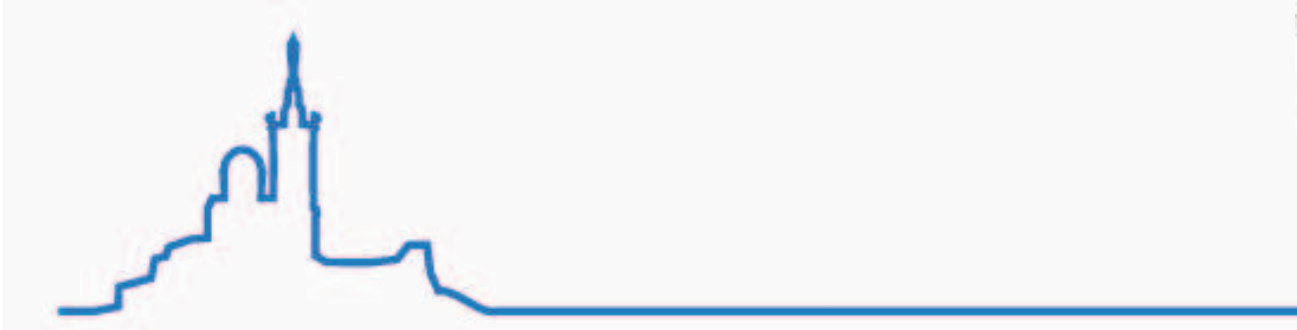
PEBBLES at STACS



Automata with **Nested Pebbles** capture
FO Logic with **Transitive Closure**

Marseille Feb 06

Joost Engelfriet
Hendrik Jan Hoogeboom
Leiden NL

two classic characterizations

strings: [non]deterministic logarithmic space

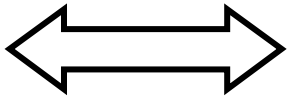
Immerman

First-Order Logic
+ transitive closure

$\varphi^*(\underline{x},\underline{y})$

Multi-Head Automata
(two-way)

arity k



k heads

Bargury&Makowsky

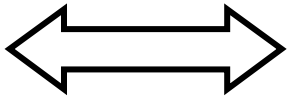
our STACS paper

on strings, trees, grids, toruses, mazes, …

First-Order Logic
+ transitive closure

Multi-Head Automata
+ ‘nested pebbles’

$\varphi^*(\underline{x},\underline{y})$

arity k  k heads

starting with …

on trees,

single head

First-Order Logic
+ transitive closure

Multi-Head Automata
+ ‘nested pebbles’

