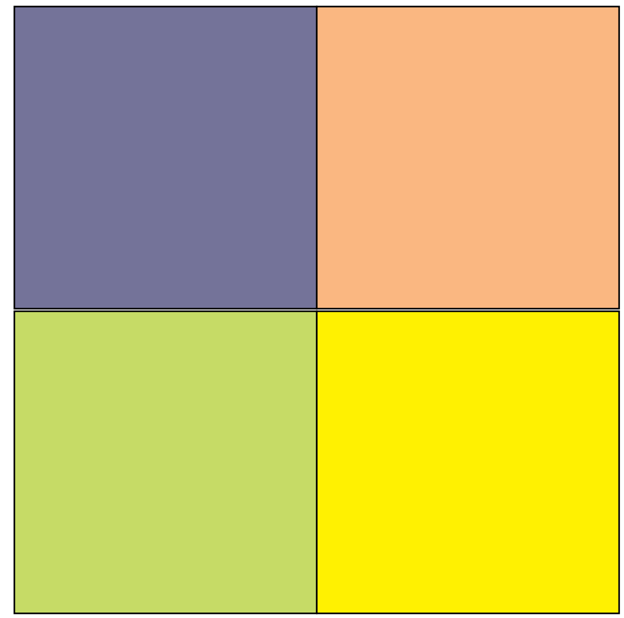
0	1	1
1	0	1
1	1	0

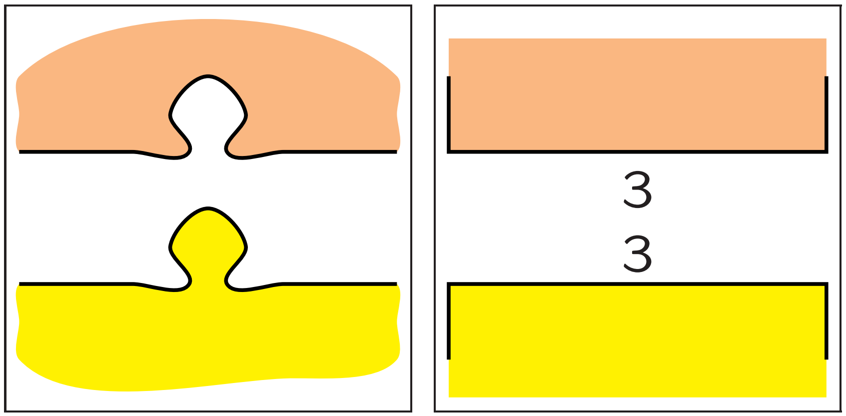
Spelregels



Penrose (1974)



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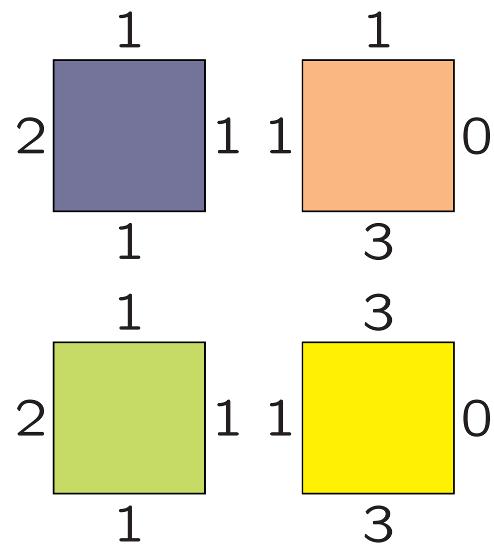
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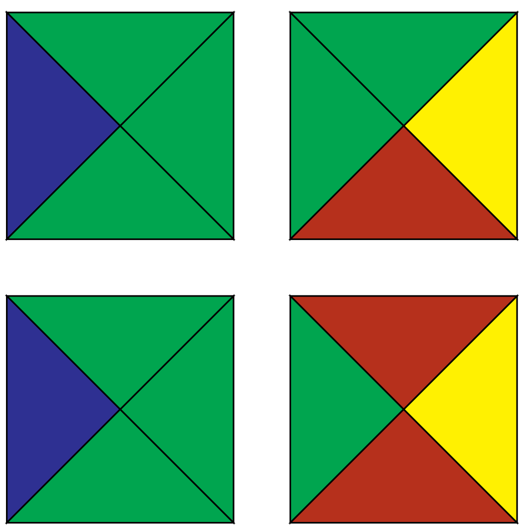
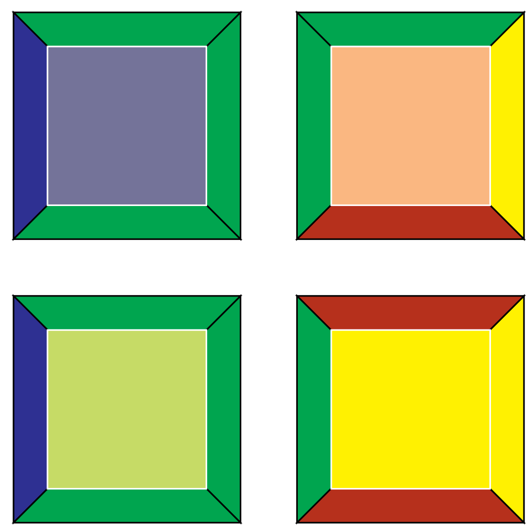
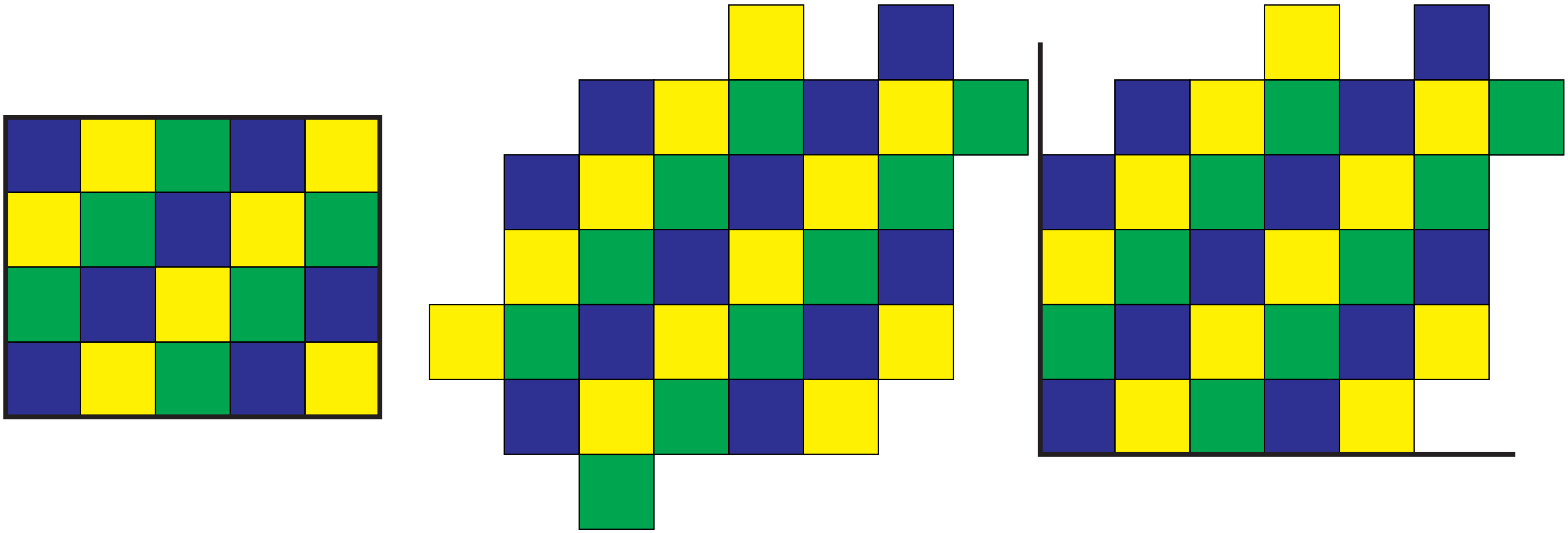
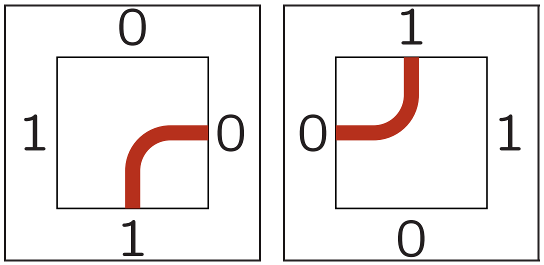




