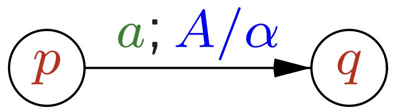
Famous Automata

(Examples / Exercises)

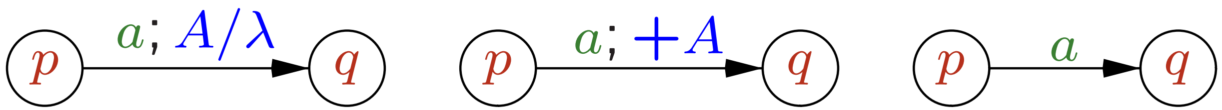
general form $(\, p, a, A, q, \alpha \,)$

$$(p, a, A) \mapsto (q, \alpha)$$



intuitive	*formalized as*	
pop A	$(\, p, a, A, q, \lambda \,)$	$\alpha = \lambda$
push A	$(\, p, a, X, q, AX \,)$	for all $X \in \Gamma$
read a	$(\, p, a, X, q, X \,)$	for all $X \in \Gamma$

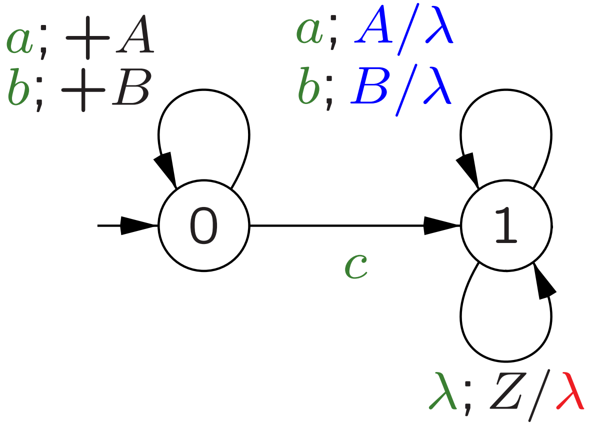
our convention



empty stack

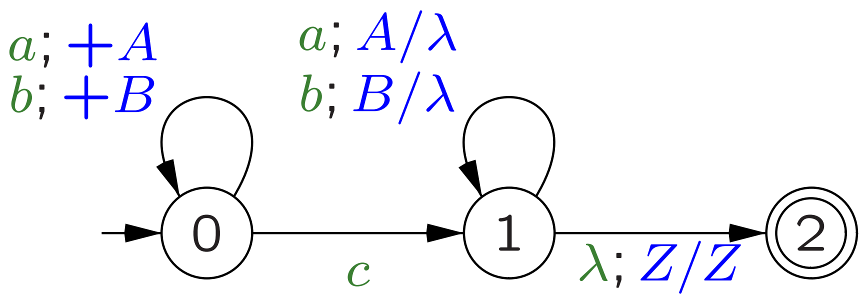
$N(\mathcal{A}_2) = L$

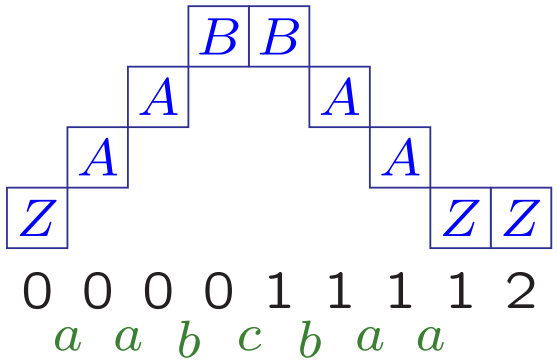
$$L = \{\, w\, c\, w^R \mid w \in \{a,b\}^* \,\}$$