unary

$\varphi^*(x,y)$

arity k

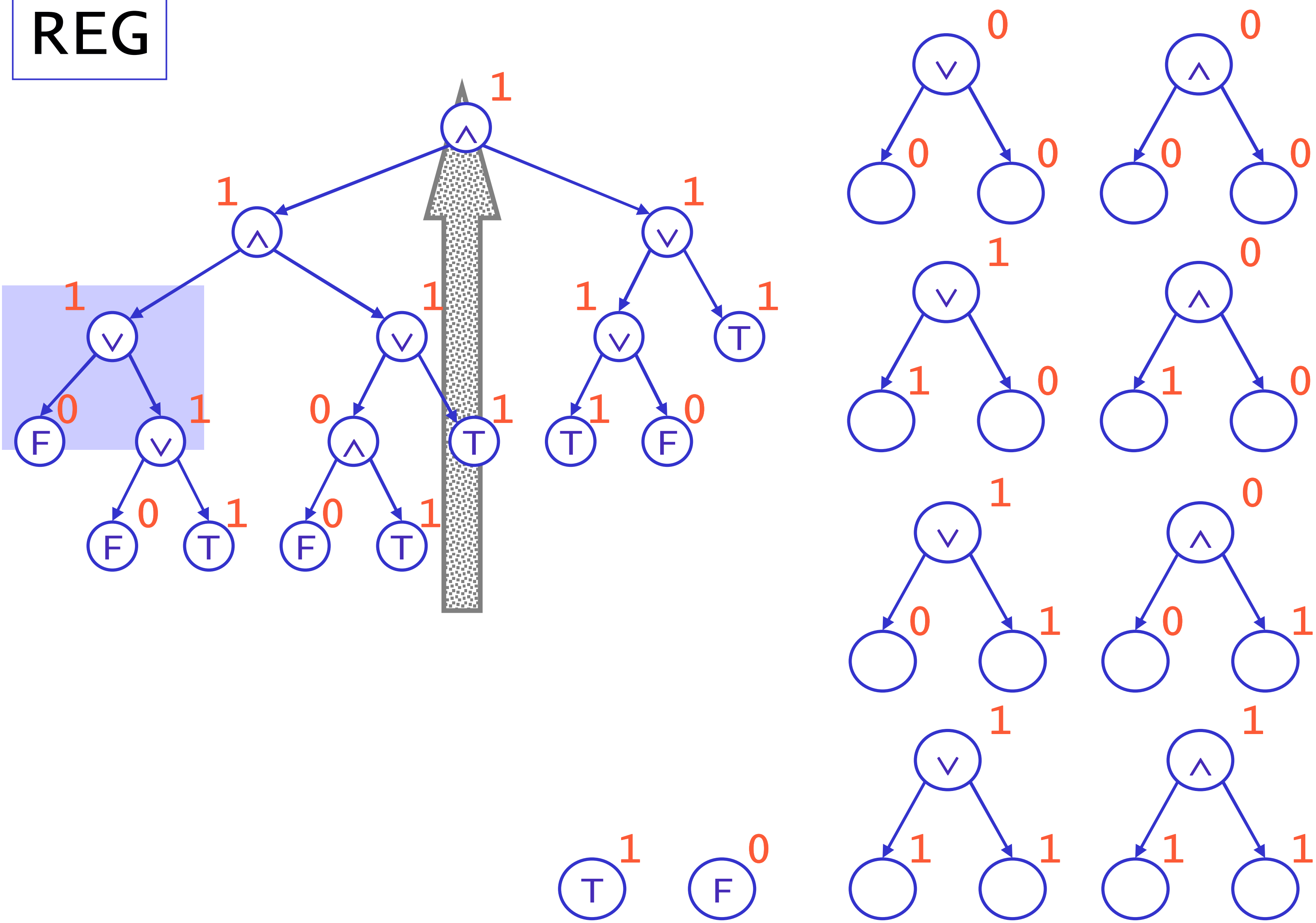
k heads

background

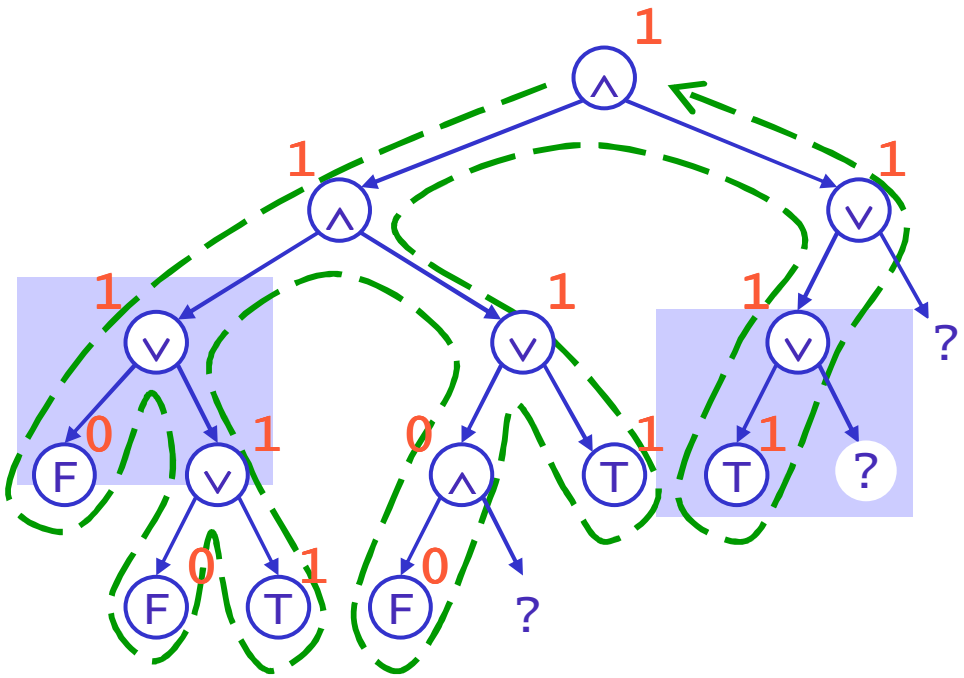
- transitive closure
  - descriptive complexity
  - strings, trees, n-dim grids, …
- XML document transformation
  - single head on (unranked) trees
- graph exploration
  - many heads on graphs ‘robots’
  - grids, toruses, mazes, …