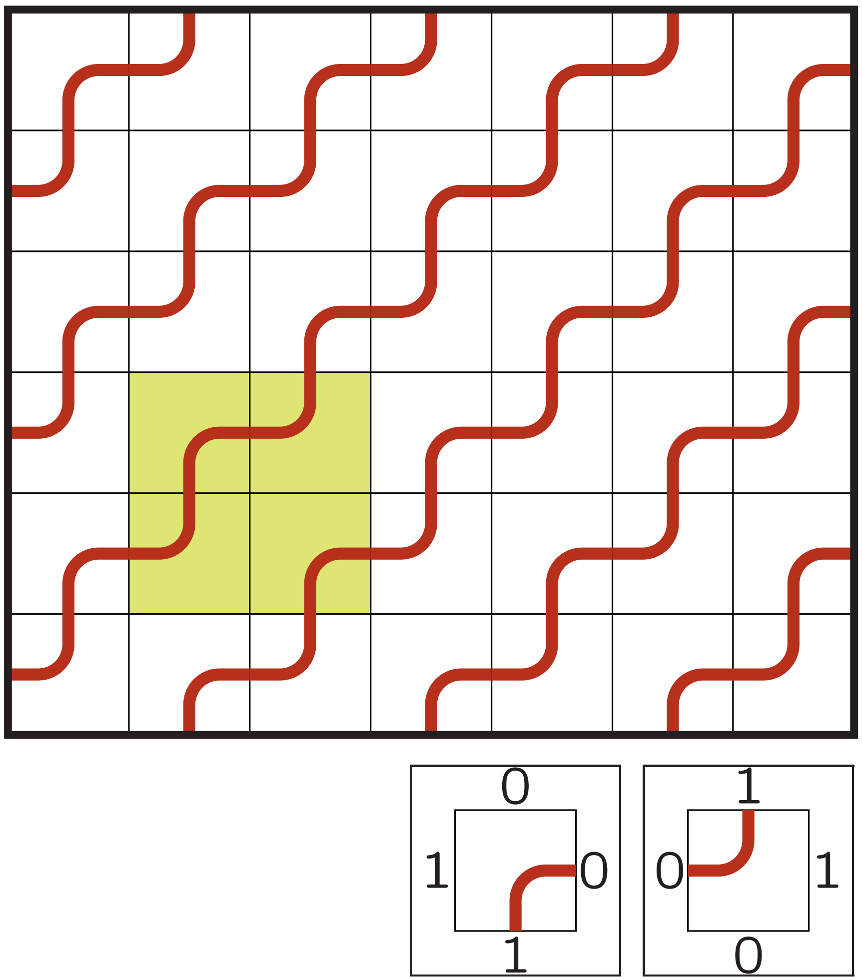


- rechthoek
- (half) vlak
- kwadrant

■ Patronen



twee tegels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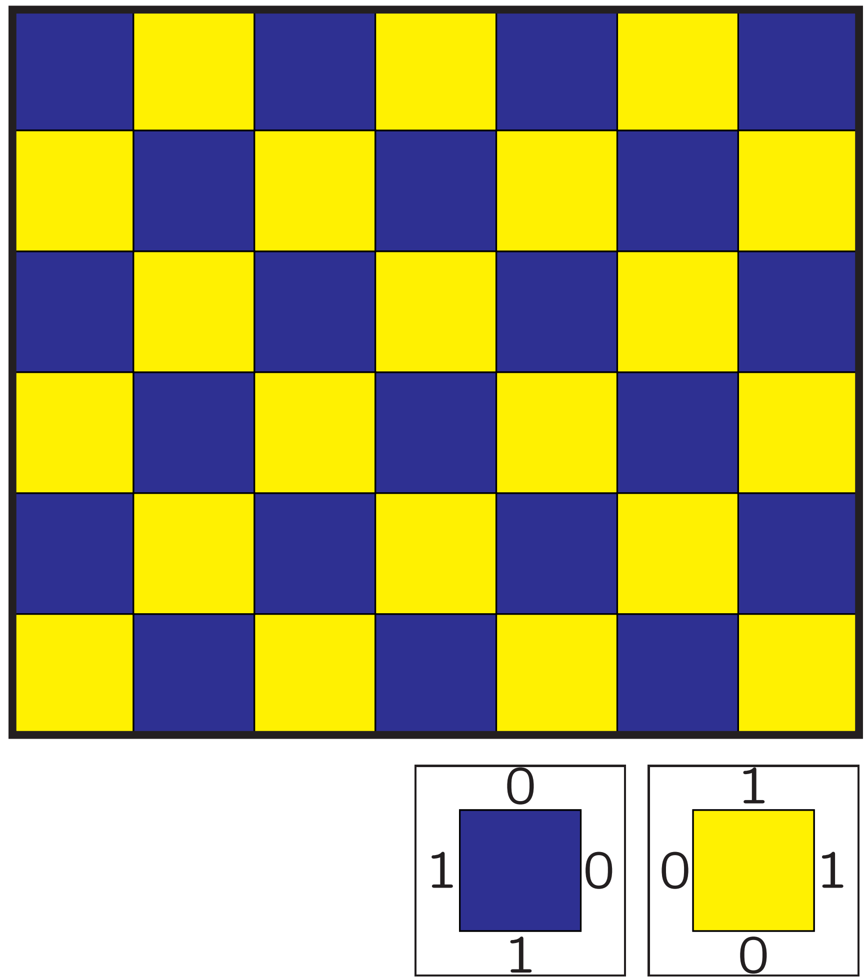
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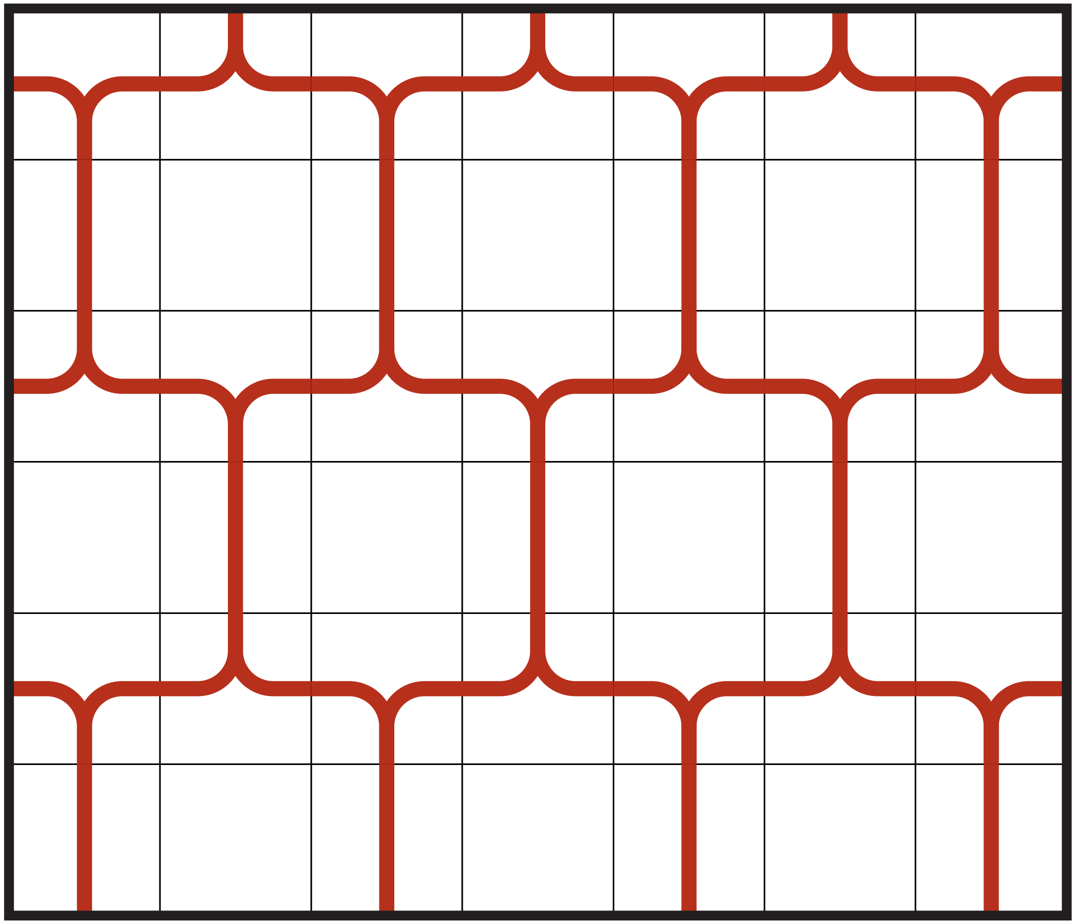
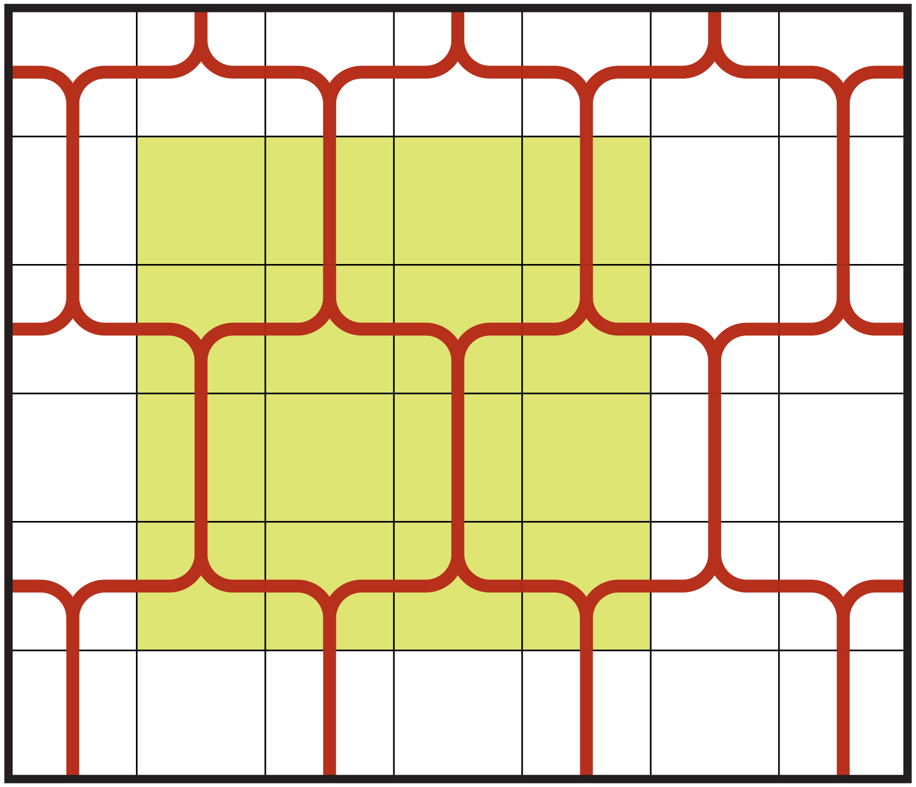
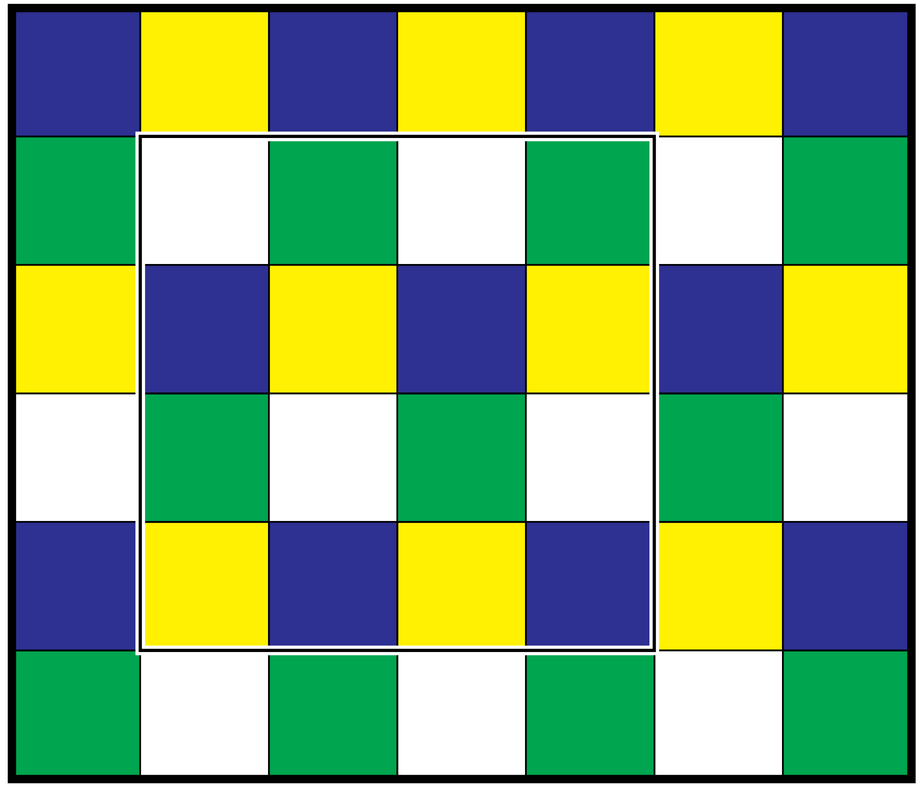
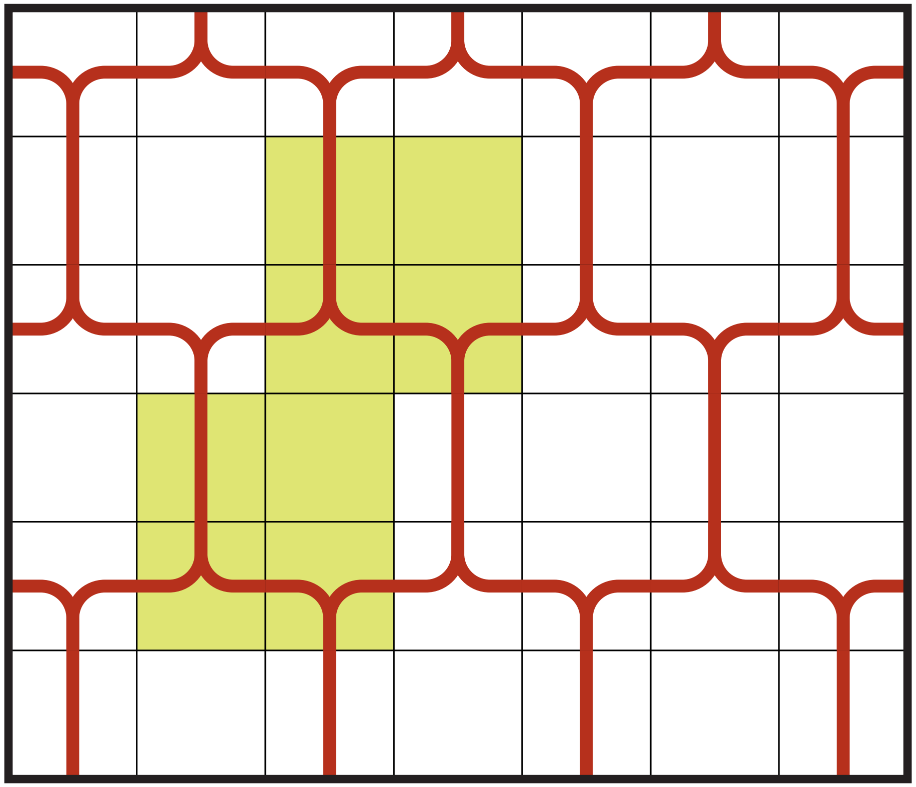
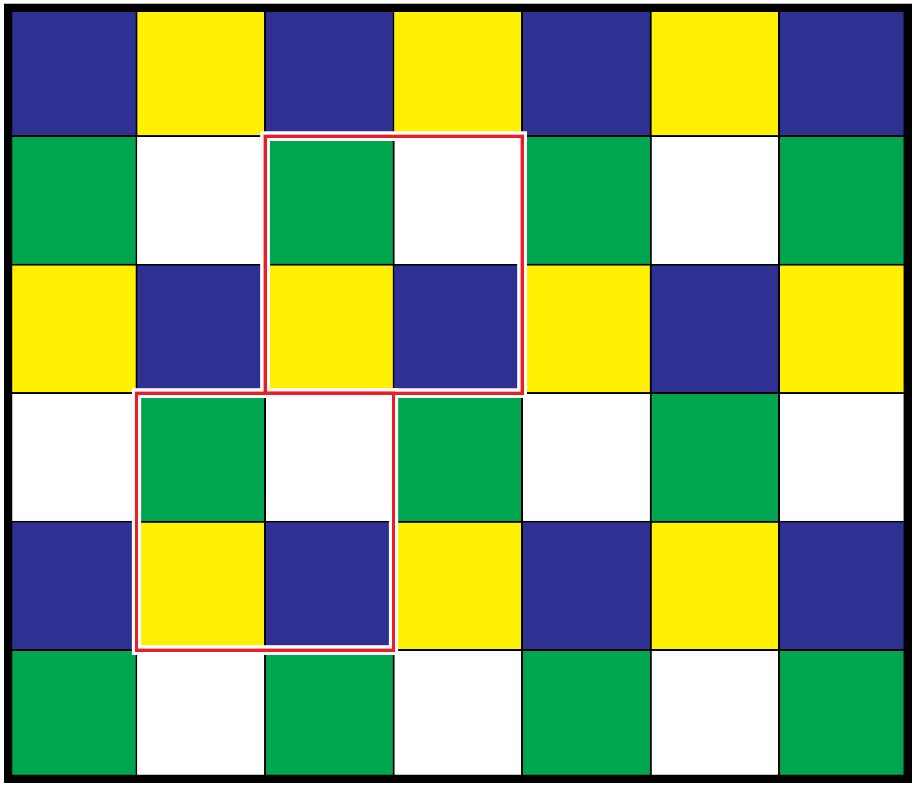
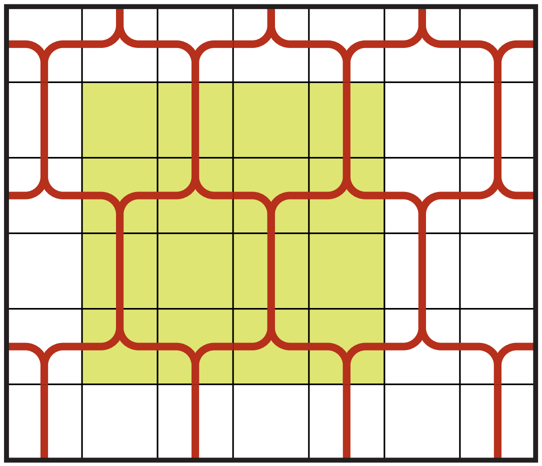


