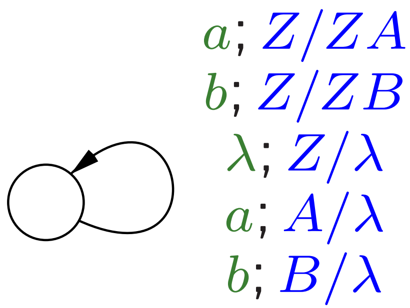
Determinism



$Z \to aZA$
$Z \to bZB$
$Z \to \lambda$
$A \to a$
$B \to b$

$P = \{\, ww^R \mid w \in \{a,b\}^* \,\}$ guessing the middle

$(aabbaa, Z) \vdash (aabbaa, \lambda) \nvdash$

$\top$

$(abbaa, ZA) \vdash (abbaa, A) \vdash (bbaa, \lambda) \nvdash$

$\top$

$(bbaa, ZAA) \vdash (bbaa, AA) \nvdash$

$\top$

$(baa, ZBAA) \vdash (baa, BAA) \vdash (aa, AA) \vdash (a, A) \vdash (\lambda, \lambda)$ ok.

$\top$

$(aa, ZBBAA) \vdash (aa, BBAA) \nvdash$

$\top$

$(a, ZABBAA) \vdash (a, ABBAA) \vdash (\lambda, BBAA) \nvdash$

$\top$

$(\lambda, ZAABBAA) \vdash (\lambda, ABBAA) \nvdash$

also $\{\, a^n b^n \mid n \in \mathbb{N} \,\} \cup \{\, a^n b^\ell c^n \mid \ell, n \in \mathbb{N} \,\}$

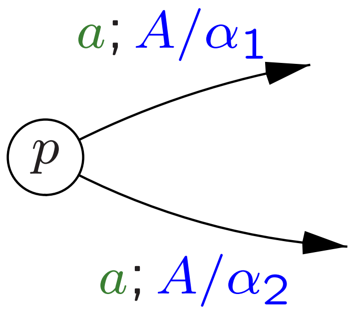
determinism means ‘no choice’ . . .

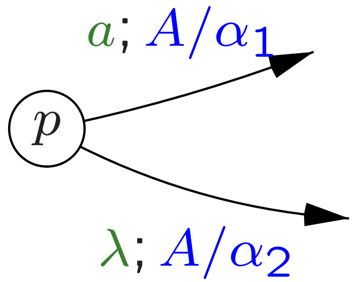
. . . where to start (ok)

. . . between two actions
with same *tape & stack* symbols

. . . between letter or λ

<u>not</u> allowed





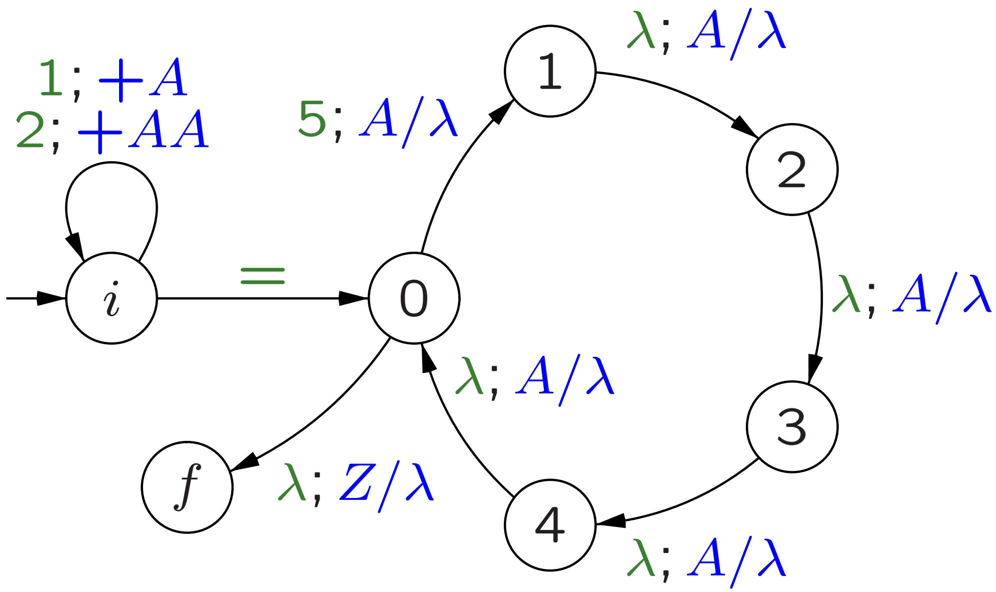
$(\, p, a, A, q_1, \alpha_1 \,)$ $(\, p, a, A, q_1, \alpha_1 \,)$

