Pushdown as Storage

Abstract Families of Automata

families of languages

relations between closure properties

trio (=faithful cone)		λ-free morphism inverse morphism intersection regular lang.
full trio (cone)	&	(arbitrary) morphism
[full] semi-AFL	&	union
[full] AFL	&	concatenation plus [star]

example ∗ every full trio is closed under quotient with regular lang.

∗ FAM_1 and FAM_2 full trio $\Rightarrow$ $\text{HOM}(\,\text{FAM}_1 \wedge \text{FAM}_2\,)$ full trio

$$\text{FAM}_1 \wedge \text{FAM}_2 = \{\, K \cap L \mid K \in \text{FAM}_1 \wedge L \in \text{FAM}_2 \,\}$$
$$\neq \text{FAM}_1 \cap \text{FAM}_2$$

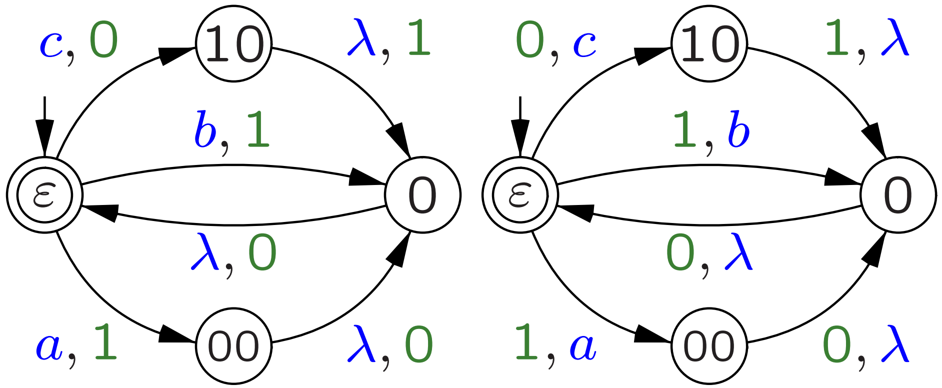
	RLIN REG	DPDℓ	CF PDn	DLBA	MON LBA	REC	TYPE0 RE
intersection	+	−	−	+	+	+	+
complement	+	+	−	+	+	+	−
union	+	−	+	+	+	+	+
concatenation	+	−	+	+	+	+	+
star, plus	+	−	+	+	+	+	+
λ-free morphism	+	−	+	+	+	+	+
morphism	+	−	+	−	−	−	+
inverse morphism	+	+	+	+	+	+	+
intersect reg lang	+	+	+	+	+	+	+
mirror	+	−	+	+	+	+	+
	fAFL		fAFL	AFL	AFL	AFL	fAFL

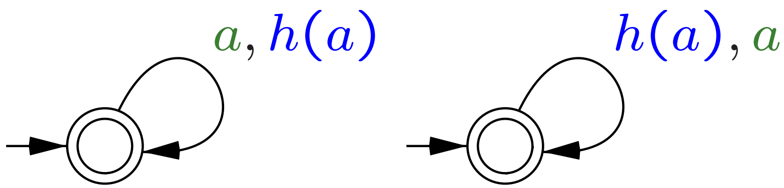
$\cap$ ^{c} $\cup$ – boolean operations

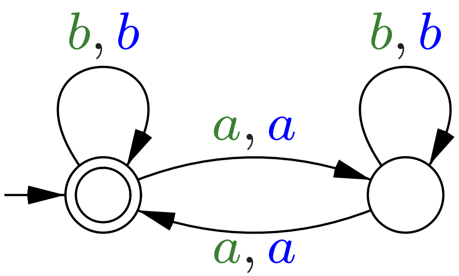
$\cup$ $\cdot$ * – regular operations

h h^{-1} $\cap R$ – (full) trio operations

$$h : \begin{cases} a \mapsto 100 \\ b \mapsto 10 \\ c \mapsto 010 \end{cases}$$







> every 'basic' trio operation is FST

> FST's are closed under composition

⇒ sequence of trio op's is FST

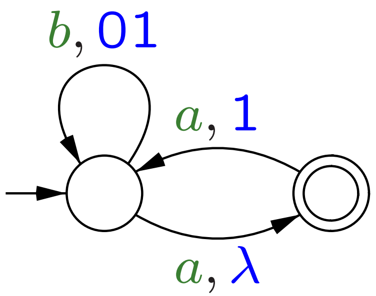
every full trio operation is a fs transduction

thm every FST is composition of full trio op's

$R_{\mathcal{M}}$ regular language over 'transitions'