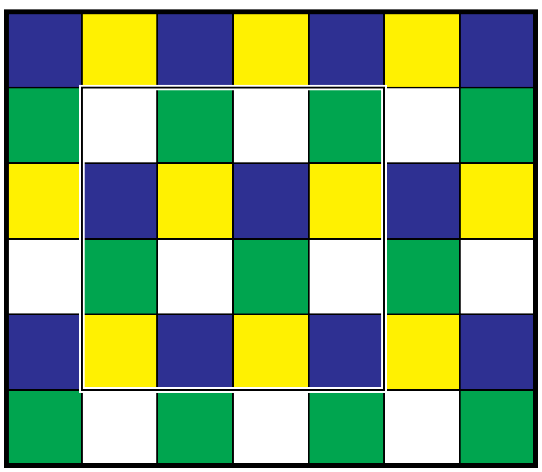


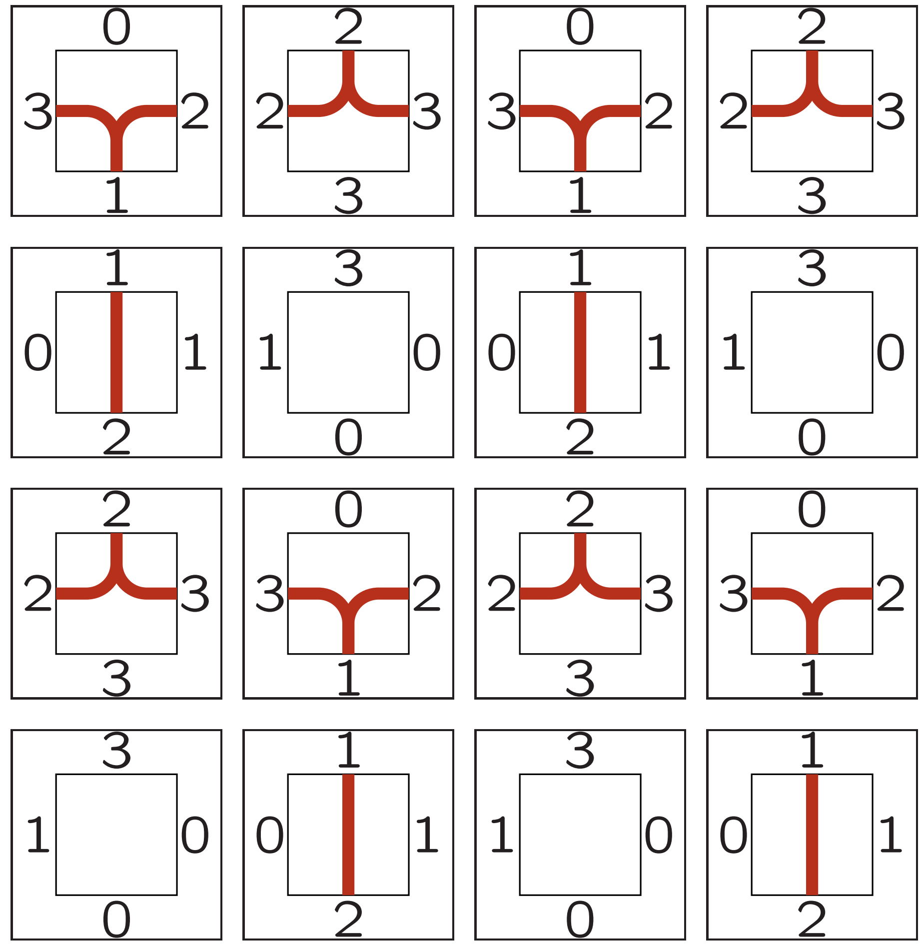






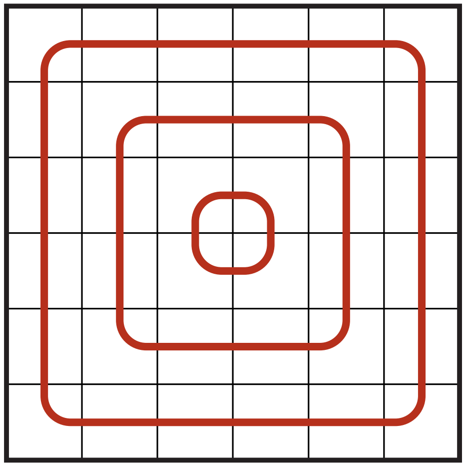


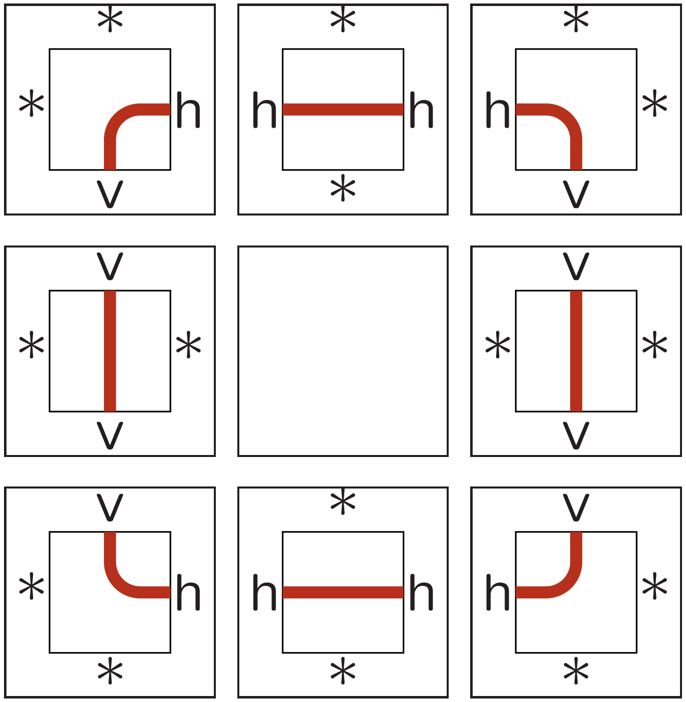




vier tegels

(en eventueel randjes)





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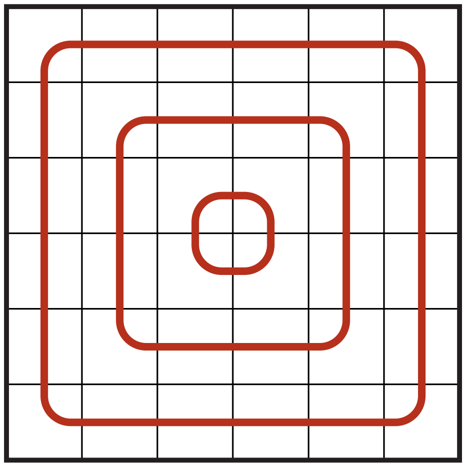

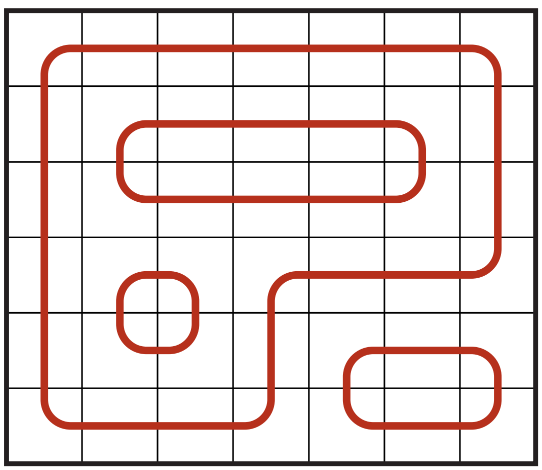

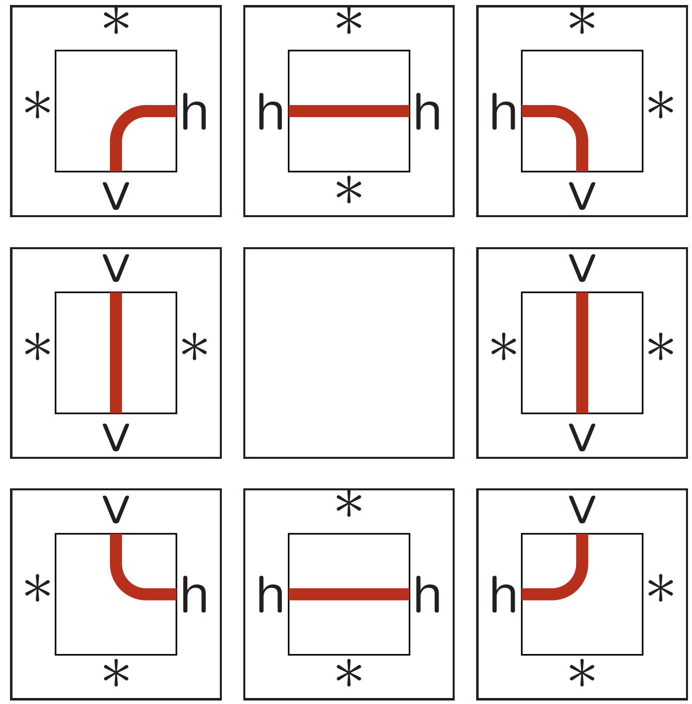

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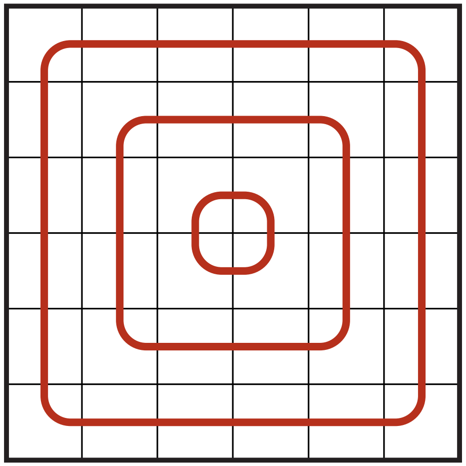