$(\, p, a, A, q_2, \alpha_2 \,)$ $(\, p, \lambda, A, q_2, \alpha_2 \,)$

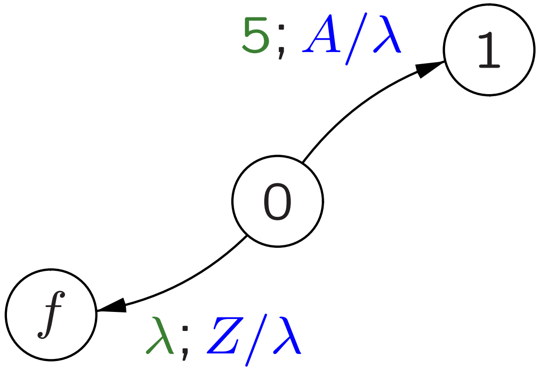
FSA = DFSA = RLIN	
PDn = PDℓ = CF	
DPDn $\subset$ DPDℓ $\subset$ CF	

final state DPDℓ deterministic CF languages

empty stack DPDn



in particular we allow

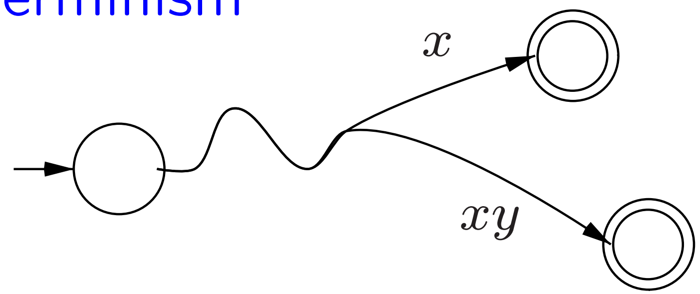


$$\{ \ a^m b^n \mid m \geq n \ \}$$

$$\{ \ a^n b^m a^n \mid m, n \in \mathbb{N} \ \}$$

language L $\quad x \in L$, $xy \in L$

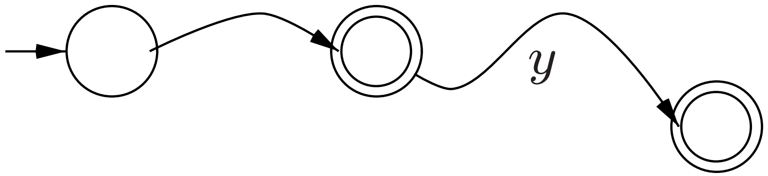
- nondeterminism



$\underline{a^n}\ \underline{b^n}$

$\underline{a^n}\ b^m\ \underline{c^n}$ $\quad$ different behaviour on b's

- determinism



computation on xy and on x must coincide!