final state

$L(\mathcal{A}_1) = L$

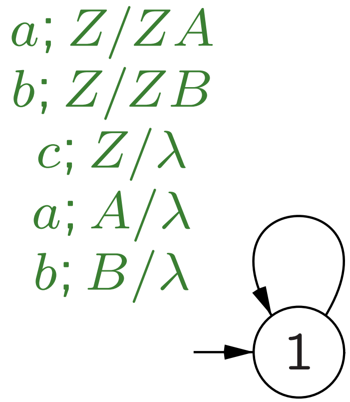


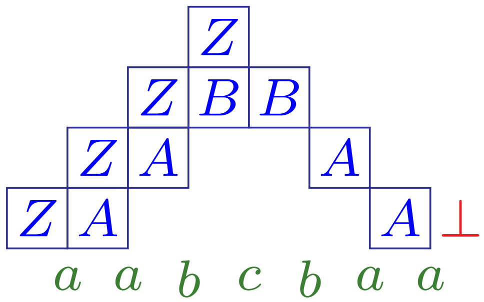


$$\begin{array}{l}
(0, aabcbaa, Z) \vdash \\
(0, abcbaa, AZ) \vdash \\
(0, bcbaa, AAZ) \vdash \\
(0, cbaa, BAAZ) \vdash \\
(1, baa, BAAZ) \vdash \\
(1, aa, AAZ) \vdash \\
(1, a, AZ) \vdash \\
(1, \lambda, Z) \vdash \\
(2, \lambda, Z)
\end{array}$$

$$L = \{\ w\,c\,w^R \mid w \in \{a,b\}^*\ \}$$

single state :- stack codes state



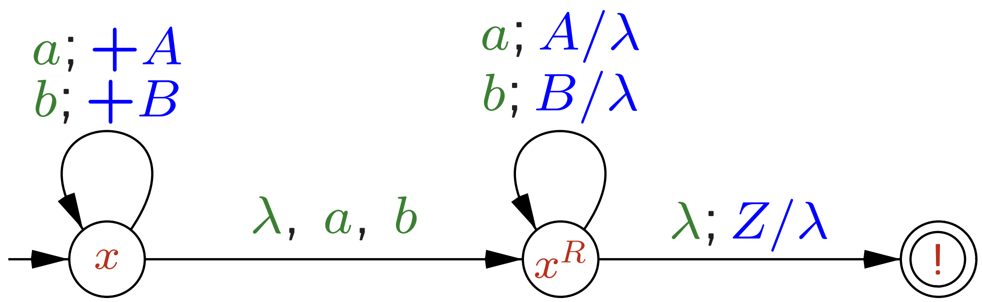


$$\begin{aligned}
&(1, aabcbaa, Z) \vdash \\
&(1, abcbaa, ZA) \vdash \\
&(1, bcbaa, ZAA) \vdash \\
&(1, cbaa, ZBAA) \vdash \\
&(1, baa, BAA) \vdash \\
&(1, aa, AA) \vdash \\
&(1, a, A) \vdash \\
&(1, \lambda, \lambda)
\end{aligned}$$

$$L = \{\ x \in \{a,b\}^* \mid x = x^R\ \}$$

x^R 'reverse', mirror image

even ww^R and odd lengths $w\sigma w^R$



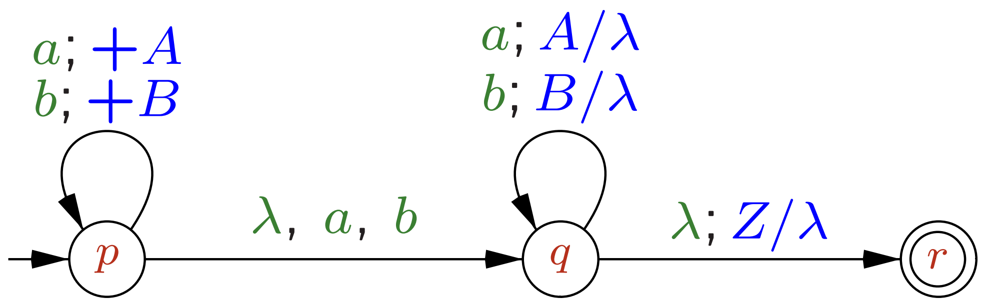
stacksymbols A, B, Z

final state and/or empty stack

(nondeterministic!)