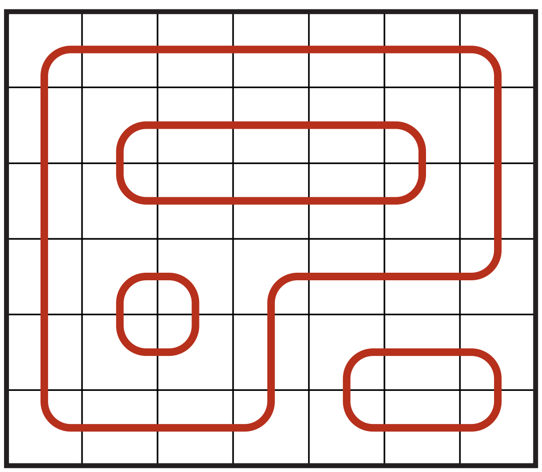

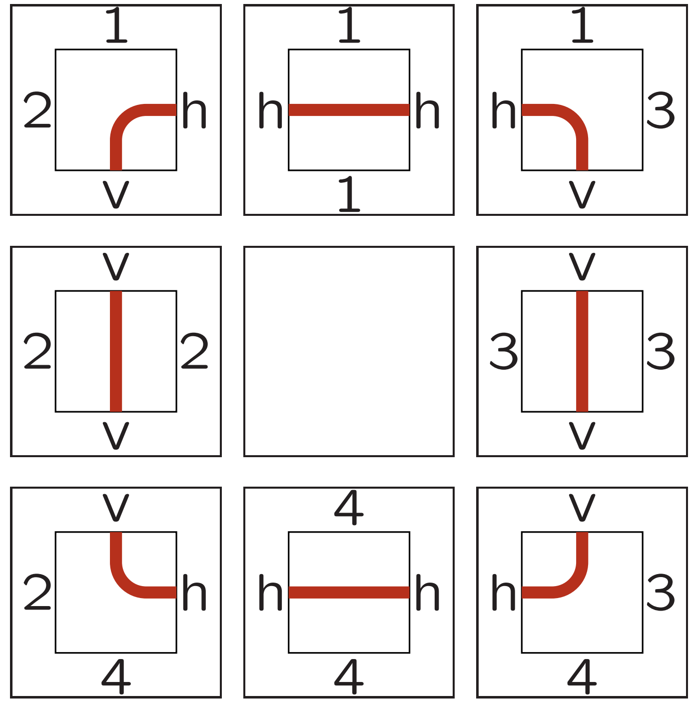

~~zes~~acht tegels

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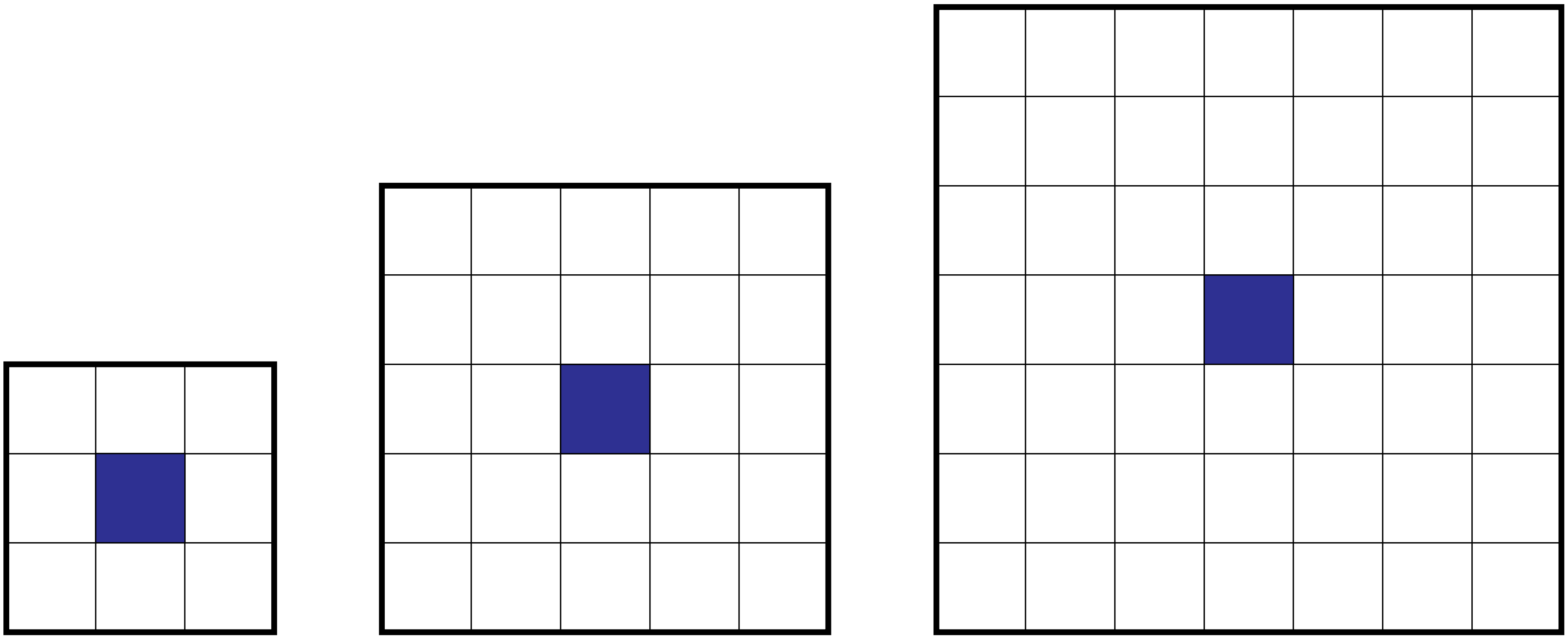
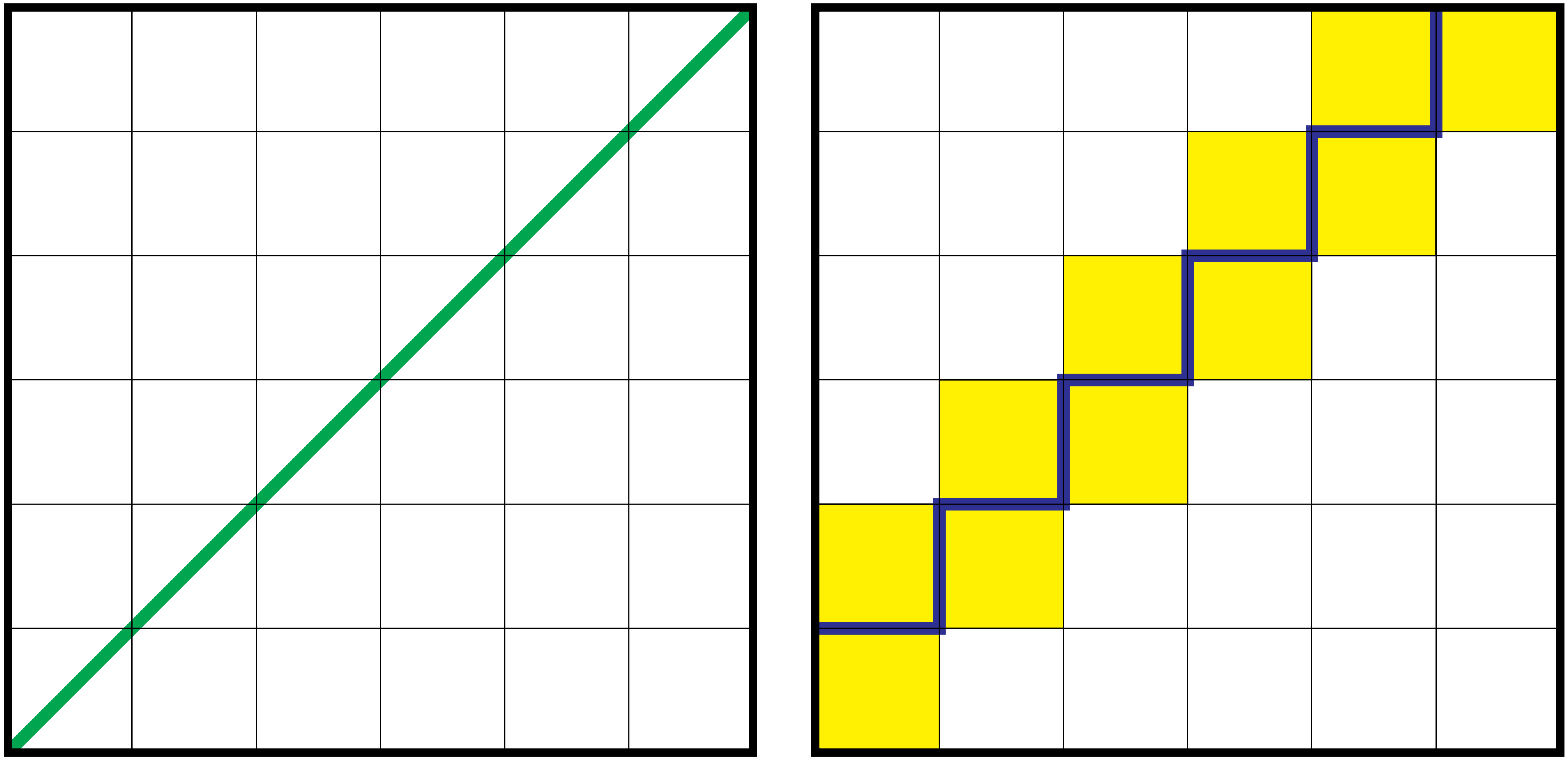
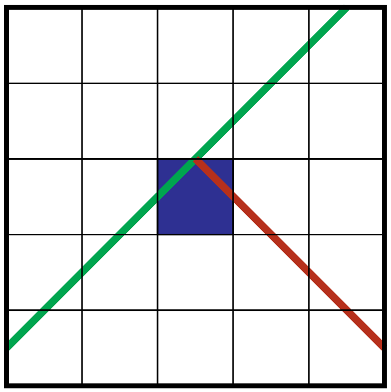
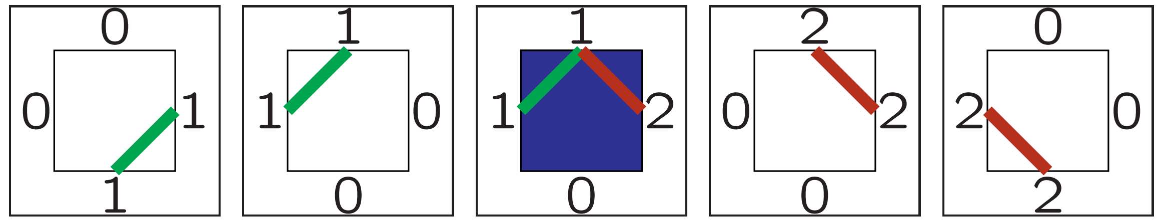
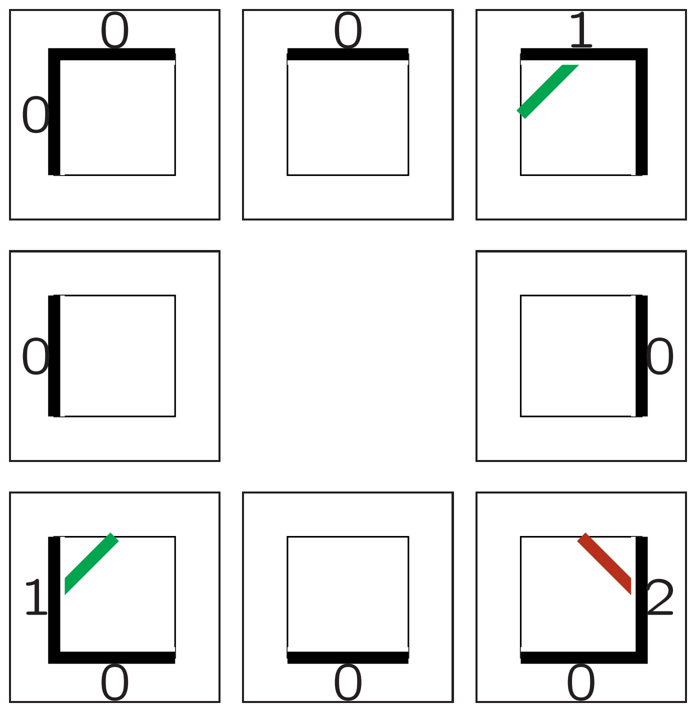


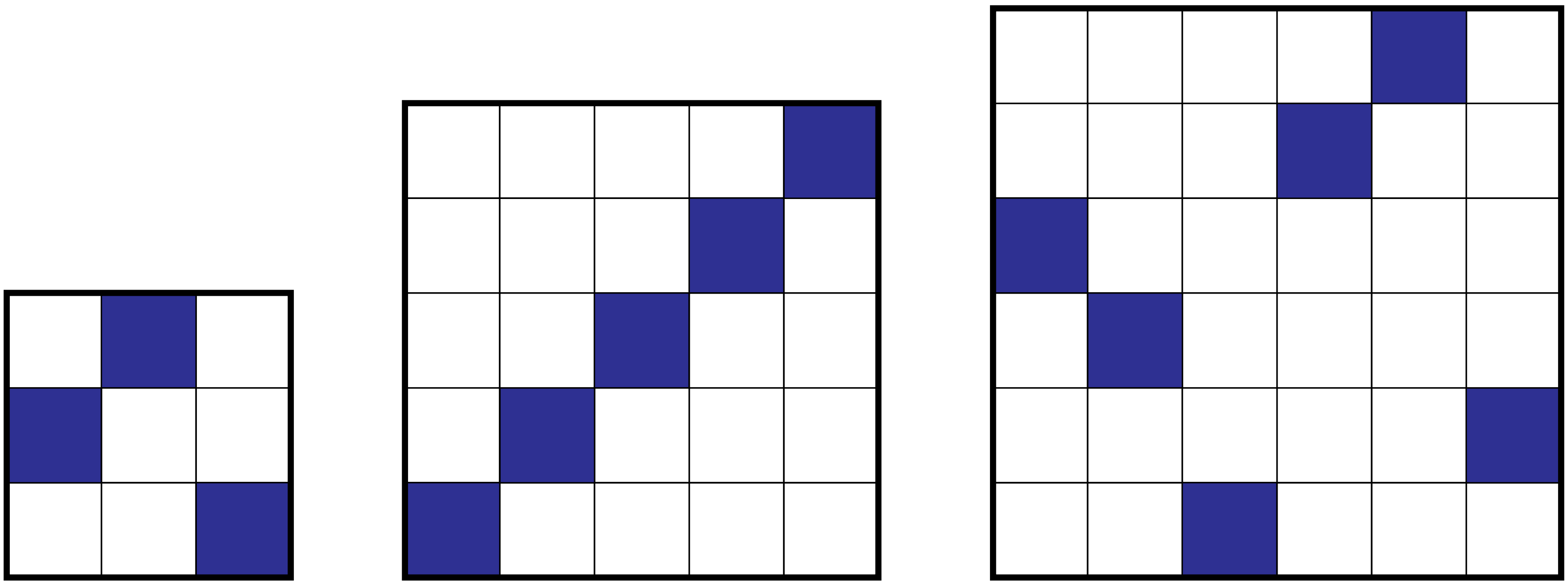


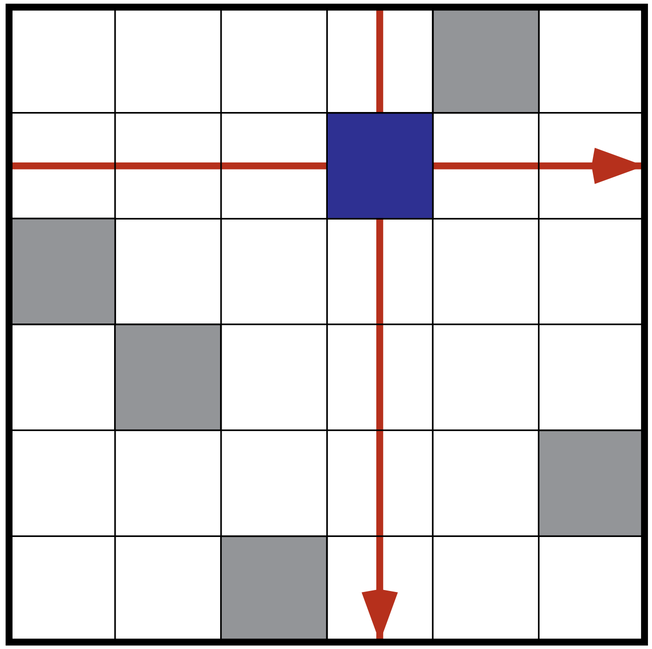


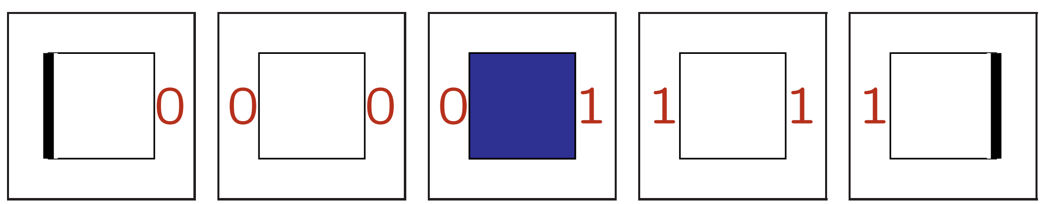


randen





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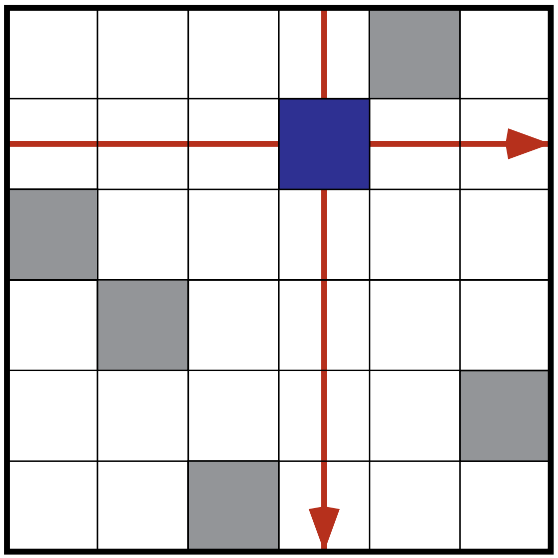
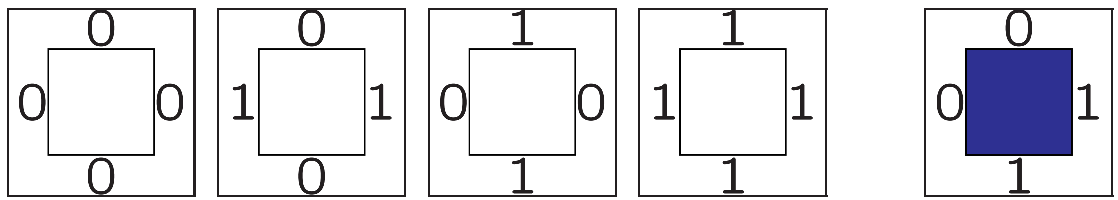
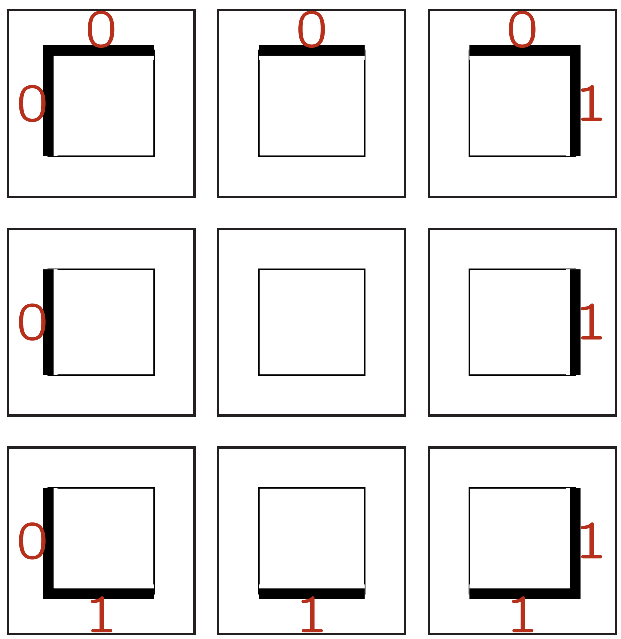