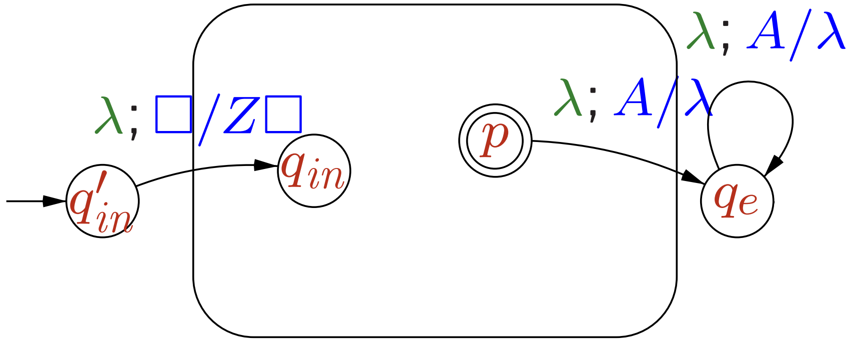
apply this to:

$$\text{haspref}(L) = \{\, xy \mid \underline{x} \in L, \underline{x}y \in L, y \neq \lambda \,\}$$

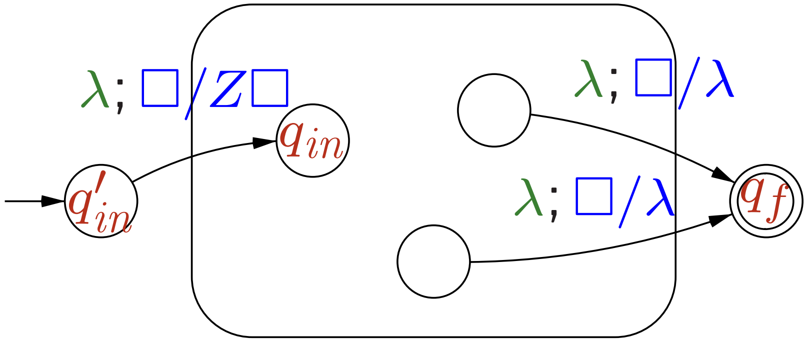
PDℓ $\subseteq$ PDn PDn $\subseteq$ PDℓ

what about determinism?

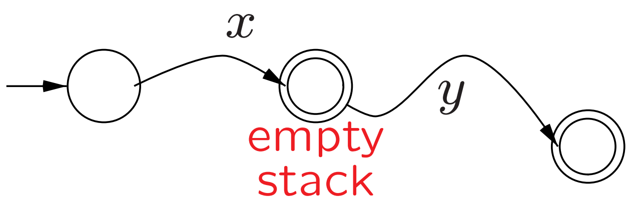
$\times$ $f \rightsquigarrow e$



$\surd$ $e \rightsquigarrow f$



DPDn $\subseteq$ DPDℓ



blocking on empty stack:

DPDn languages are *prefix-free*

$$x, xy \in L \Rightarrow y = \lambda$$

REG $\not\subseteq$ DPDn a^*, $\{\lambda, a\}$

REG $\subset$ DPDℓ

$$\text{haspref}(L) = \{ \, xy \mid x \in L, xy \in L, y \neq \lambda \, \}$$

$$L_0 = \{ \, a^n b^n \mid n \geq 1 \, \} \cup \{ \, a^n b^m c^n \mid m, n \geq 1 \, \}$$

$$\text{haspref}(L_0) = \{ \, a^n b^m c^n \mid m \geq n \geq 1 \, \} \notin \text{CF}$$

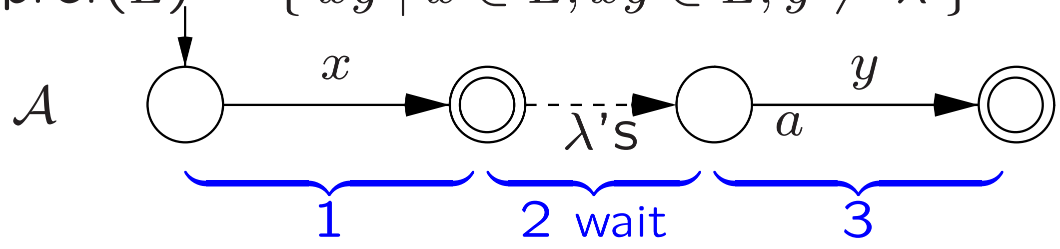
- ※ CF = PDℓ is not closed under haspref
- ※ DPDℓ *is* closed under haspref