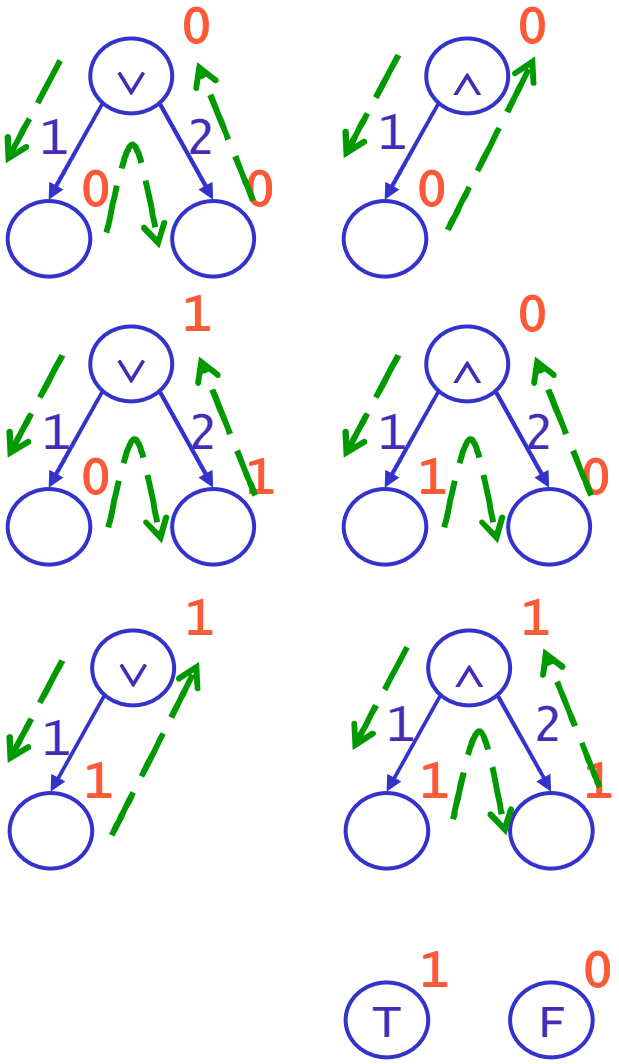
bottom-up tree automata

REG



walking along the tree



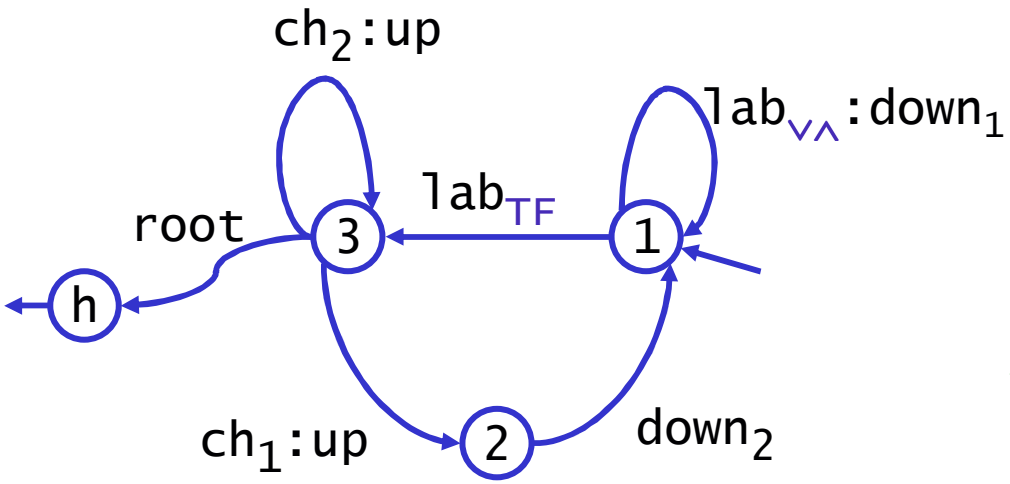


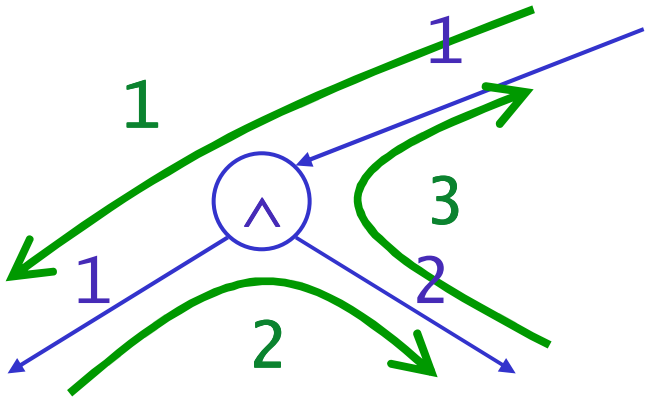
cf. two-way finite state automaton

tree walking automaton

example: tree traversal

TW



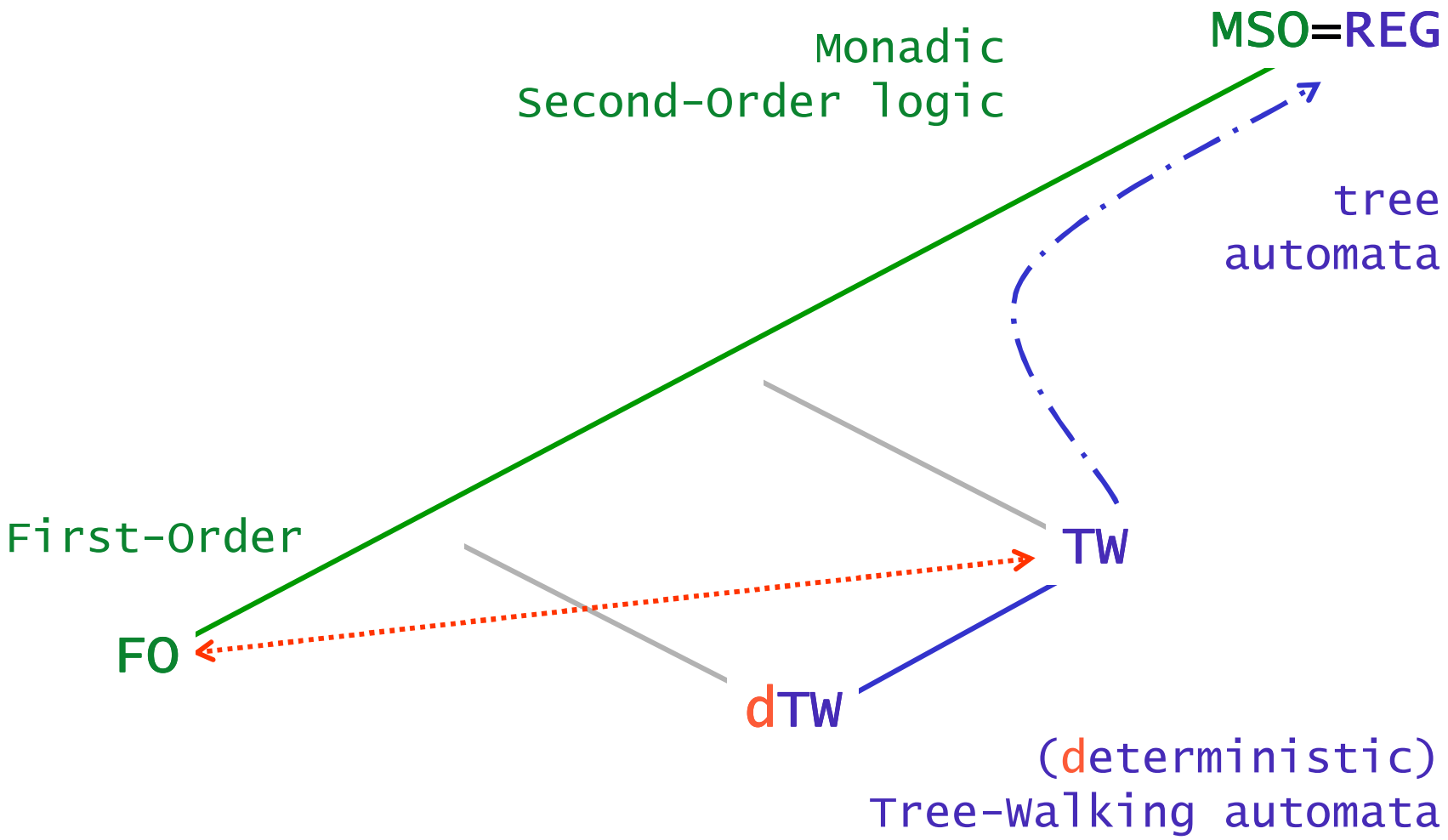


walk along edges, moves based on

- state
- node label
- child number
  (= incoming edge)