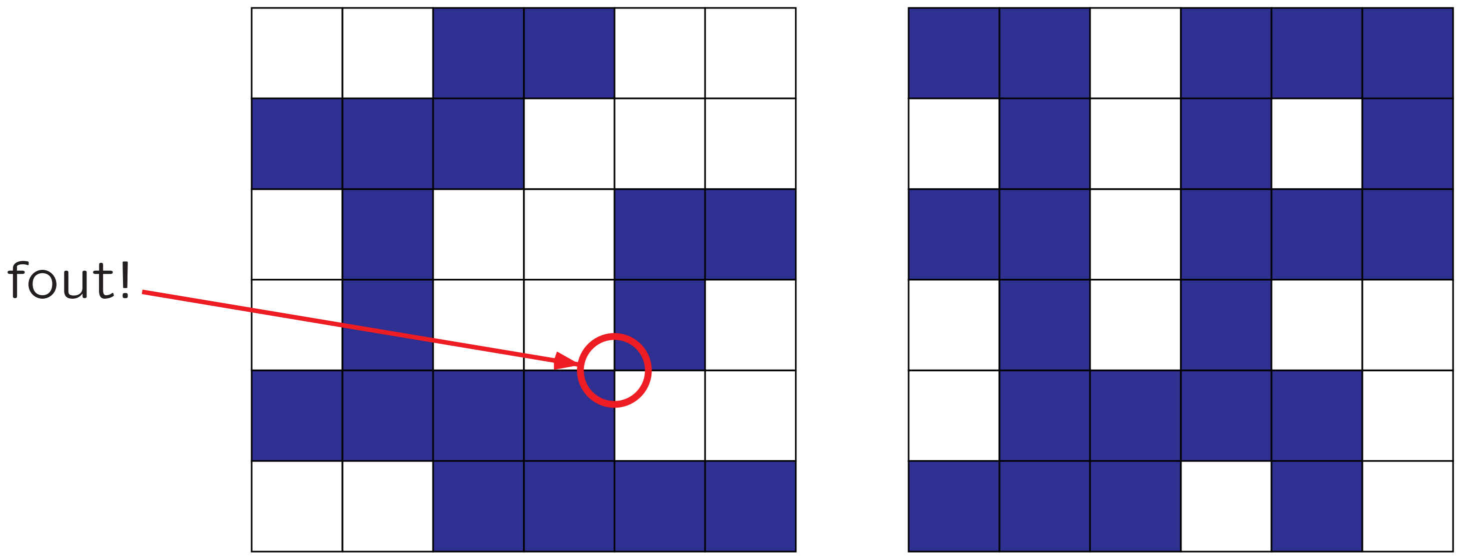


links/boven 0 rechts/onder 1

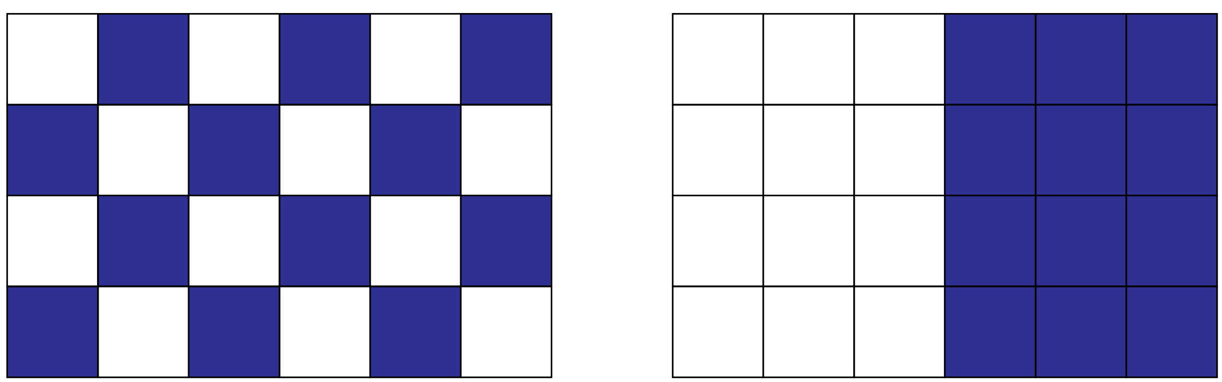


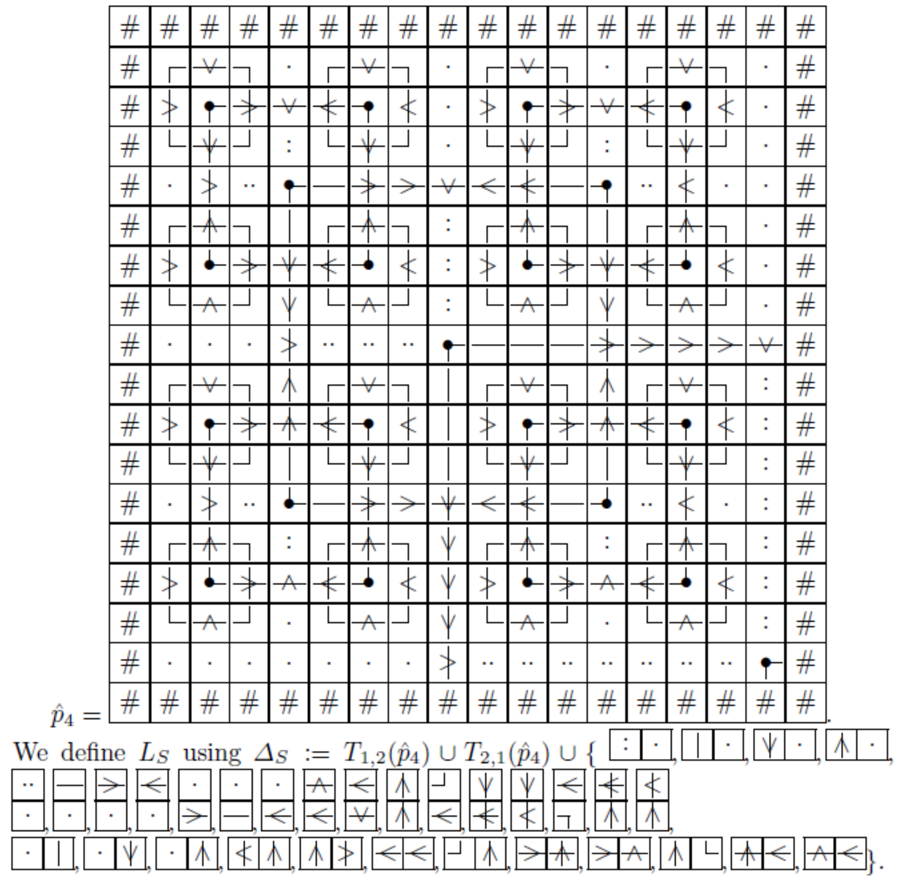
Hoeveel tegels bij twee torens per rij/kolom?

- alle blauwe tegels zijn verbonden (*moeilijk*)



- evenveel van beide kleuren (*heel moeilijk*)