[proof follows]

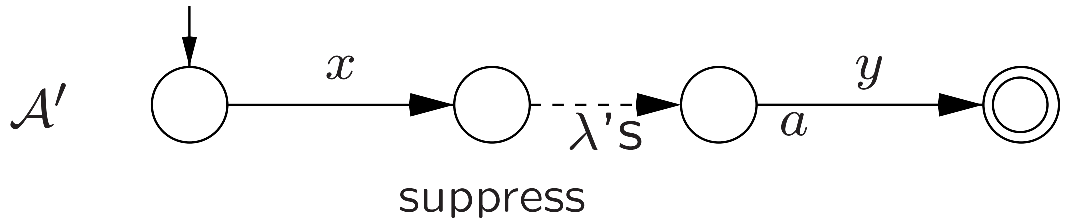
consequences

- ※ DPDℓ ⊂ PDℓ = CF $\quad L_0 \in$ CF − DPDℓ
- ※ DPDℓ is not closed under union
- ※ also $\{ \, ww^R \mid w \in \{a, b\}^* \, \} \notin$ DPDℓ

haspref$(L) = \{ \, xy \mid x \in L, xy \in L, y \neq \lambda \, \}$



▷ intuition



▷ construction

$Q' = Q \times \{1, 2, 3\}$
$q'_{in} = \langle q_{in}, 1 \rangle$
$F' = F \times \{3\}$

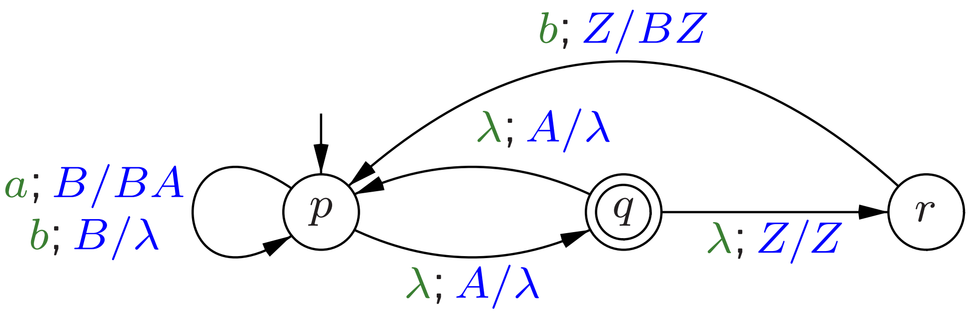
δ	δ'	when
$(\, p, a, A, q, \alpha \,)$	$(\, \langle p, 1 \rangle, a, A, \langle q, 1 \rangle, \alpha \,)$	$q \notin F$
	$(\, \langle p, 1 \rangle, a, A, \langle q, 2 \rangle, \alpha \,)$	$q \in F$
	$(\, \langle p, 2 \rangle, \lambda, A, \langle q, 2 \rangle, \alpha \,)$	$a = \lambda$
	$(\, \langle p, 2 \rangle, a, A, \langle q, 3 \rangle, \alpha \,)$	$a \neq \lambda$
	$(\, \langle p, 3 \rangle, a, A, \langle q, 3 \rangle, \alpha \,)$	