single head on trees

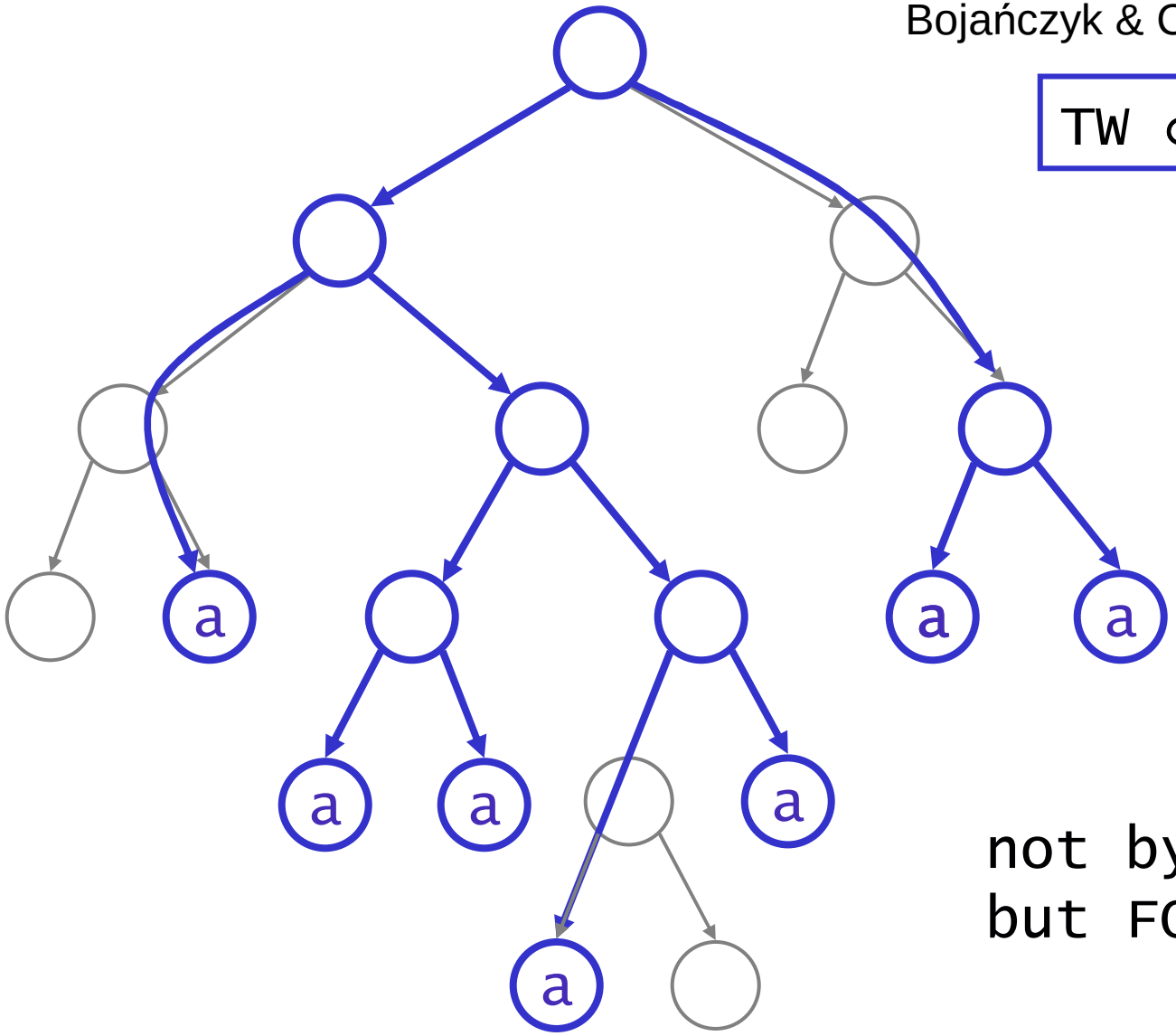
Doner; Thatcher & Wright



‘branching structure’ of even length

Bojańczyk & Colcombet

TW ⊂ REG



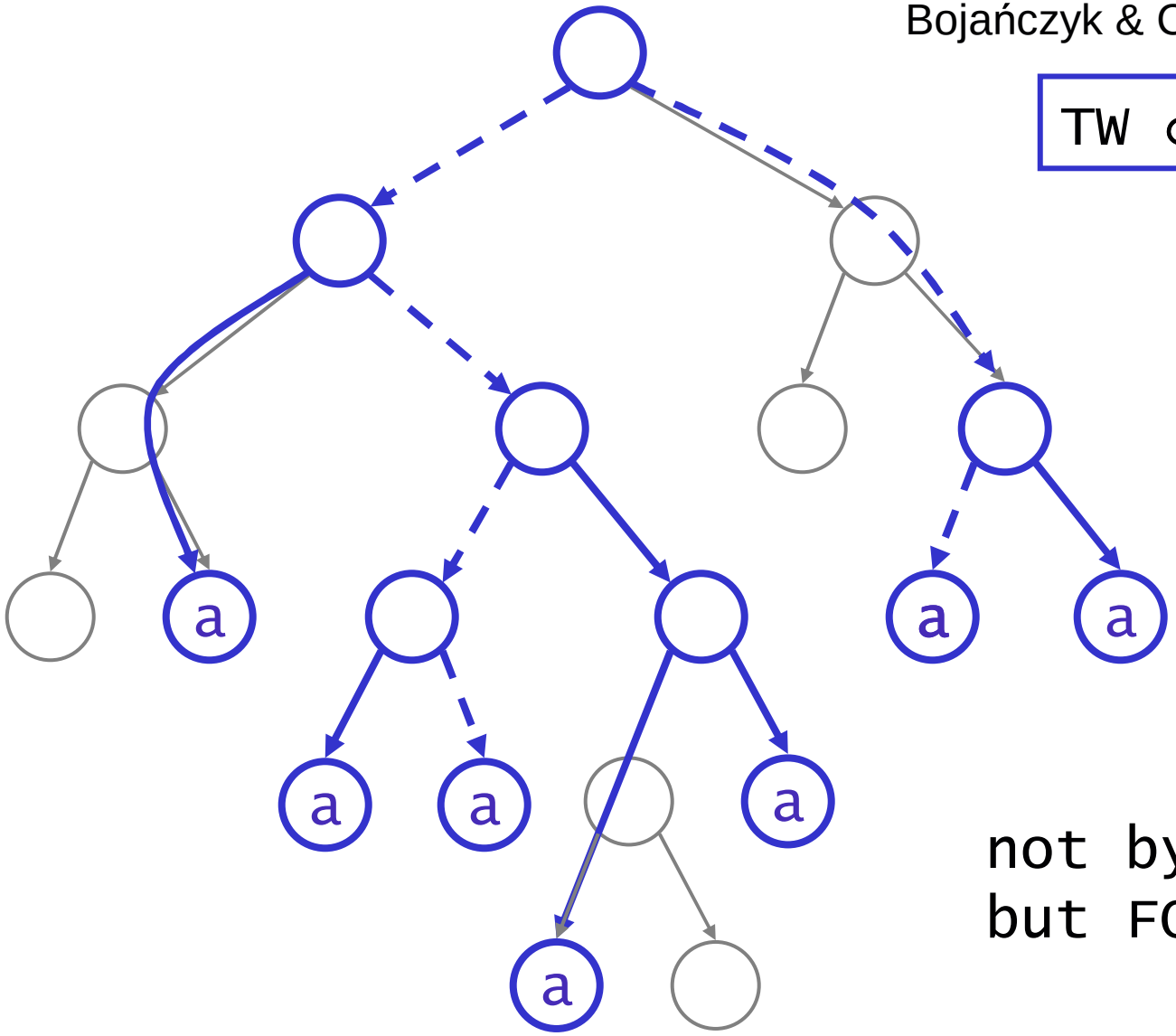
not by TWA
but FO (!)