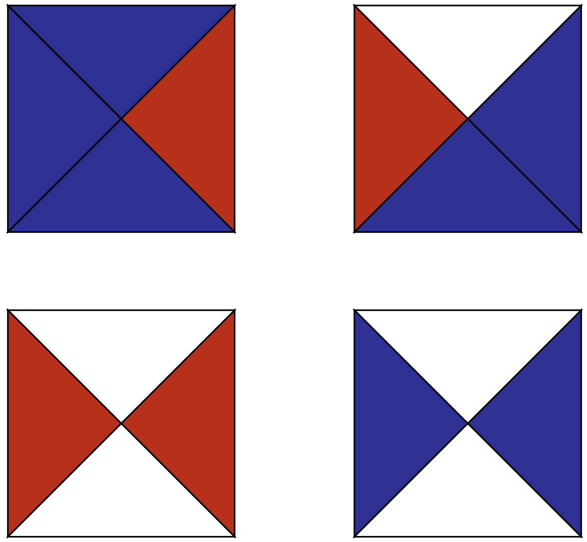


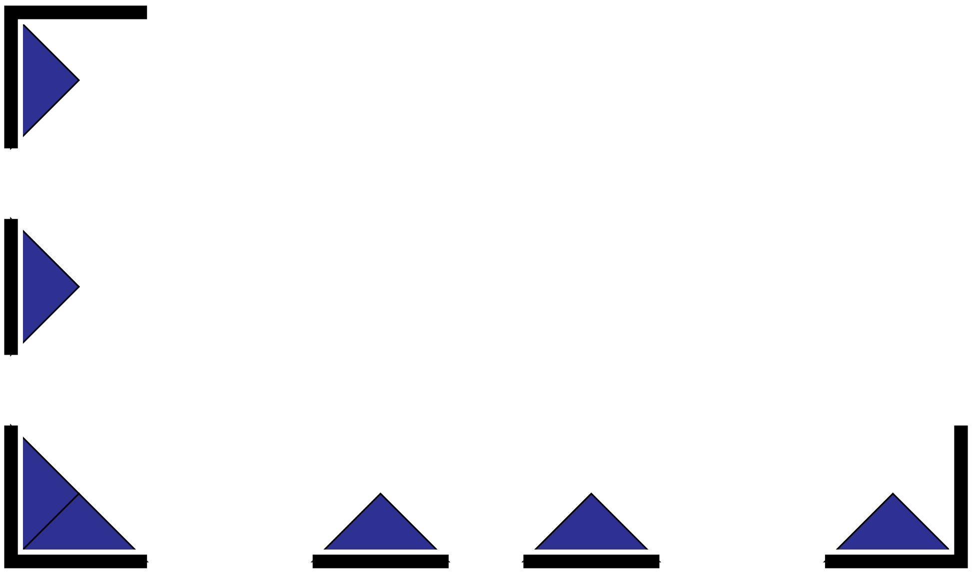


The $\#a = \#b$ Pictures are Recognizable

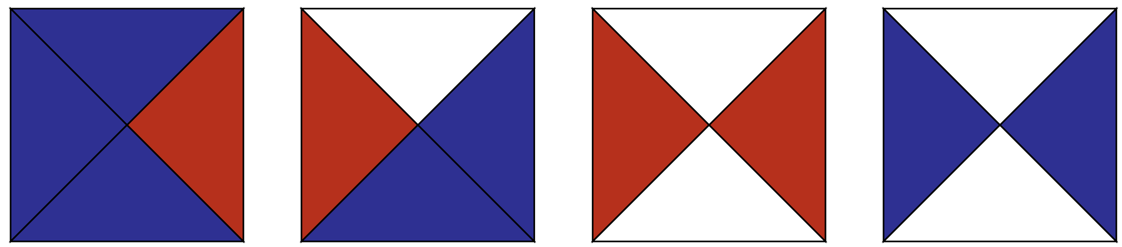
Klaus Reinhardt, STACS 2001, LNCS 2010

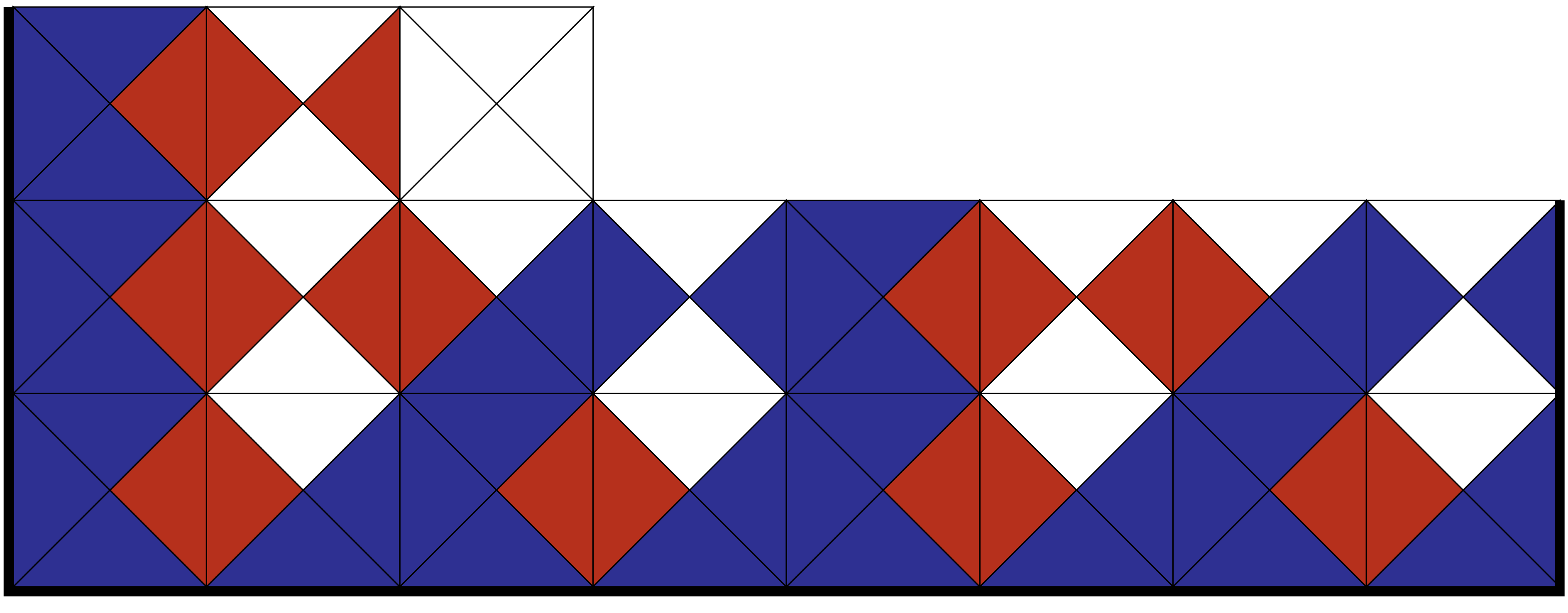
Rekenen

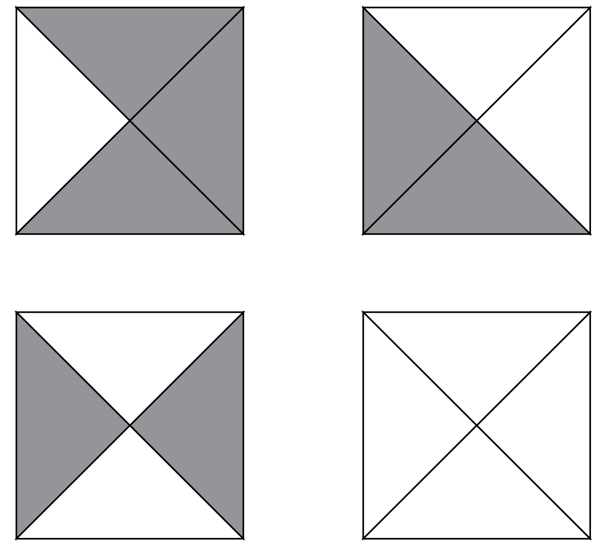


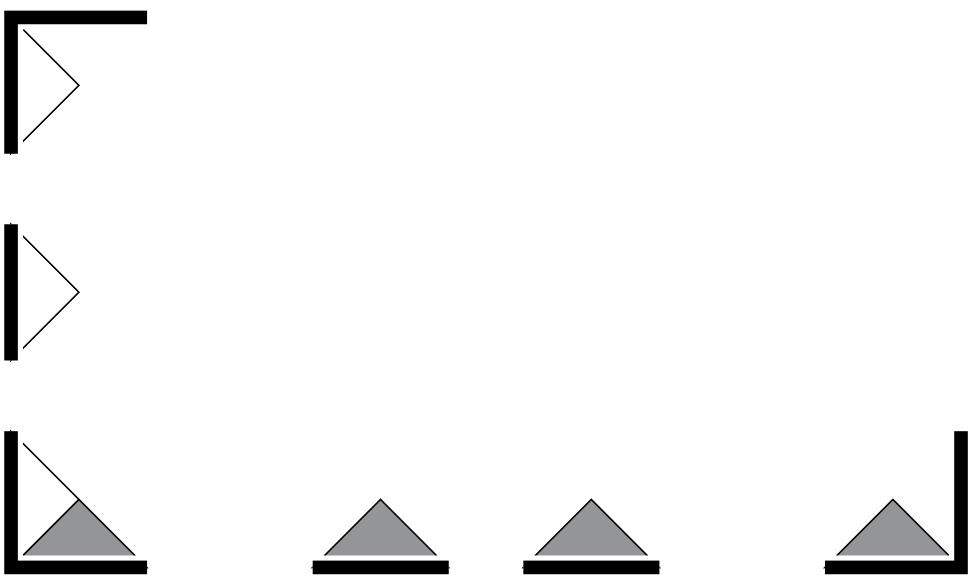


blauw onder en rechts
begin linksonder



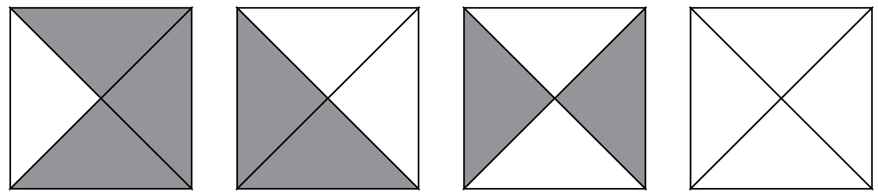


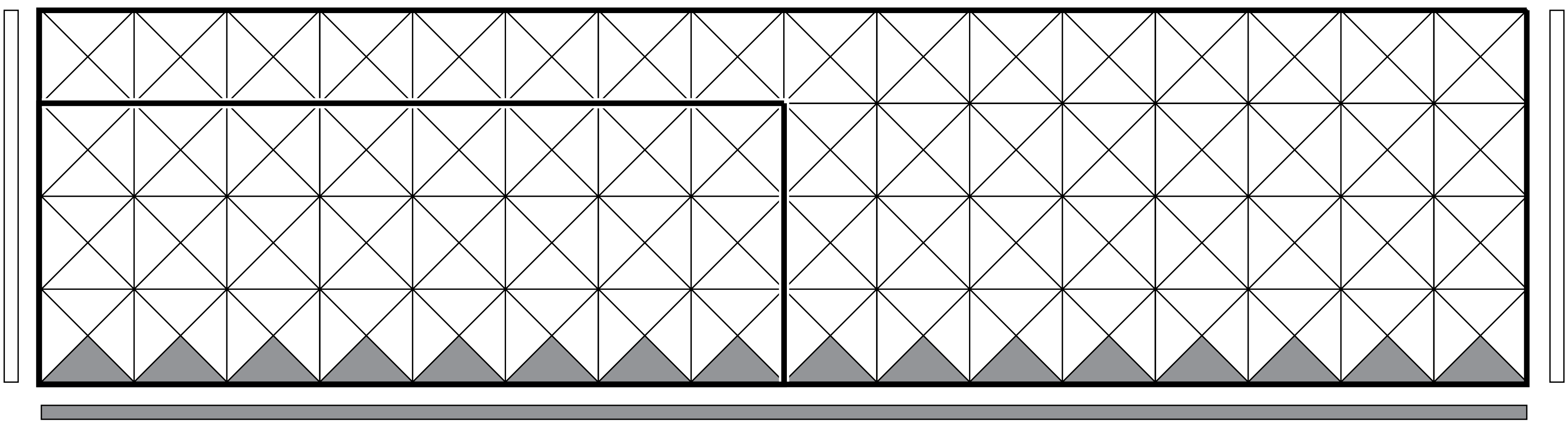


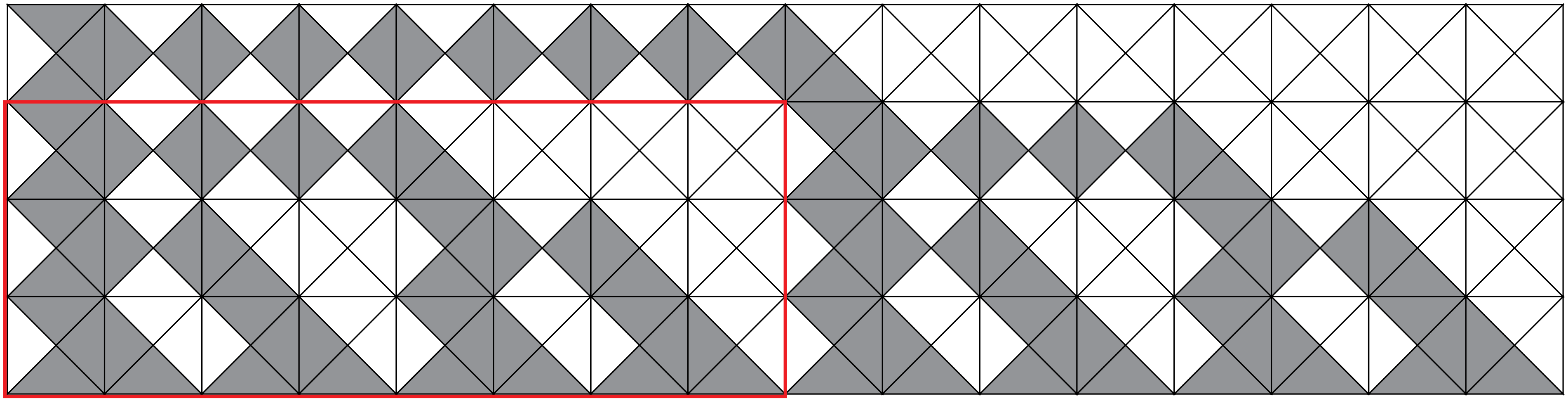


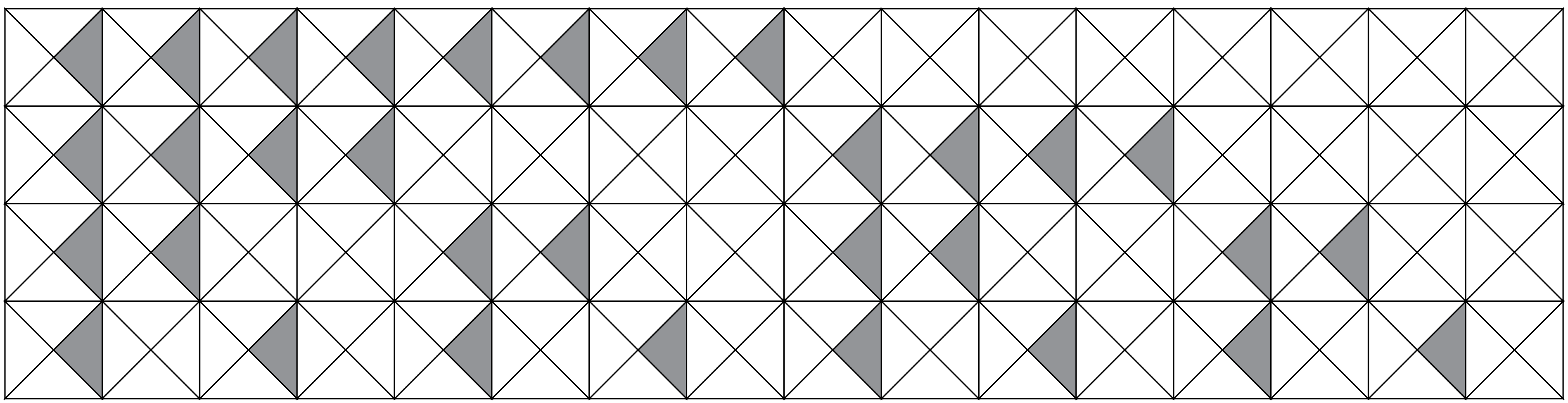
rechthoeken:
8×3 & 16×4

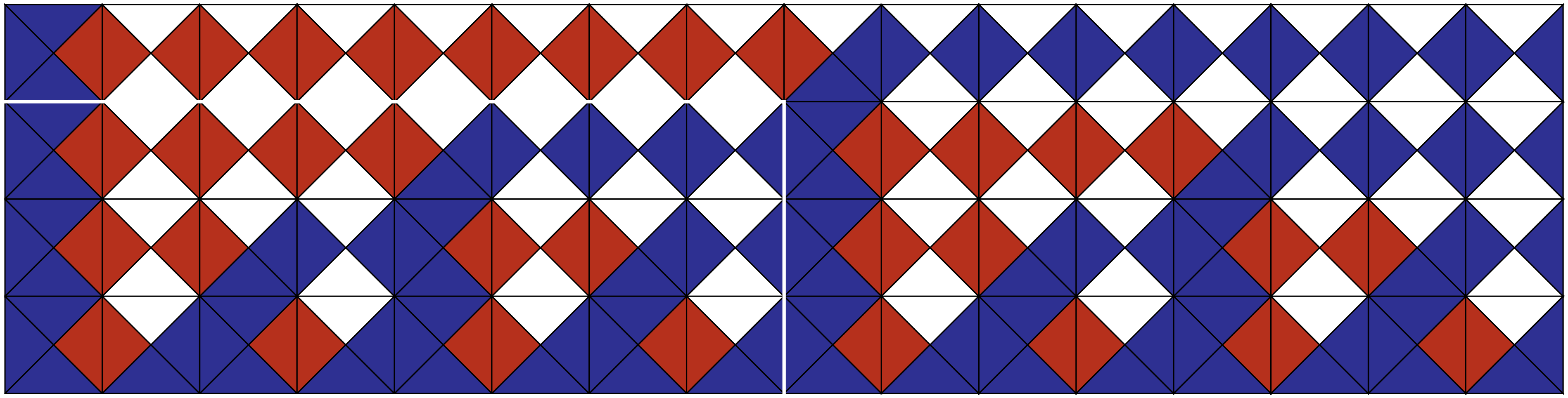