$\{\ a{:}\lambda,\ a{:}1,\ b{:}01\ \}$

h and g select input and output



$$
\begin{array}{lccccccc}
K & \ni & b & b & a & a & b & a \\
 & & & \uparrow h & & & & \\
R_{\mathcal{M}} & \ni & b{:}01 & b{:}01 & a{:}\lambda & a{:}1 & b{:}01 & a{:}\lambda \\
 & & & \downarrow g & & & & \\
T_{\mathcal{M}}(K) & \ni & 01 & 01 & \lambda & 1 & 01 & \lambda
\end{array}
$$

$$\underset{\text{input}}{x} \xleftarrow{h} \underset{\text{computation}}{R} \xrightarrow{g} \underset{\text{output}}{y}$$

$$\boxed{T_{\mathcal{M}}(K) = g(\ h^{-1}(K)\ \cap\ R_{\mathcal{M}}\ )}$$

families of languages

$$\underset{\text{input}}{x} \xleftarrow{h} \underset{\text{computation}}{R} \xrightarrow{g} \underset{\text{output}}{y}$$

$$\boxed{T_{\mathcal{M}}(K) = g(\, h^{-1}(K) \,\cap\, R_{\mathcal{M}} \,)}$$

trio		λ-free morphism
(=*faithful cone*)		inverse morphism
		intersection regular lang.
	$\Leftrightarrow$	λ-free fs transductions
full trio (*cone*)	&	(arbitrary) morphism
	$\Leftrightarrow$	(arbitrary) fs transductions

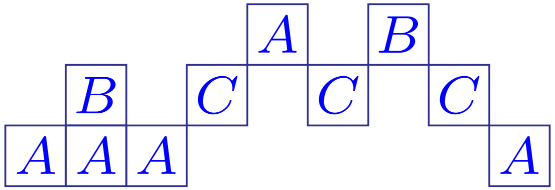
A, $\bar{A}$ push A, pop A $A \in \Sigma$

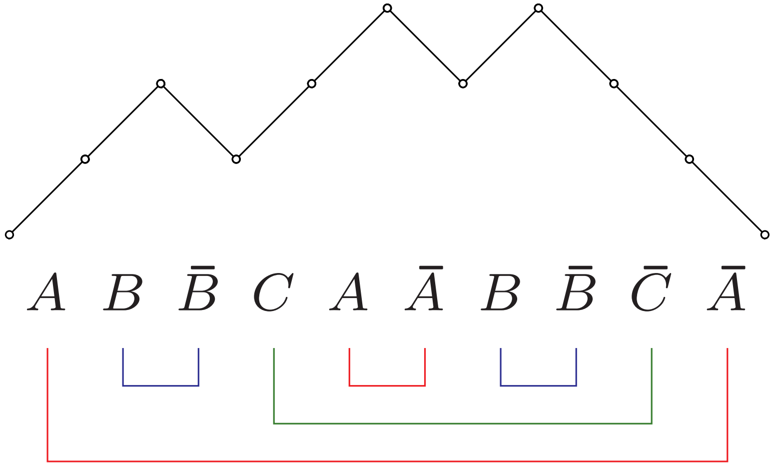
'legal' PDA instructions

$S \to A\,S\,\bar{A}$, $A \in \Sigma$

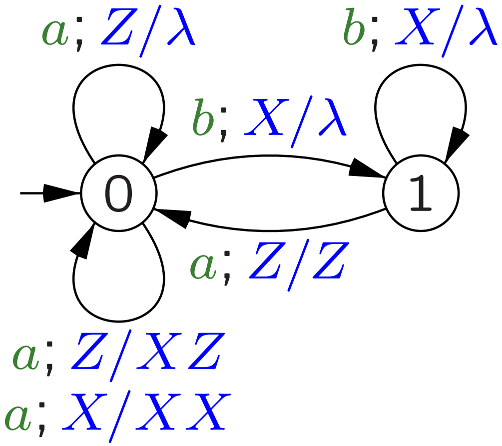
$S \to S\,S \mid \lambda$

Dyck language D_Σ





two pair brackets $D_{\{A,B\}}$ or D_2

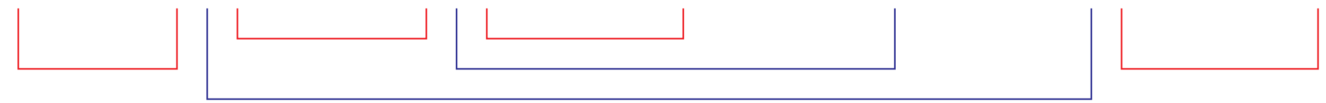


AFA	Automata	abstract storage
AFL	Languages	closure properties

σ sequence transitions

legal iff $h(\sigma) \in D_2$ iff $\sigma \in h^{-1}(D_2)$

	a	a	b	b	a	a	$\triangle$
		$\uparrow$					$\uparrow g$
Z	$a;\, Z/XZ$	$a;\, X/XX$	$b;\, X/\lambda$	$b;\, X/\lambda$	$a;\, Z/Z$	$a;\, Z/\lambda$	R
		$\downarrow$					$\downarrow h$
Z	$\bar{Z}ZX$	$\bar{X}XX$	$\bar{X}$	$\bar{X}$	$\bar{Z}Z$	$\bar{Z}$	D_2