(aa)*

‘branching structure’ of even length

Bojańczyk & Colcombet

TW ⊂ REG

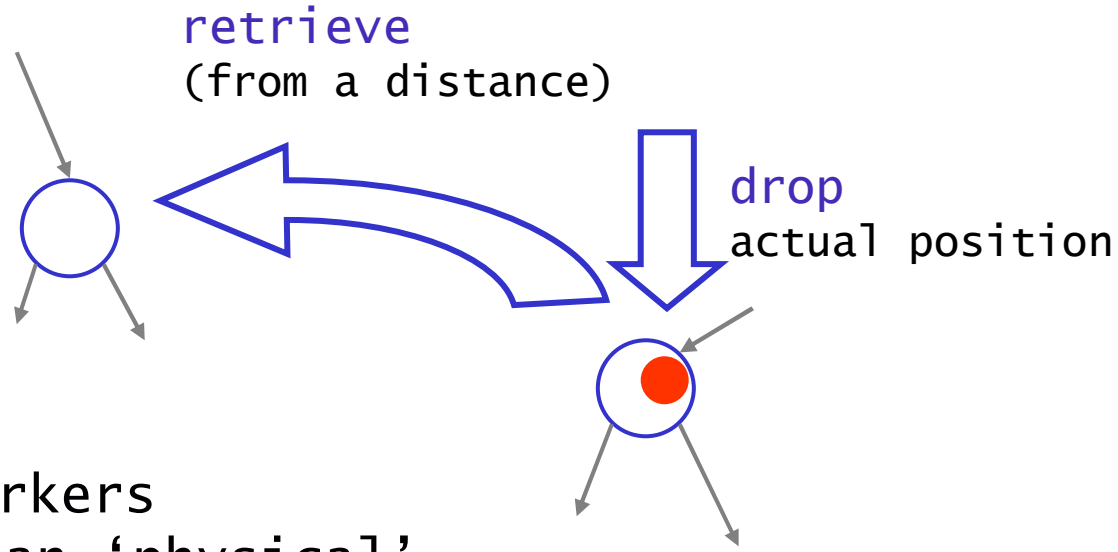


not by TWA
but FO (!)

(aa)*

adding nested pebbles

pebble: marks a node

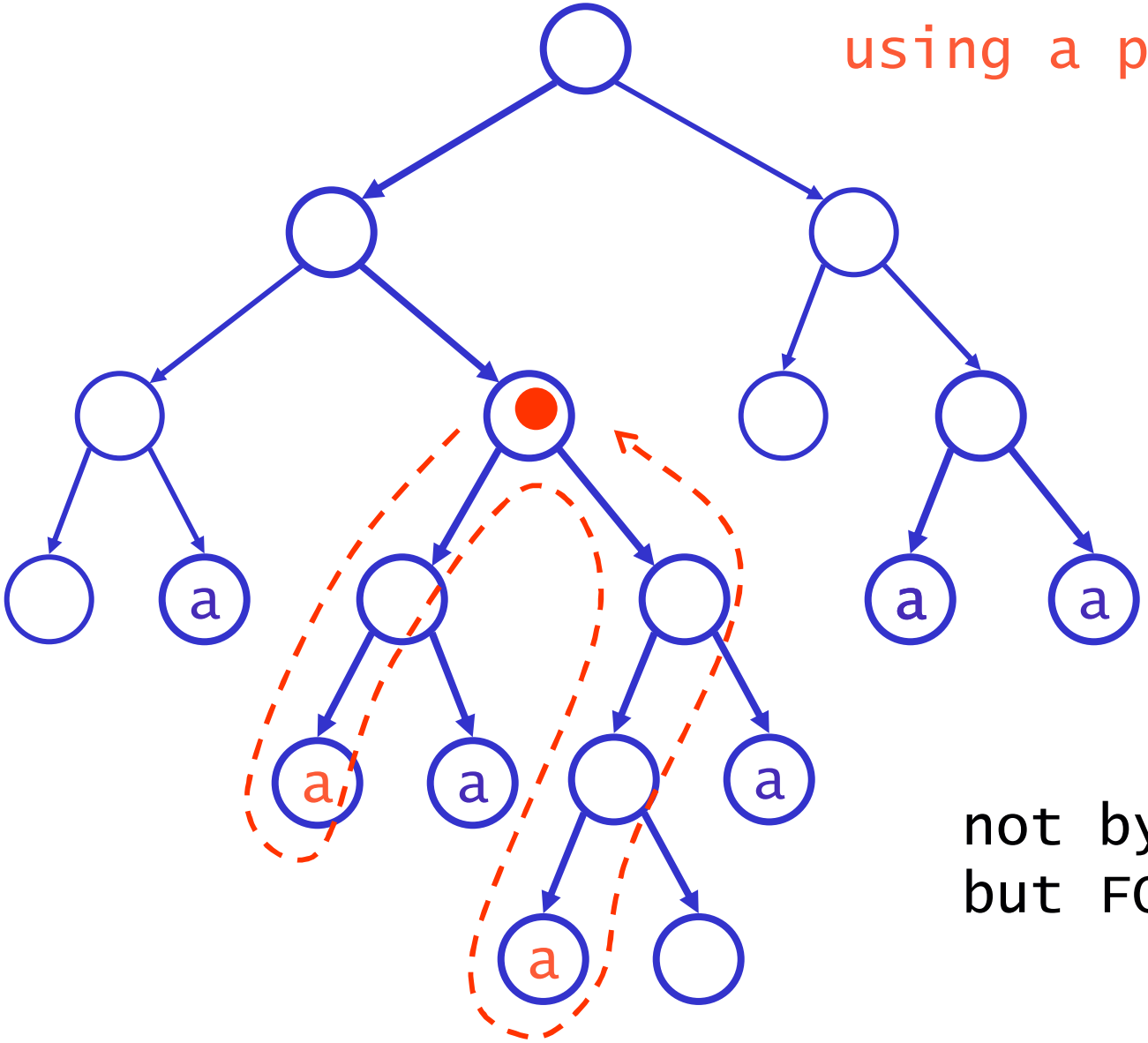


- ‘abstract’ markers
  rather than ‘physical’
- *nested lifetimes* LIFO
- fixed number for automaton
- can be distinguished