$S \to aSa \mid bSb$

$S \to \lambda \mid a \mid b$



useful: $[\pi, A, q]\ [\pi, B, q]\ [\pi, Z, r] \qquad \pi \in \{p, q\}$

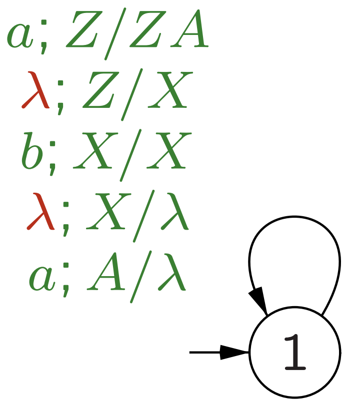
$$
\begin{array}{lll}
(p, a, Z, p, AZ) & [p, Z, r] \to a\,[p, A, q][q, Z, r] & \\
(p, a, X, p, AX) & [p, X, q] \to a\,[p, A, q][q, X, q] & X \in \{A, B\} \\
(p, b, Z, p, BZ) & [p, Z, r] \to b\,[p, B, q][q, Z, r] & \\
(p, b, X, p, BX) & [p, X, q] \to b\,[p, B, q][q, X, q] & X \in \{A, B\} \\
(p, \sigma, Z, q, Z) & [p, Z, r] \to \sigma\,[q, Z, r] & \sigma \in \{\lambda, a, b\} \\
(p, \sigma, X, q, X) & [p, X, q] \to \sigma\,[q, X, q] & \\
(q, a, A, q, \lambda) & [q, A, q] \to a & \\
(q, b, B, q, \lambda) & [q, B, q] \to b & \\
(q, \lambda, Z, r, \lambda) & [q, Z, r] \to \lambda &
\end{array}
$$

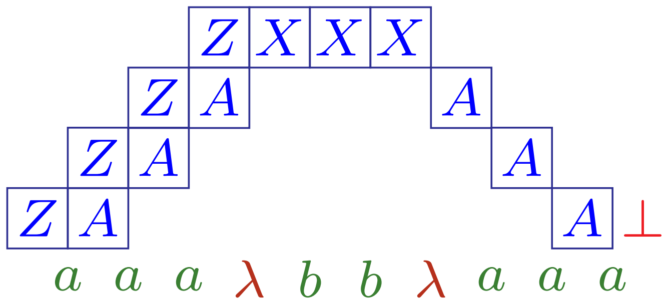
rename & substitute

$[p, Z, r] \rightsquigarrow Z$
$[p, A, q] \rightsquigarrow A$
$[p, B, q] \rightsquigarrow B$
$[q, A, q] \rightsquigarrow a$
$[q, B, q] \rightsquigarrow b$
$[q, Z, r] \rightsquigarrow \lambda$

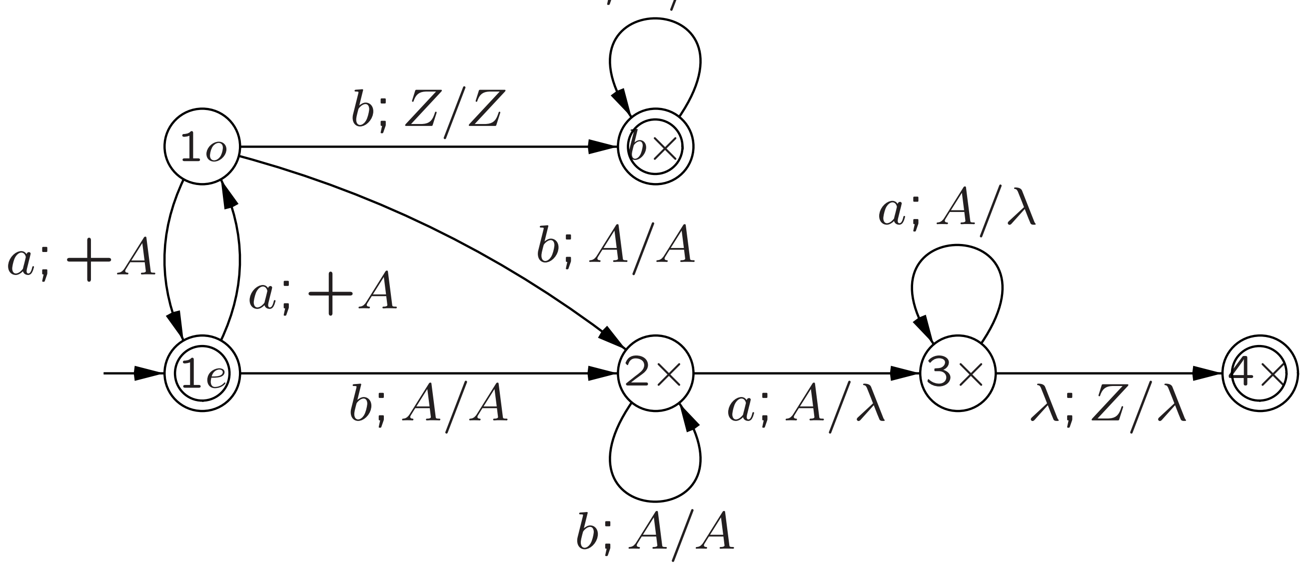
$Z \to aA,\ A \to aAa,\ B \to aAb,$
$Z \to bB,\ A \to bBa,\ B \to bBb,$
$Z \to \lambda \mid a \mid b$
$A \to a \mid aa \mid ba$
$B \to b \mid ab \mid bb$

$$L = \{\ a^nb^ma^n \mid m, n \in \mathbb{N}\ \}$$





deterministic:

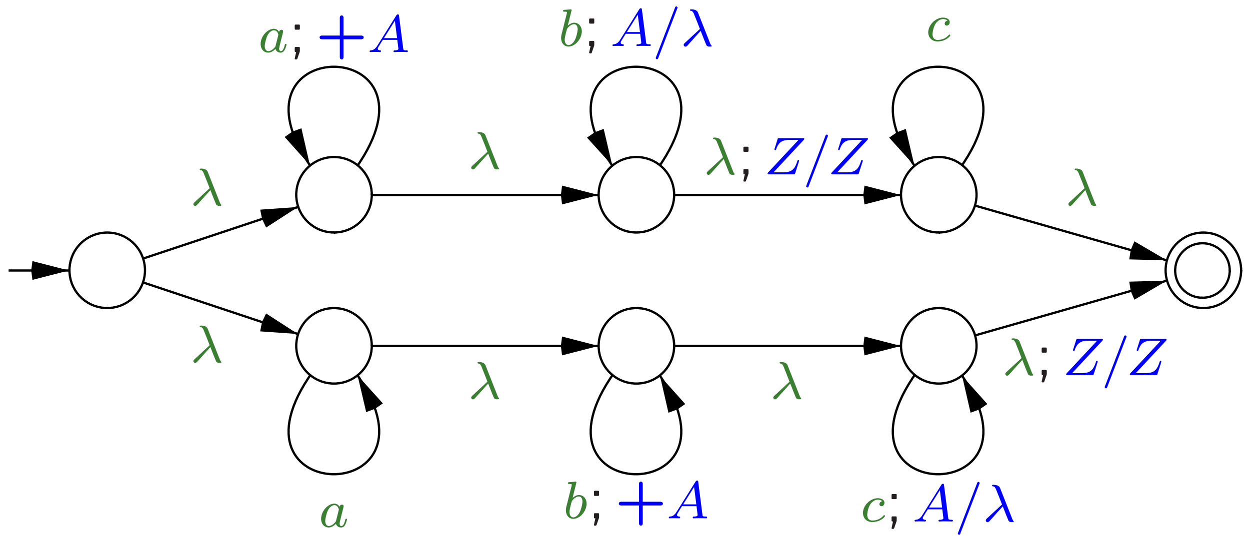


see:

Closure and Determinism

$$L = \{\ a^i b^j c^k \mid i = j \text{ or } j = k\ \}$$

choose what to push



initial stack symbol Z

$S \rightarrow AB \mid DC$

$A \rightarrow aA \mid \lambda$

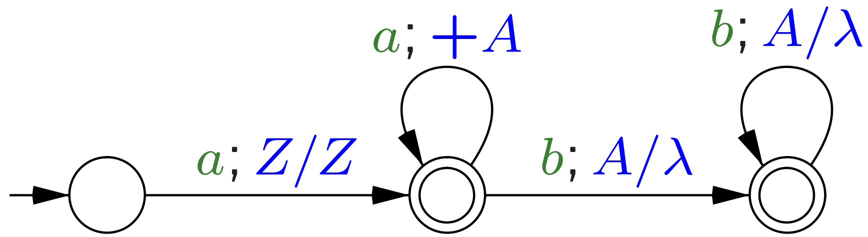
$B \rightarrow bBc \mid \lambda$

$C \rightarrow cC \mid \lambda$

$D \rightarrow aDb \mid \lambda$

$$L = \{\ a^m b^n \mid m > n\ \}$$

first a is not ‘counted’