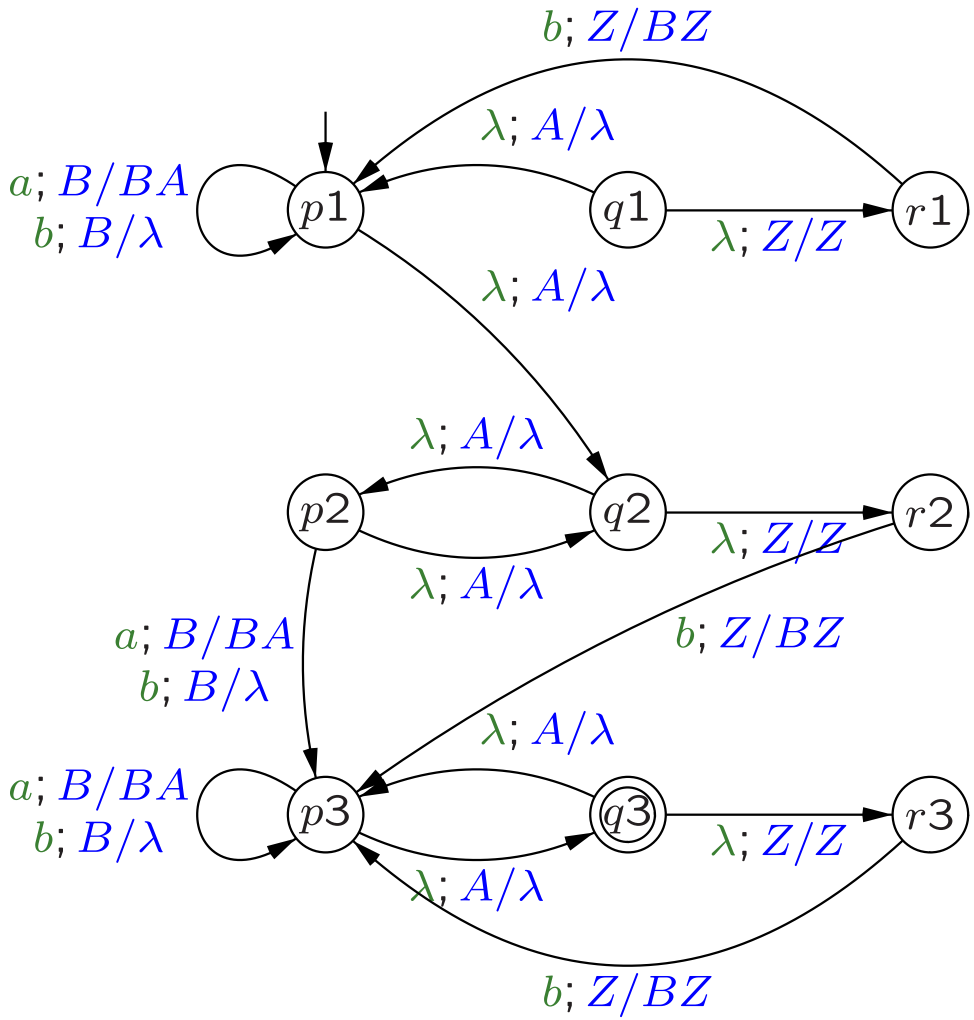
example haspref construction

$Q' = Q \times \{1, 2, 3\}$
$q'_{in} = \langle q_{in}, 1 \rangle$
$F' = F \times \{3\}$

$(\, p, a, A, q, \alpha \,) \in \delta$

δ' contains	when
$(\, \langle p, 1 \rangle, a, A, \langle q, 1 \rangle, \alpha \,)$	$q \notin F$
$(\, \langle p, 1 \rangle, a, A, \langle q, 2 \rangle, \alpha \,)$	$q \in F$
$(\, \langle p, 2 \rangle, \lambda, A, \langle q, 2 \rangle, \alpha \,)$	$a = \lambda$
$(\, \langle p, 2 \rangle, a, A, \langle q, 3 \rangle, \alpha \,)$	$a \neq \lambda$
$(\, \langle p, 3 \rangle, a, A, \langle q, 3 \rangle, \alpha \,)$	