links&rechts wit, onder zwart

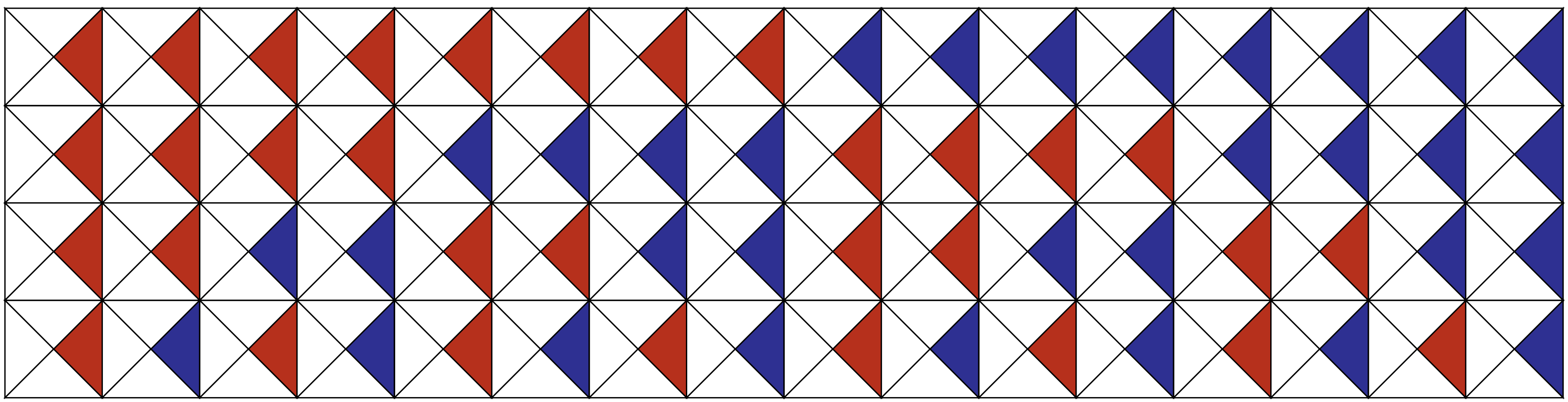












13	
6	1
3	0
1	1
0	1

tientallig: machten van tien

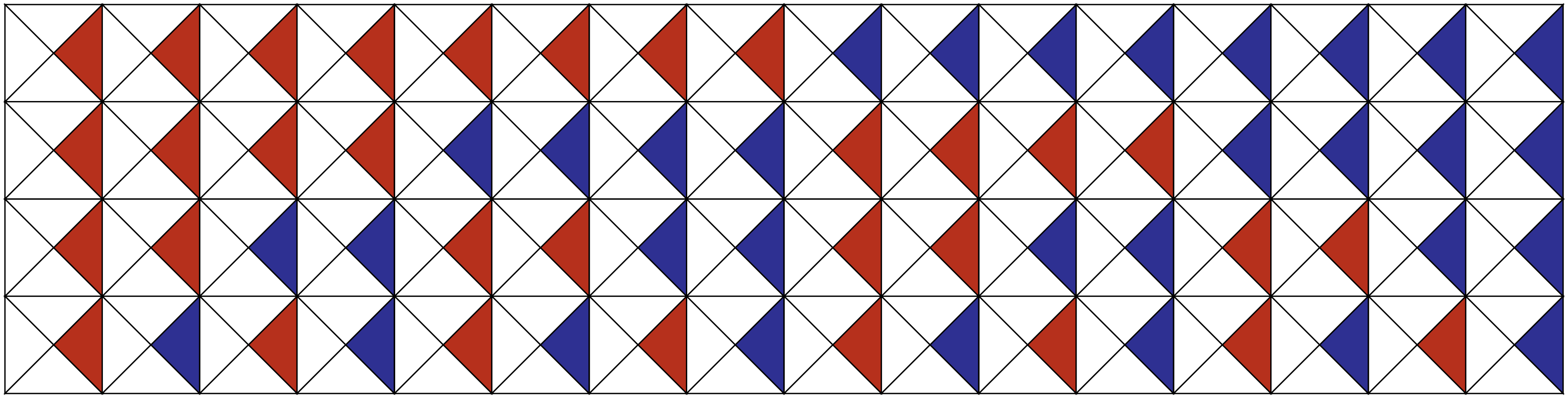
$3418_{10} = 3 \cdot 10^3 + 4 \cdot 10^2 + 1 \cdot 10^1 + 8 \cdot 10^0$

binair: machten van twee

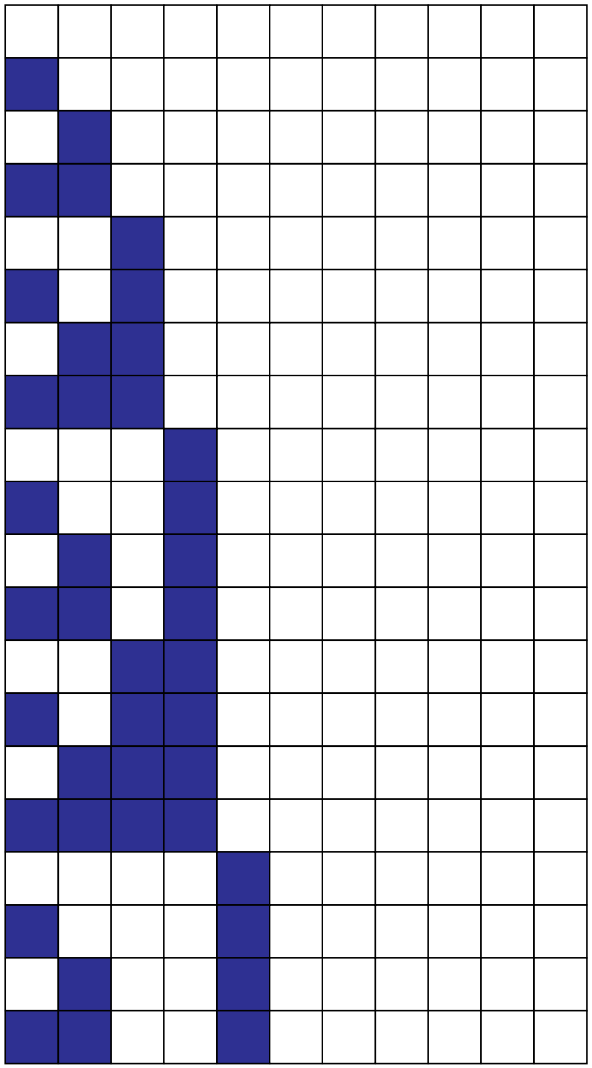
32, 16, 8, 4, 2, 1

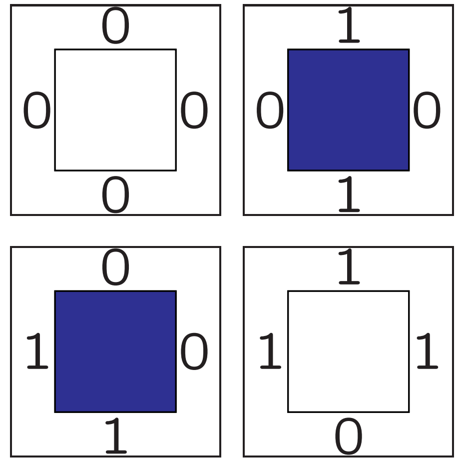
$1101_2 = 2^3 + 2^2 + 2^0 = 8 + 4 + 1 = 13$

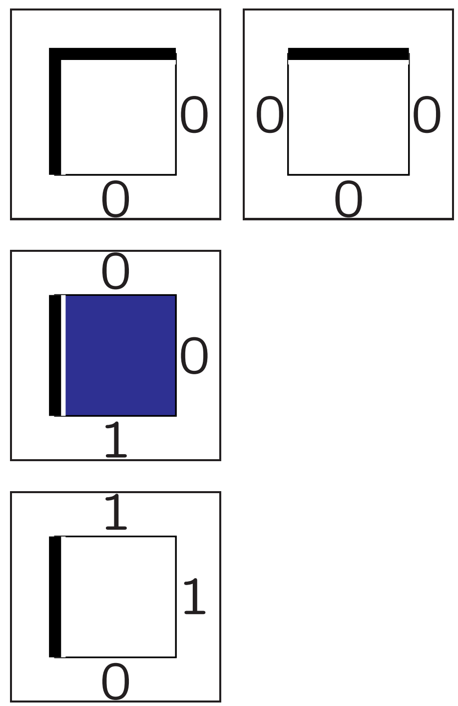
0, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111, 10000, . . .



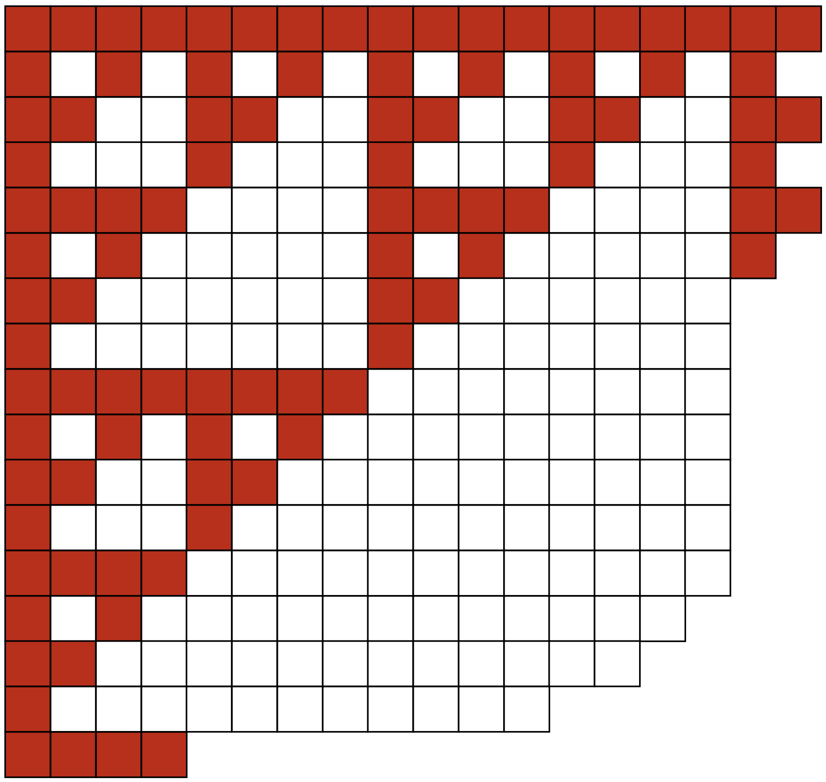
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