initial stack symbol Z

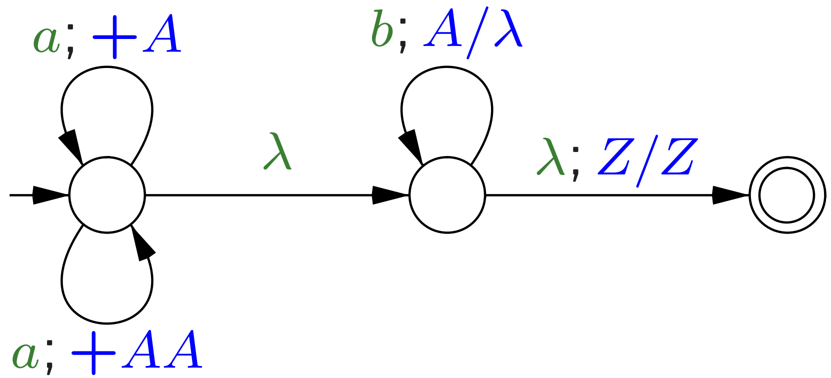
$S \to aSb \mid aS \mid a$

‘blocks’ on $a^m b^n$, $m \leq n$

deterministic

$$L = \{ a^m b^n \mid m \le n \le 2m \}$$

nondeterministic

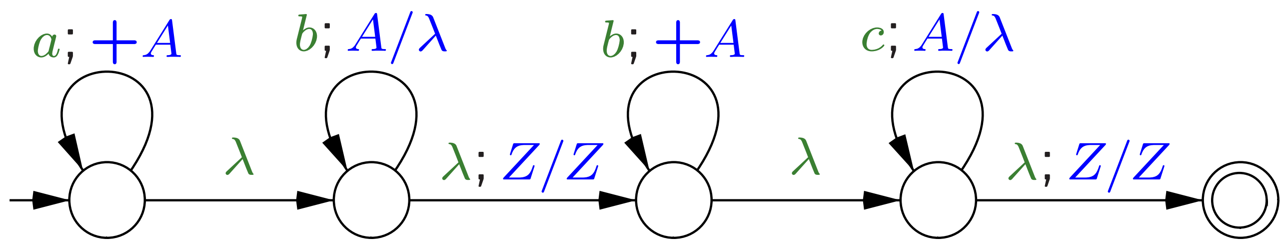


$S \to aSbb \mid aSb \mid \lambda$

initial stack symbol Z

$$L = \{ a^ib^jc^k \mid j = i + k \}$$
$$= \{ a^ib^i\, b^kc^k \mid i, k \in \mathbb{N} \}$$

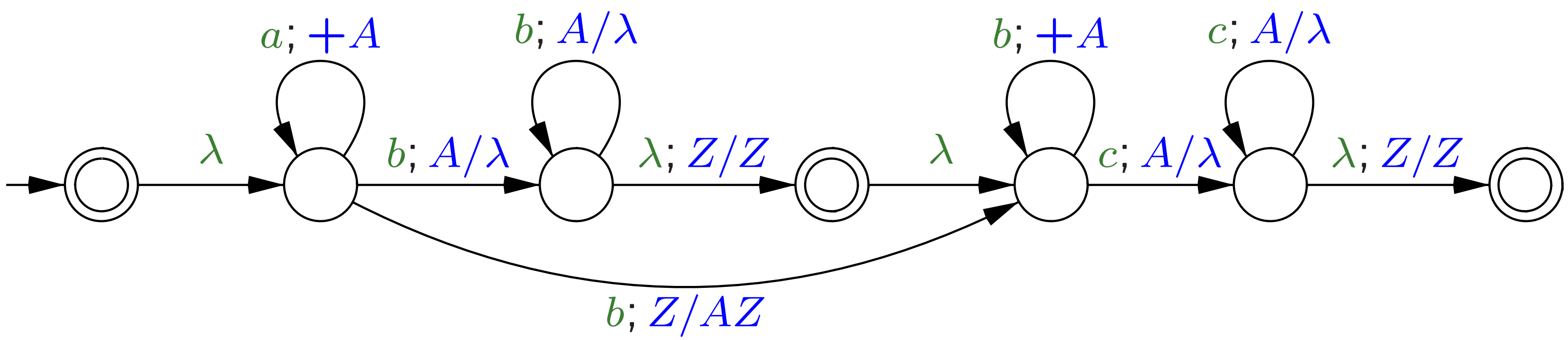
counting . . .