CF $= \{\, g\,(h^{-1}(D_2) \cap R) \mid$ g, h morphism, R regular $\}$

g λ-free iff PDA real-time

CS, REC	trio
REG, LIN, CF, RE	full trio

$\mathcal{L}$ language family

$\mathcal{M}(\mathcal{L})$ smallest *trio* containing $\mathcal{L}$

$\Leftrightarrow$ closed λ-free morphisms, inverse morphisms, intersection regular

$\Leftrightarrow$ closed λ-free FTS's

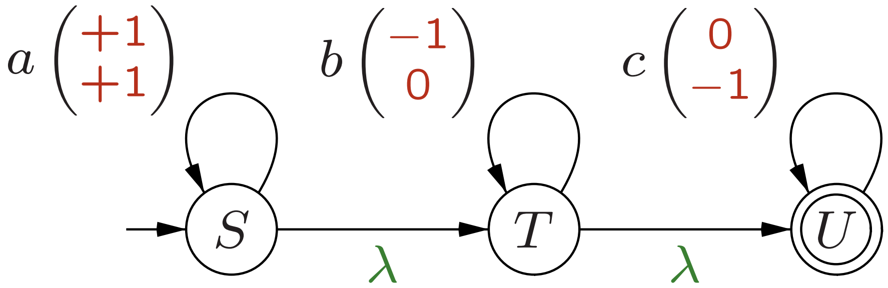
$$\Leftrightarrow \{\, g\,(h^{-1}(\mathcal{L}) \cap R) \mid g \text{ } \lambda\text{-free morphism, } h \text{ morphism, } R \text{ regular } \}$$

$\hat{\mathcal{M}}(\mathcal{L})$ smallest *full trio* containing $\mathcal{L}$

[λ-free] $\Rightarrow$ arbitrary

Greibach* $\quad \hat{\mathcal{M}}(D_2) = \text{CF} \overset{*}{=} \mathcal{M}(D_2)$

$S \to aS$	$+1,+1$
$S \to T$	$0,0$
$T \to bU$	$-1,0$
$T \to U$	$0,0$
$U \to cU$	$0,-1$
$U \to \lambda$	$0,0$



blind: no zero test (except final acceptance)

k counters kBC – storage type $\mathbb{Z}^k$

legal storage operations

$$\Sigma_k = \{a_1, b_1, \ldots a_k, b_k\}$$

$$B_k = \{\ w \in \Sigma_k^* \mid \#_{a_i} w = \#_{b_i} w \ (i = 1 \ldots k)\ \}$$

Greibach $k\text{BC} = \hat{\mathcal{M}}(B_k)$ full trio

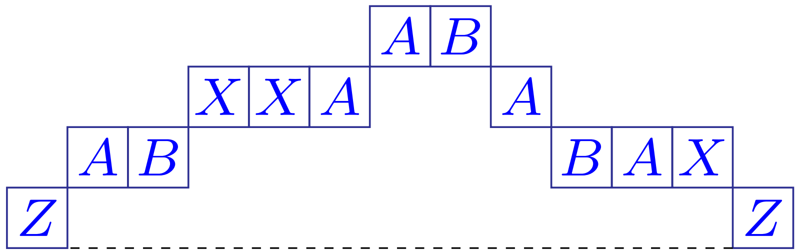
Latteux $= \mathcal{M}(B_k)$ trio, real-time