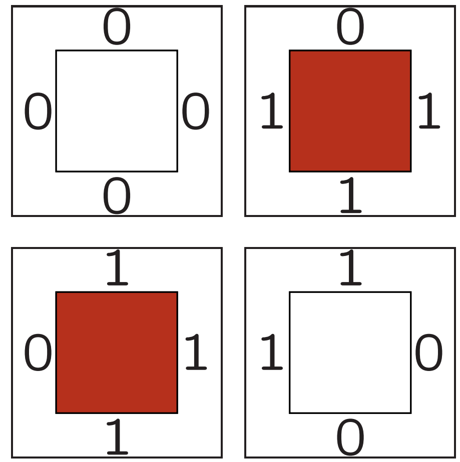


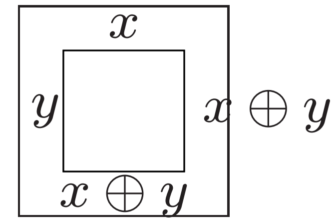

0 0 0 0
1 0 0 1
0 1 0 1
1 1 1 0


0 0
0 0 0
0 0 1
1 1 0

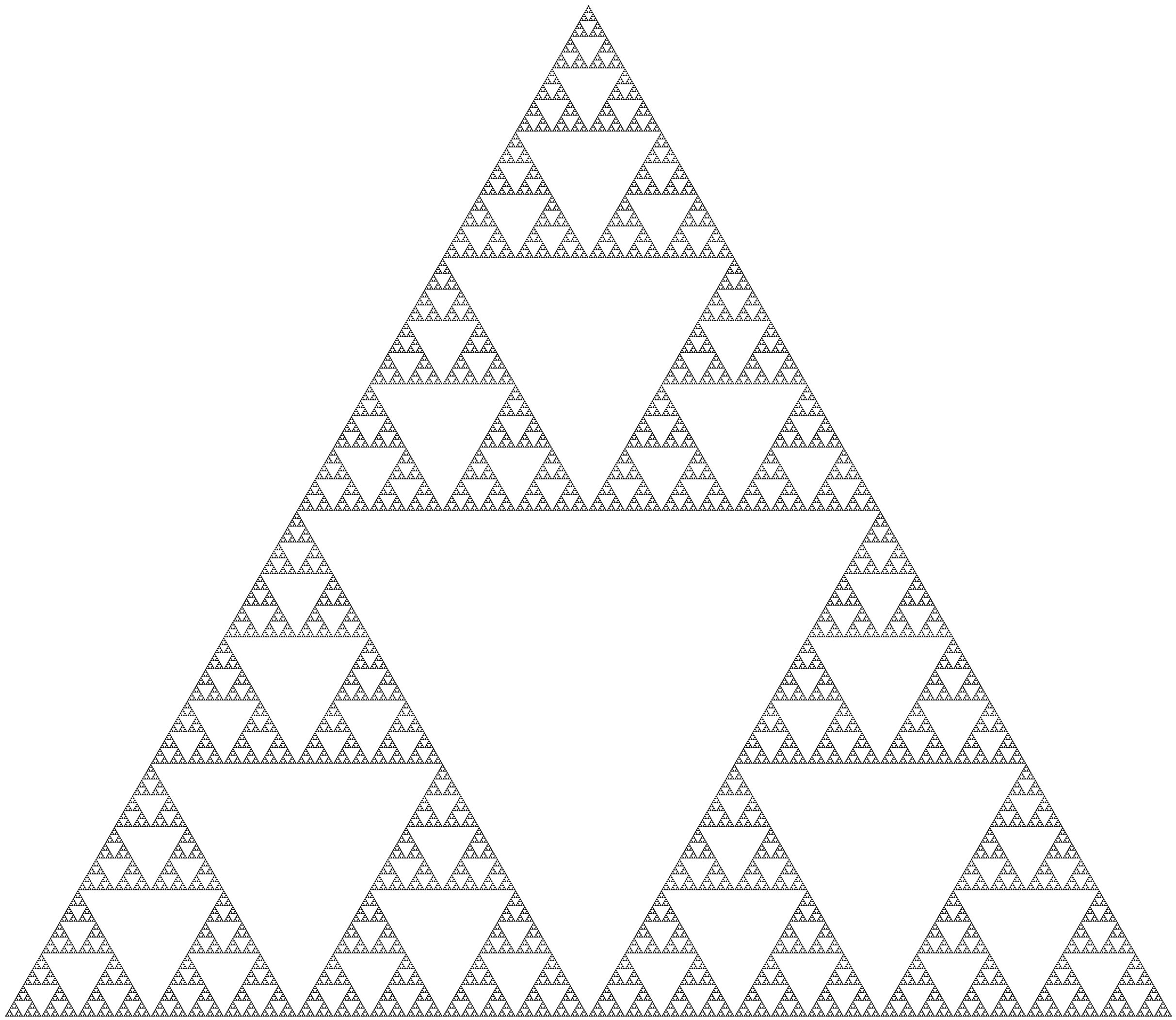
Self-Assembly





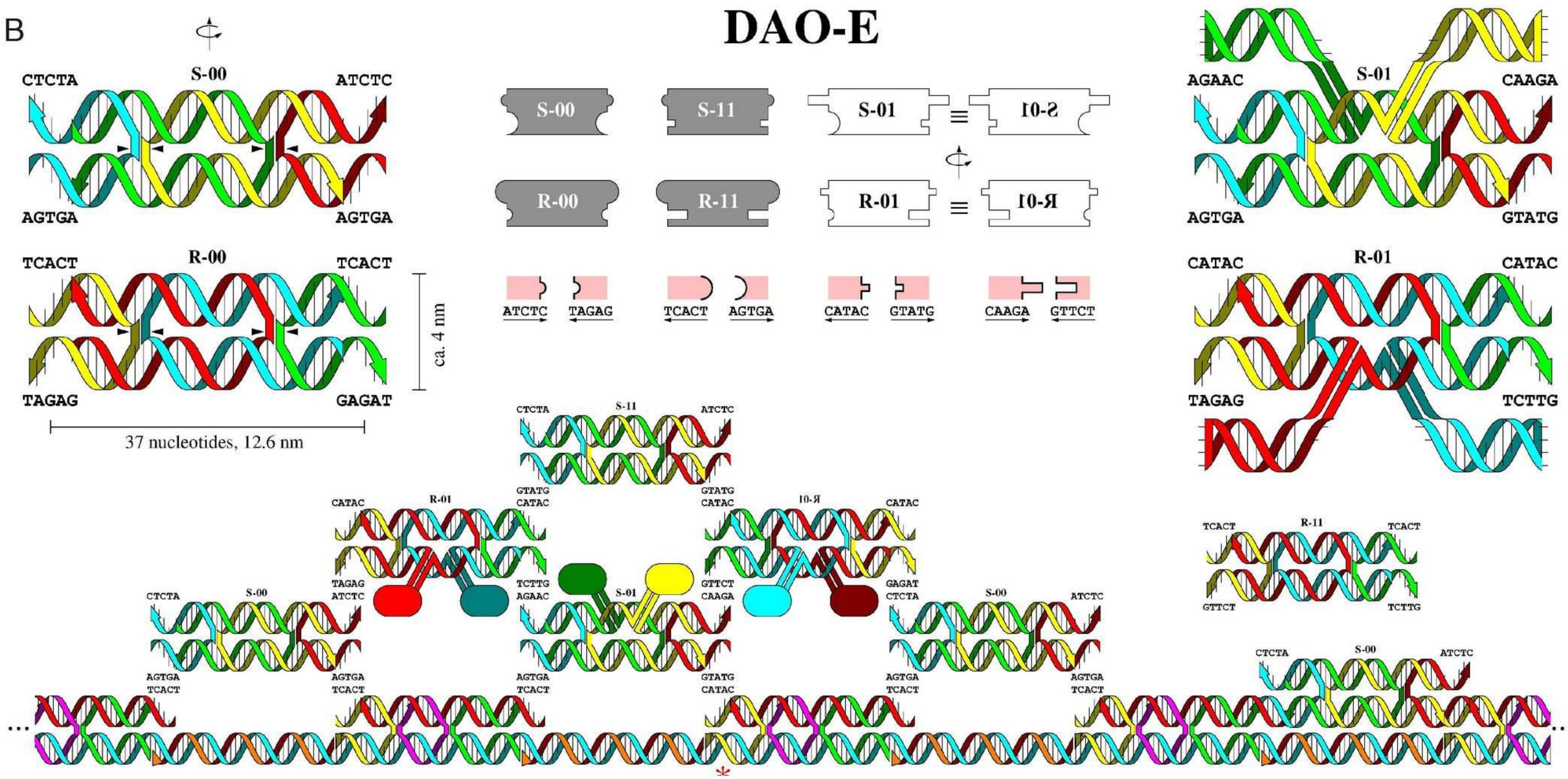


x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0



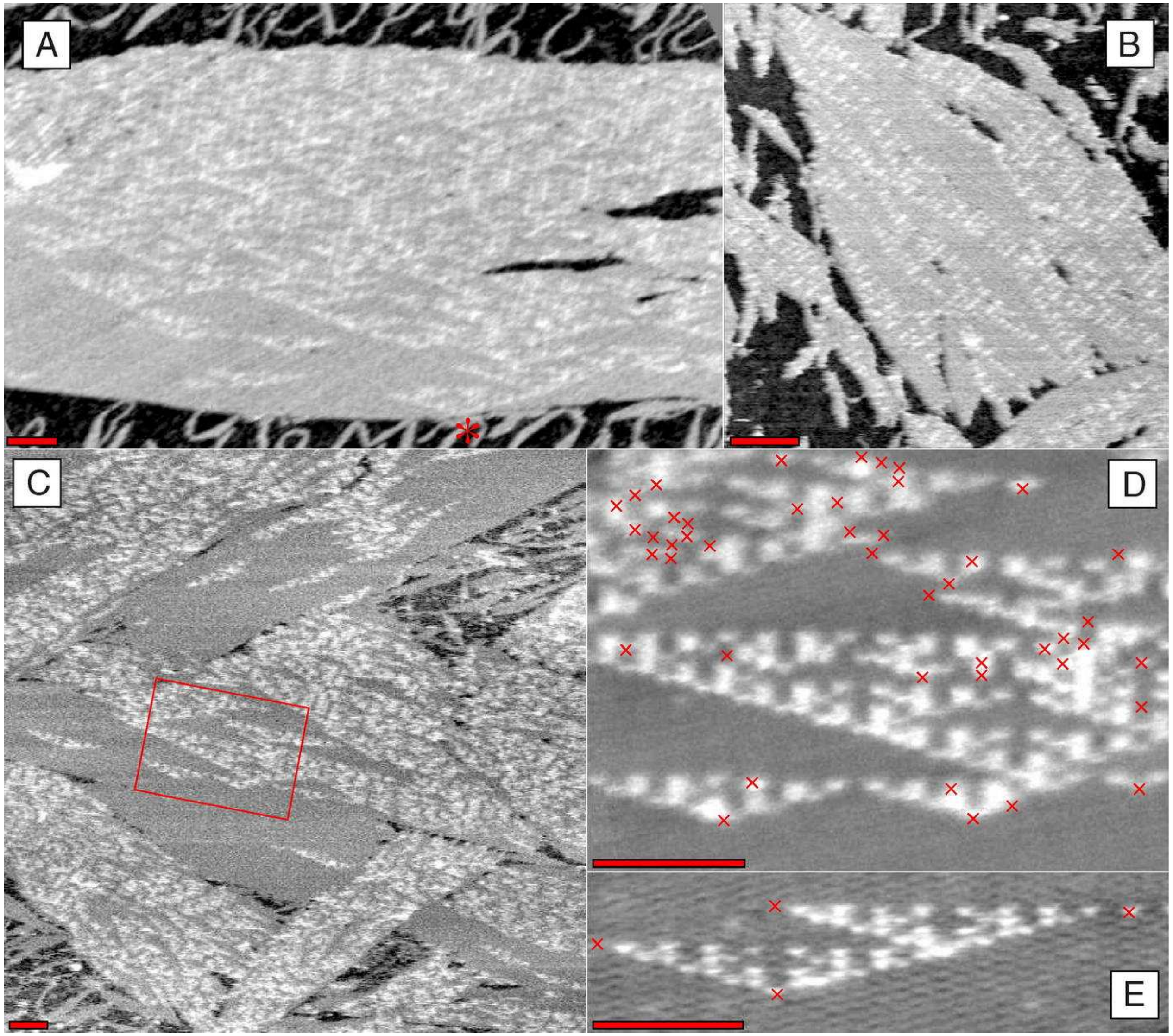
Qef's Website

wikipedia



Algorithmic Self-Assembly of DNA Sierpinski Triangles (2004)

Rothemund, Papadakis, Winfree; PLoS Biology


A
B
C
D
E

en.wikipedia.org

© Peter Ruoff, Stavanger

Conway's game of life
2-dim cellulaire automaat

Afsluiting

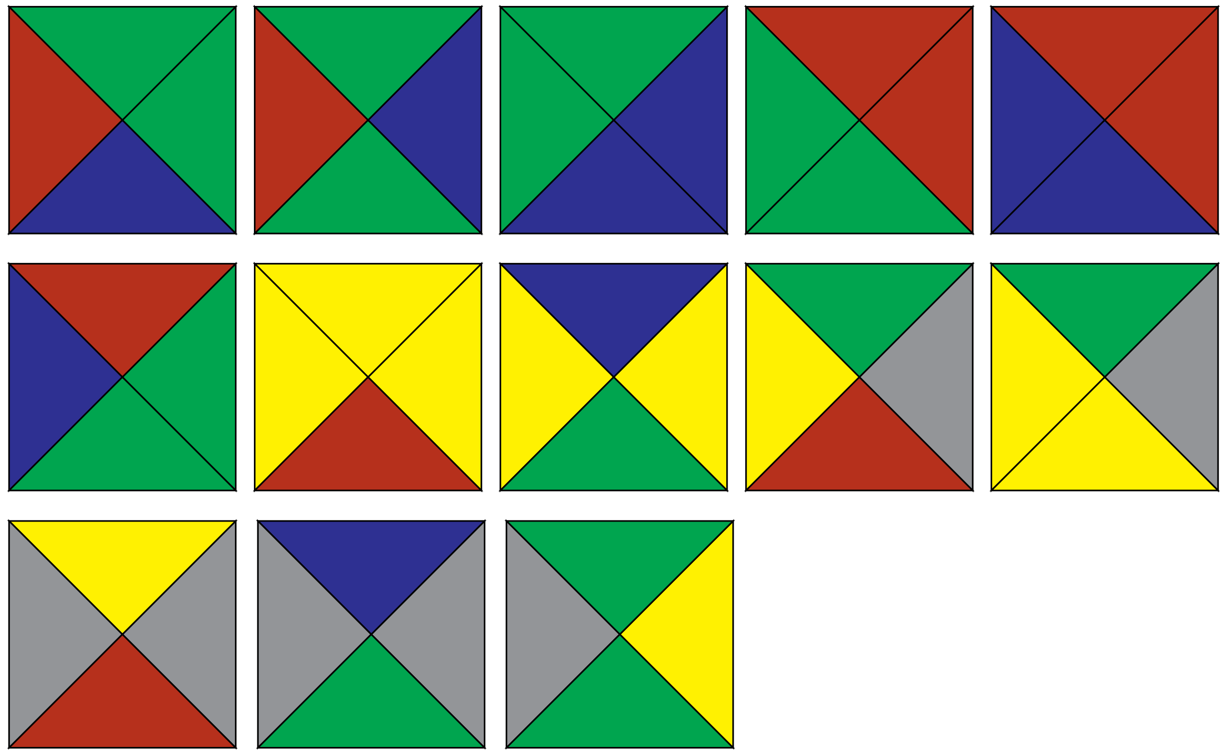
Alan Mathison Turing,
FRS OBE, 1912 – 1954

computability
wát kunnen we berekenen?
Turingmachine

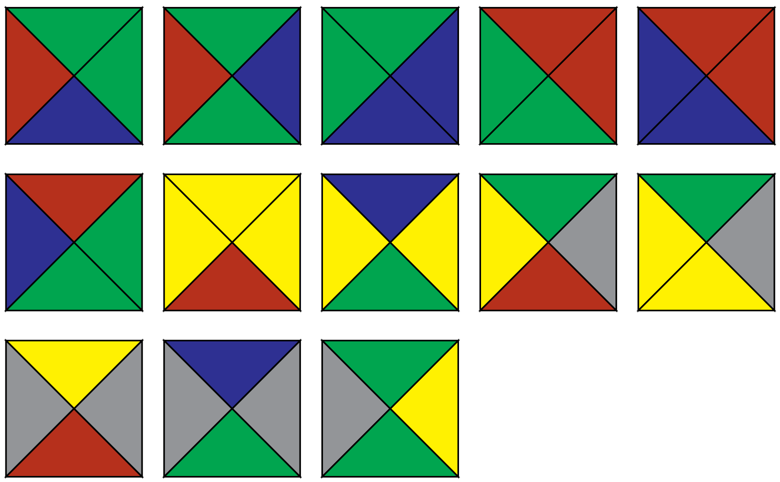
Enigma
breaking the code

artificial intelligence
Turing test

morfogenese
biologische patroonvorming



Wang tiles, 1961



patroon zonder regelmaat (lastig)

Karel Culik II, 1996

invoer: verzameling tegels

gevraagd: bestaat er een passende betegeling van het vlak (van een rechthoek) ?

er is géén algoritme dat dit probleem oplost

Berger 1966

(echt niet!)