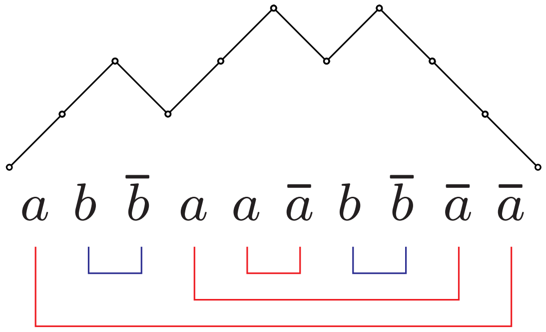
$S \to AB$
$A \to aAb \mid \lambda$
$B \to bBc \mid \lambda$

initial stack symbol Z

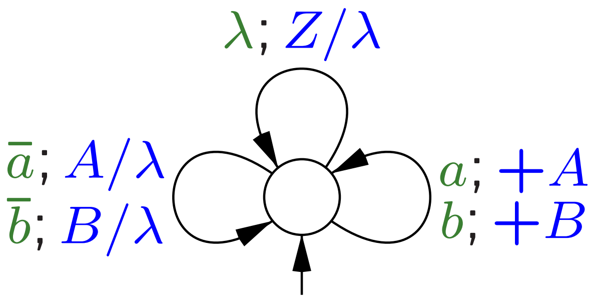
now deterministic (mind λ! etc)

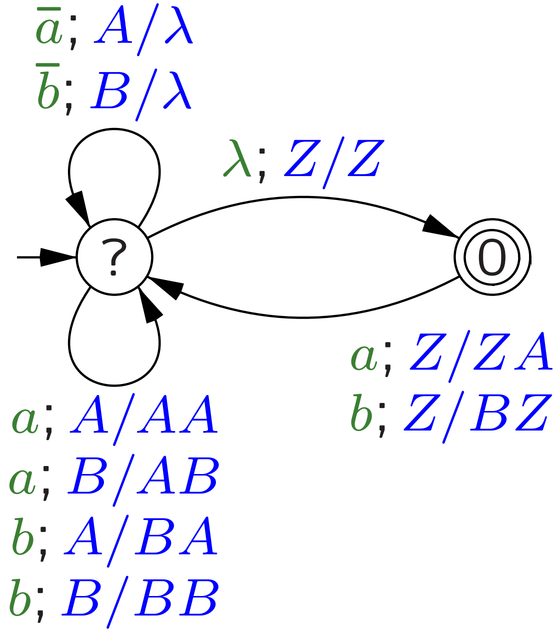


Dyck language D_2



$$S \to SS \mid \lambda$$
$$S \to aS\bar{a} \mid bS\bar{b}$$





stacksymbols A, B, Z

empty stack (left, nondeterministic)

final state (right, deterministic)