B_1 is context-free $1\text{BC} \subset \text{CF}$

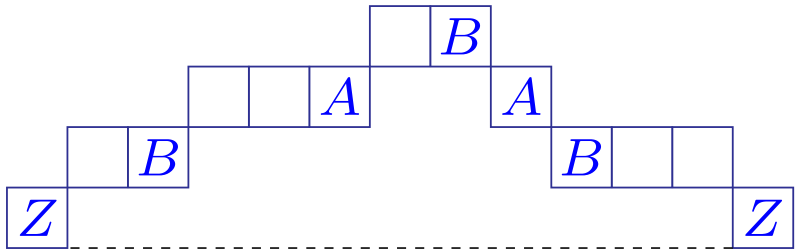
B_2 is not context-free

kBC incomparable to CF

LIN = 1tPDA



same level: by FST



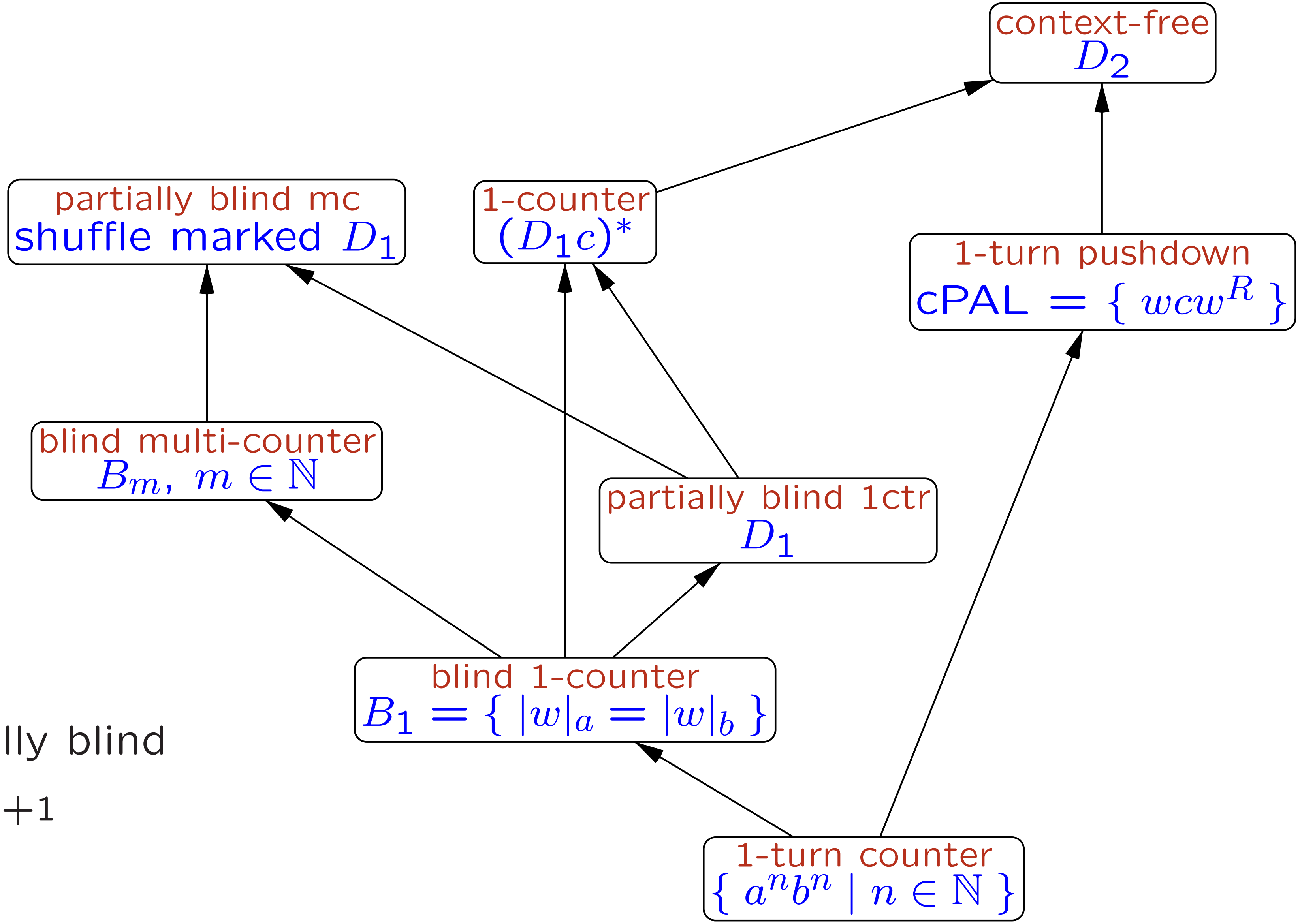
two stack symbol normal form

LIN = $\hat{\mathcal{M}}$(cPal) cPal = $\{ wcw^R \mid w \in \{a,b\}^* \}$

$$Z A \bar{A} B X \bar{X} X \bar{X} A A \bar{A} B \bar{B} \bar{A} \bar{B} A \bar{A} X \bar{X} \bar{Z}$$

$$\leftrightarrow \quad Z B A B \bar{B} \bar{A} \bar{B} \bar{Z}$$

$$\leftrightarrow \quad Z B A B \; c \; B A B Z$$



$\mathbb{Z}$ blind

$\mathbb{N}$ partially blind

$\mathbb{Z}^k \rightsquigarrow \mathbb{N}^{k+1}$

$\mathbb{N} \not\rightsquigarrow \mathbb{Z}^k$

Pushdown Automata

transparencies made for a course at the

International PhD School
in Formal Languages and Applications

Rovira i Virgili University
Tarragona, Spain

Hendrik Jan Hoogeboom, Leiden

`http://www.liacs.nl/~hoogeboo/praatjes/tarragona/`