$L \in$ DPDn $\Rightarrow$ $L \in$ DPDℓ

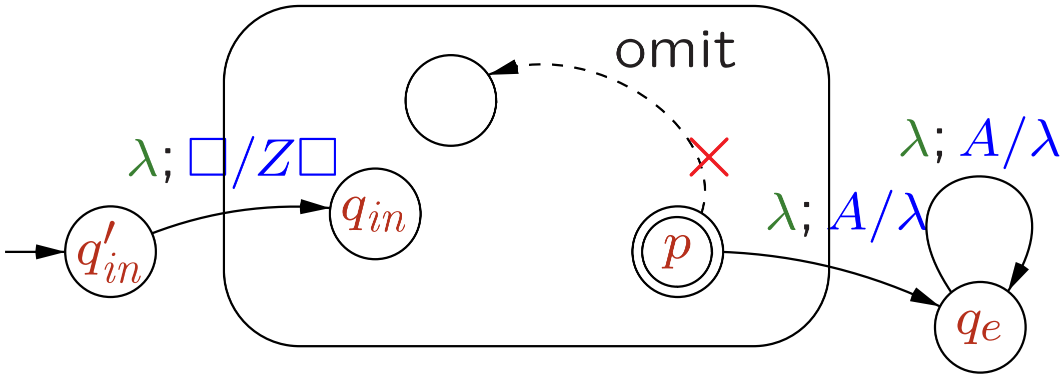
$\not\Leftarrow$

$L \in$ DPDn $\Leftrightarrow$ $L \in$ DPDℓ and prefix-free

$(p, w, \alpha) \vdash^* (q, \lambda, \lambda)$

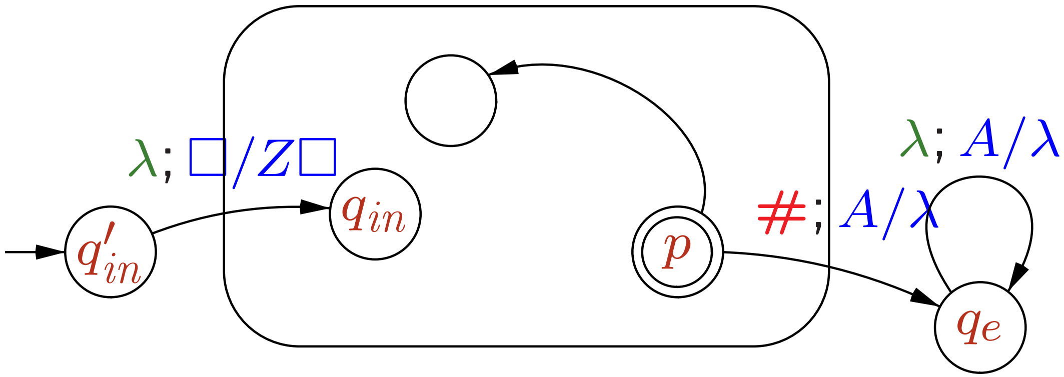
iff

$(p, wz, \alpha) \vdash^* (q, z, \lambda)$



endmarker: new letter #

$L\#$ $\in$ DPDn $\Leftarrow$ $L \in$ DPDℓ



... *only if final states have no outgoing* λ !!

CFG characterization for DPDn

Harrison-Havel 1973

CFG $G = (N, T, S, P)$ strict deterministic iff
equivalence relation $\equiv$ on $N \cup T$

- T is a class for $\equiv$
- for all $X, X' \in N$, $\alpha, w, w' \in (N \cup T)^*$
  if $X \equiv X'$, $X \to \alpha w$, $X' \to \alpha w'$
  then $w = w' = \lambda$ and $X = X'$
  or $w = Yv$, $w' = Y'v'$ for some $Y \equiv Y'$
  $Y, Y' \in (N \cup T)$

$$X \to \alpha\, w$$
$$\equiv$$
$$X' \to \alpha\, w$$

then

$$X \to \alpha$$
$$=$$
$$X' \to \alpha$$

or

$$X \to \alpha\, Y\ \ v$$
$$\equiv \qquad \equiv$$
$$X' \to \alpha\, Y'\, v'$$

Pushdown Automata

transparencies made for a course at the

International PhD School
in Formal Languages and Applications

Rovira i Virgili University
Tarragona, Spain

Hendrik Jan Hoogeboom, Leiden

`http://www.liacs.nl/~hoogeboo/praatjes/tarragona/`