$$L = \{\ w \in \{a,b\}^* \mid \#_a(w) = \#_b(w)\ \}$$

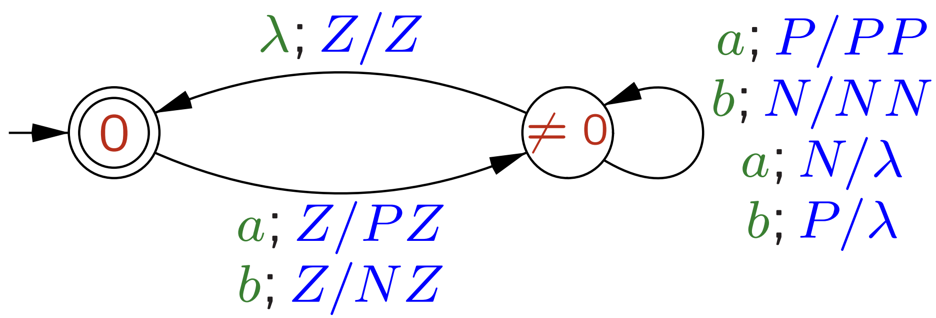
push $\#_a(w) - \#_b(w)$

Zero, Positive, Negative

deterministic: test for zero to accept

$S \to SS \mid \lambda$

$S \to aSb \mid bSa$



$L \in$ DPDℓ

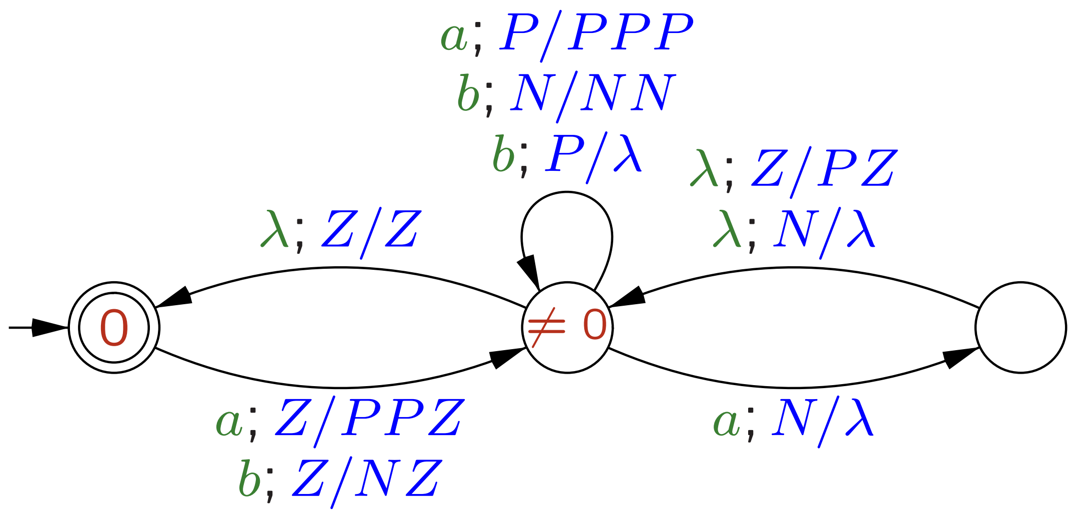
$L \notin$ DPDn (not prefix-free: $\lambda \in L$)

$$L = \{\ w \in \{a,b\}^* \mid 2\#_a(w) = \#_b(w)\ \}$$

push $2\#_a(w) - \#_b(w)$

Zero, Positive, Negative

deterministic: test for zero to accept

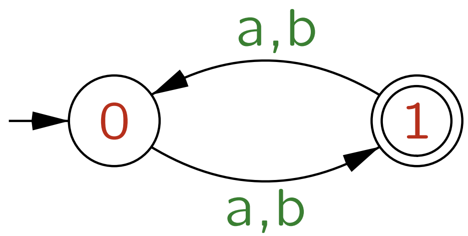


again $L \in$ DPDℓ − DPDn

google favourite ‘pushdown automata language odd length’

$$L = \{\ w \in \{a,b\}^* \mid |w| \text{ is odd } \}$$

regular! (i.e., not much stack action)