‘regular’ extension
(for single head on trees)

‘branching structure’ of even length

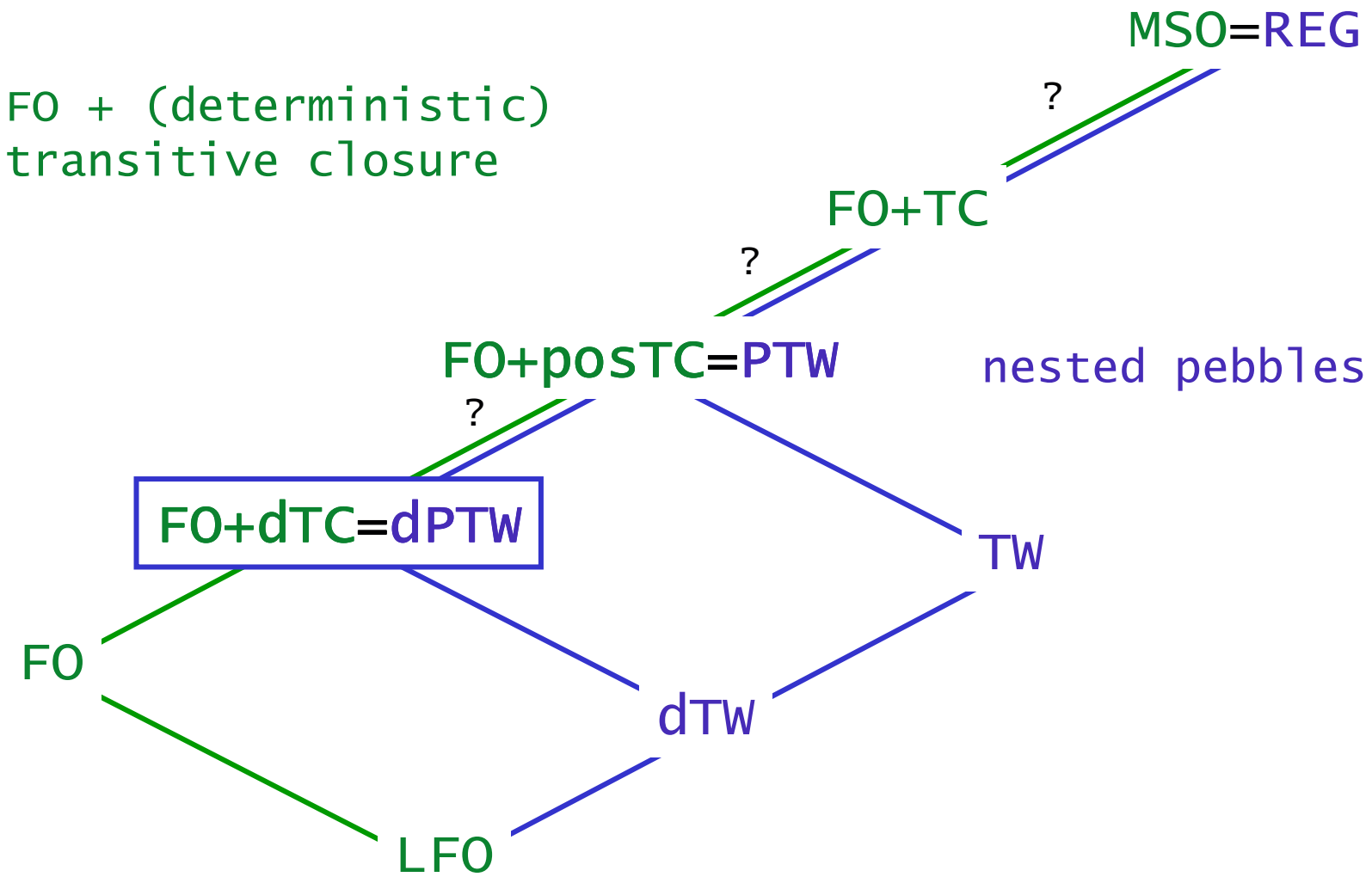
using a pebble



not by TWA
but FO (!)

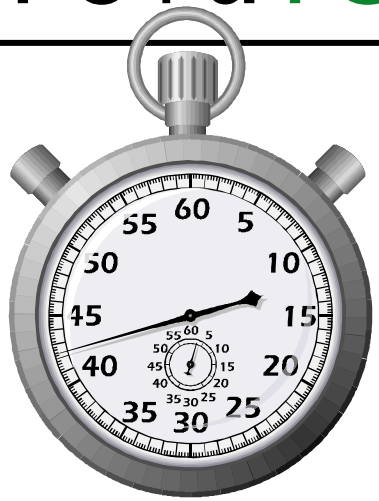
(aa)*

single head on trees



main result

$$FO{+}dTC^k = dPTW^k$$



proof summary
manager style

(deterministic,
single head,
on trees)

FO+dTC ⊆ dPTW

(1) logic to nested pebbles

$\varphi \rightarrow \mathcal{A}$

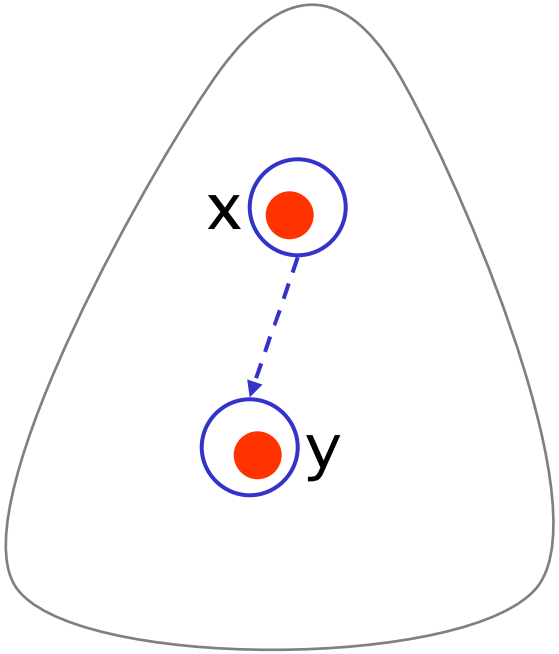
$lab_a(x)$
$edg_i(x,y)$

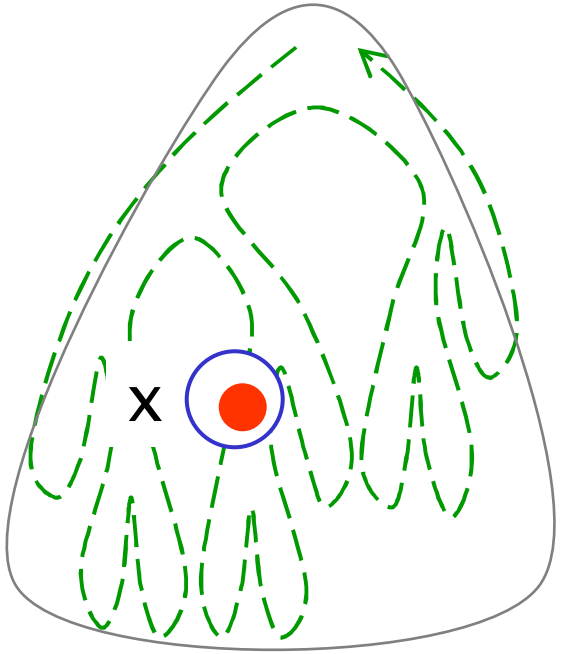
$x \leq y$
$x = y$

$\neg \ \wedge \ \vee$
$\forall x \ \exists x$

$\varphi^*(x,y)$

always halting
free variables ~
fixed pebbles



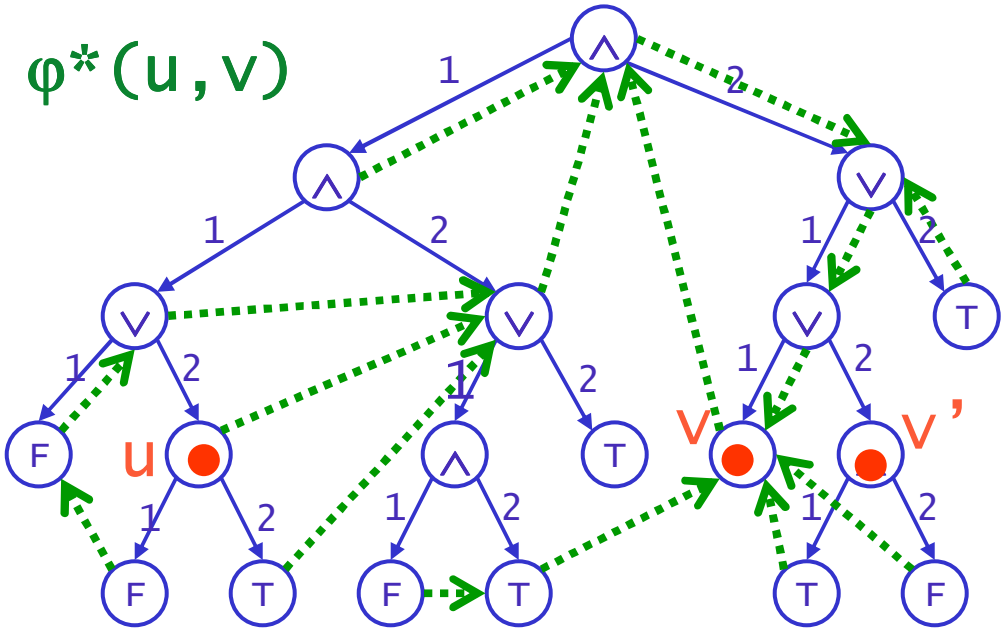


$x \leq y$

$\forall x \ \varphi(x)$ $\mathcal{A}_\varphi$

FO+dTC ⊆ dPTW

(1ctd) transitive closure

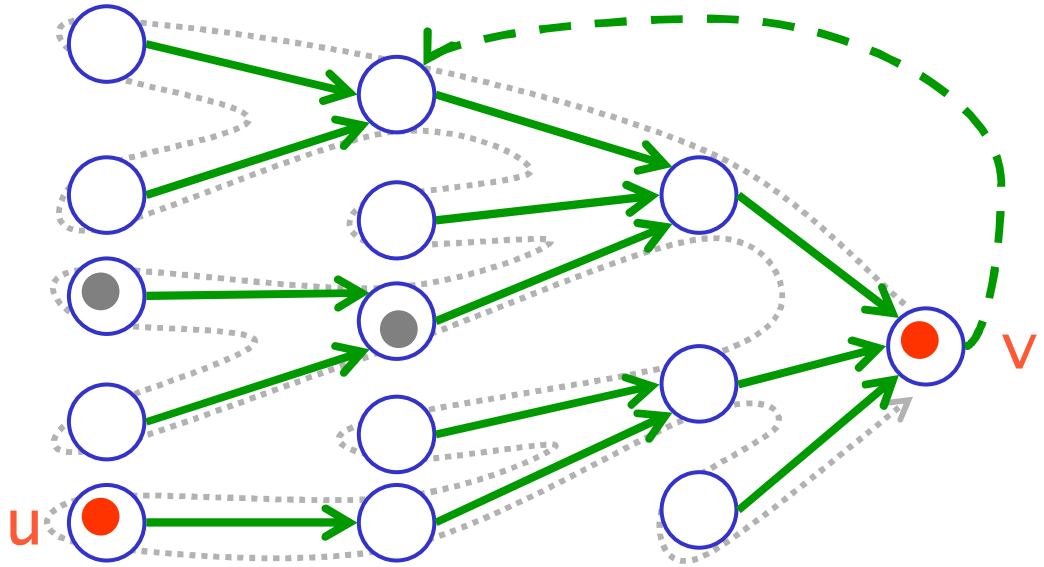


functional!