acceptance by final state

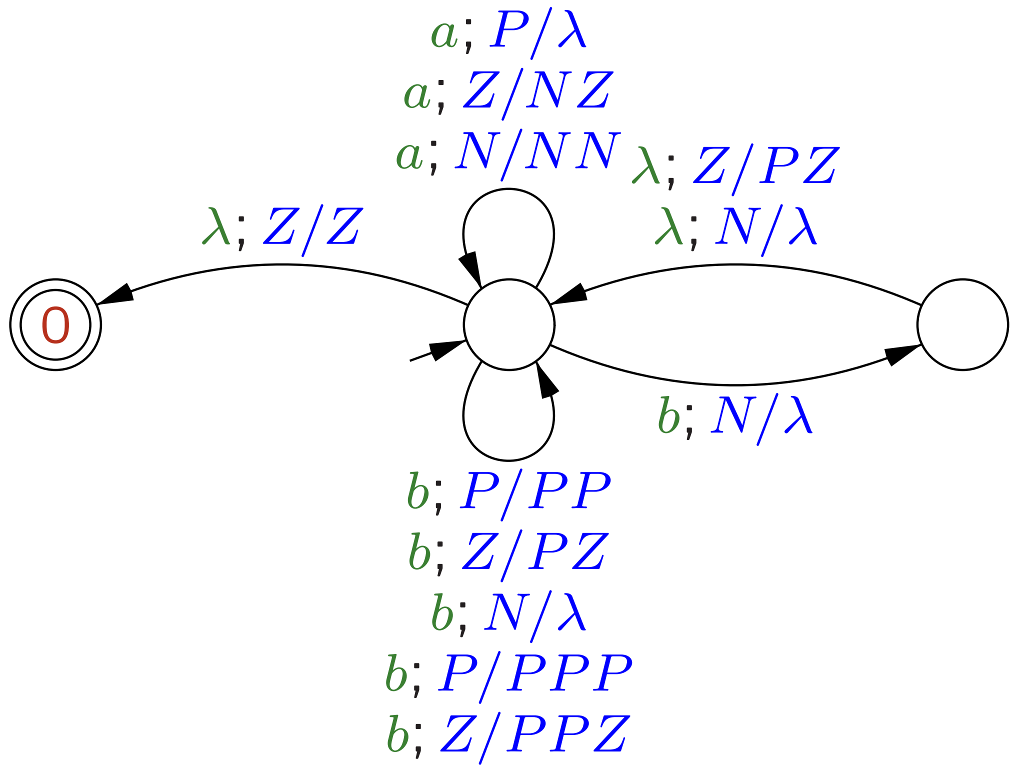
single stack symbol Z

$L = \{\ w \in \{a,b\}^* \mid \#_a(w) \le \#_b(w) \le 2\#_a(w)\ \}$

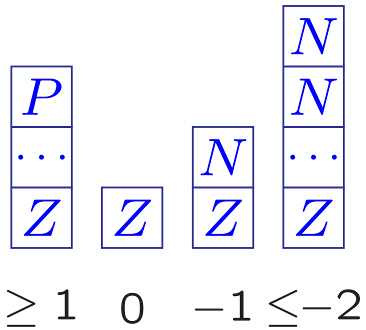
add 1 or 2 for each b; subtract 1 for each a.

Zero, Positive, Negative

*non*deterministic (choice for each b).

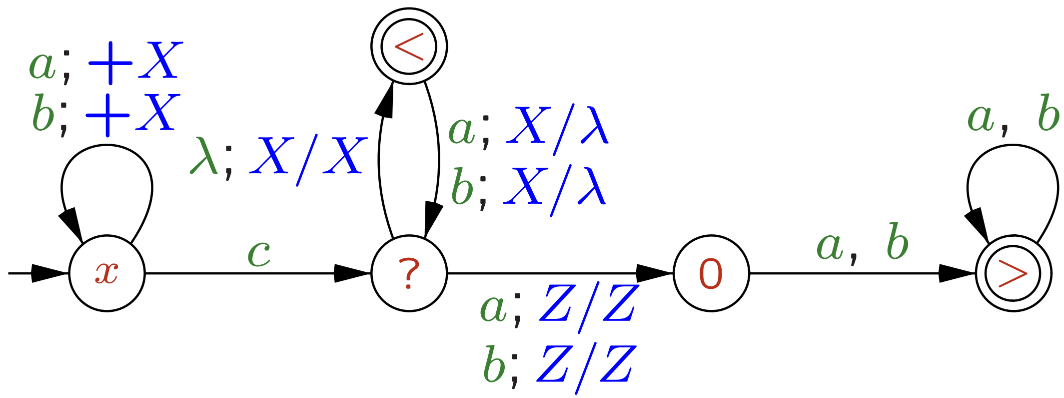


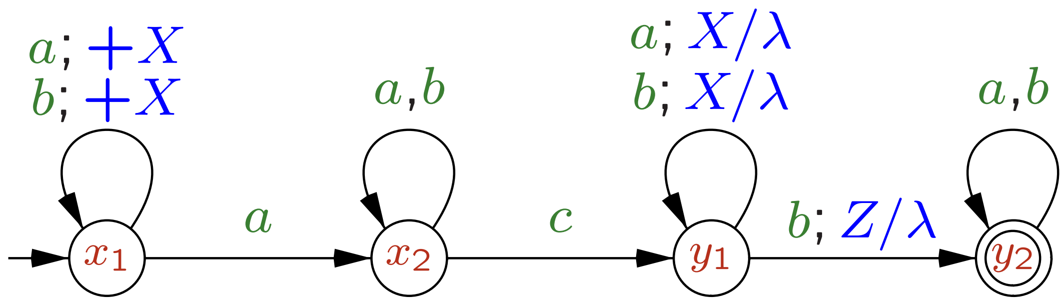
add 2 to



$$L = \{\, x\,c\,y \mid x, y \in \{a,b\}^*, x \neq y \,\}$$

$$x \neq y \iff |x| \neq |y|$$
$$\text{or} \quad x = x_1 a x_2,\ y = y_1 b y_2,\ |x_1| = |y_1|$$
$$\text{or} \quad x = x_1 b x_2,\ y = y_1 a y_2,\ |x_1| = |y_1|$$





+ variant $a \leftrightarrow b$

Pushdown Automata

transparencies made for a course at the

International PhD School
in Formal Languages and Applications

Rovira i Virgili University
Tarragona, Spain

Hendrik Jan Hoogeboom, Leiden

`http://www.liacs.nl/~hoogeboo/praatjes/tarragona/`