φ

but implicit

$\mathcal{A}\varphi$



tree walking

implicit φ-tree

reconstruct locally

backwards! Sipser

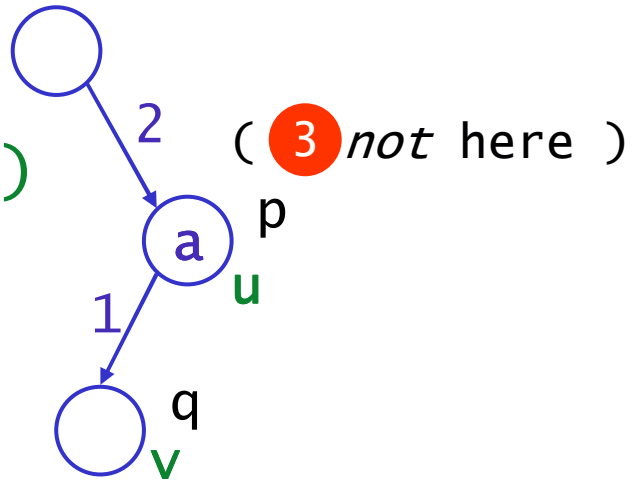
dPTW ⊆ FO+dTC

(2) nested pebbles to logic

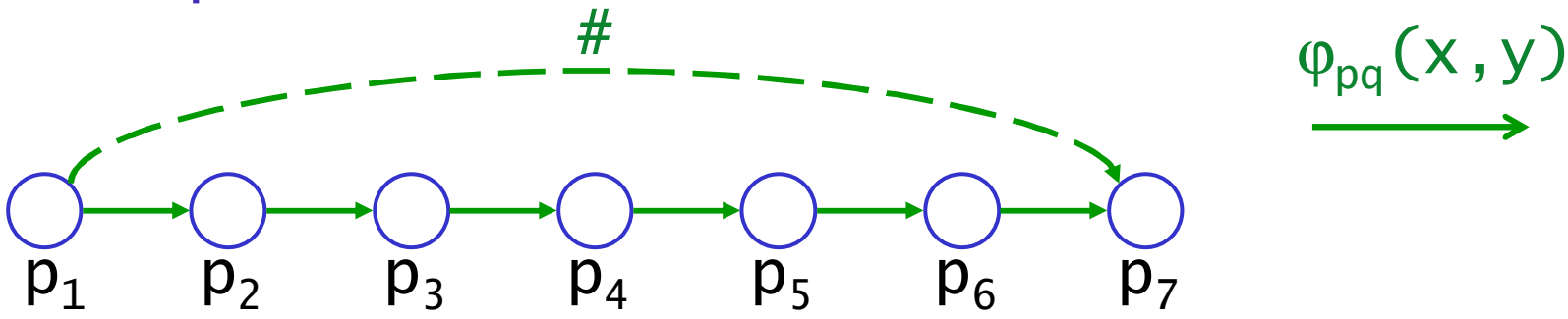
i single move $\varphi_{pq}(u,v)$

$$lab_a(u) \wedge (\exists u')edg_2(u',u) \wedge u \neq x_3 \wedge edg_1(u,v)$$

free variables for pebbles



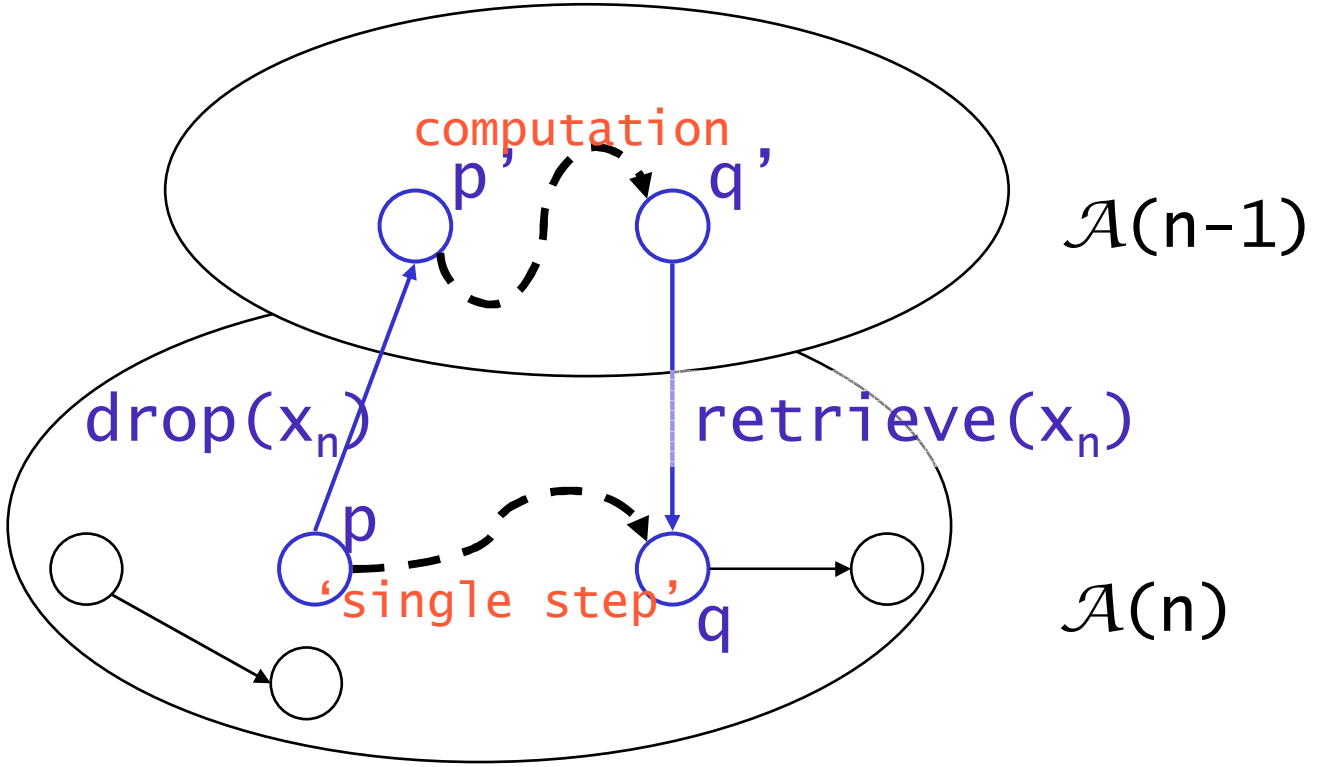
ii computation ~ tc with states



Kleene: removing states finite aut to reg expr

dPTW ⊆ FO+dTC

(2ctd) dropping pebbles

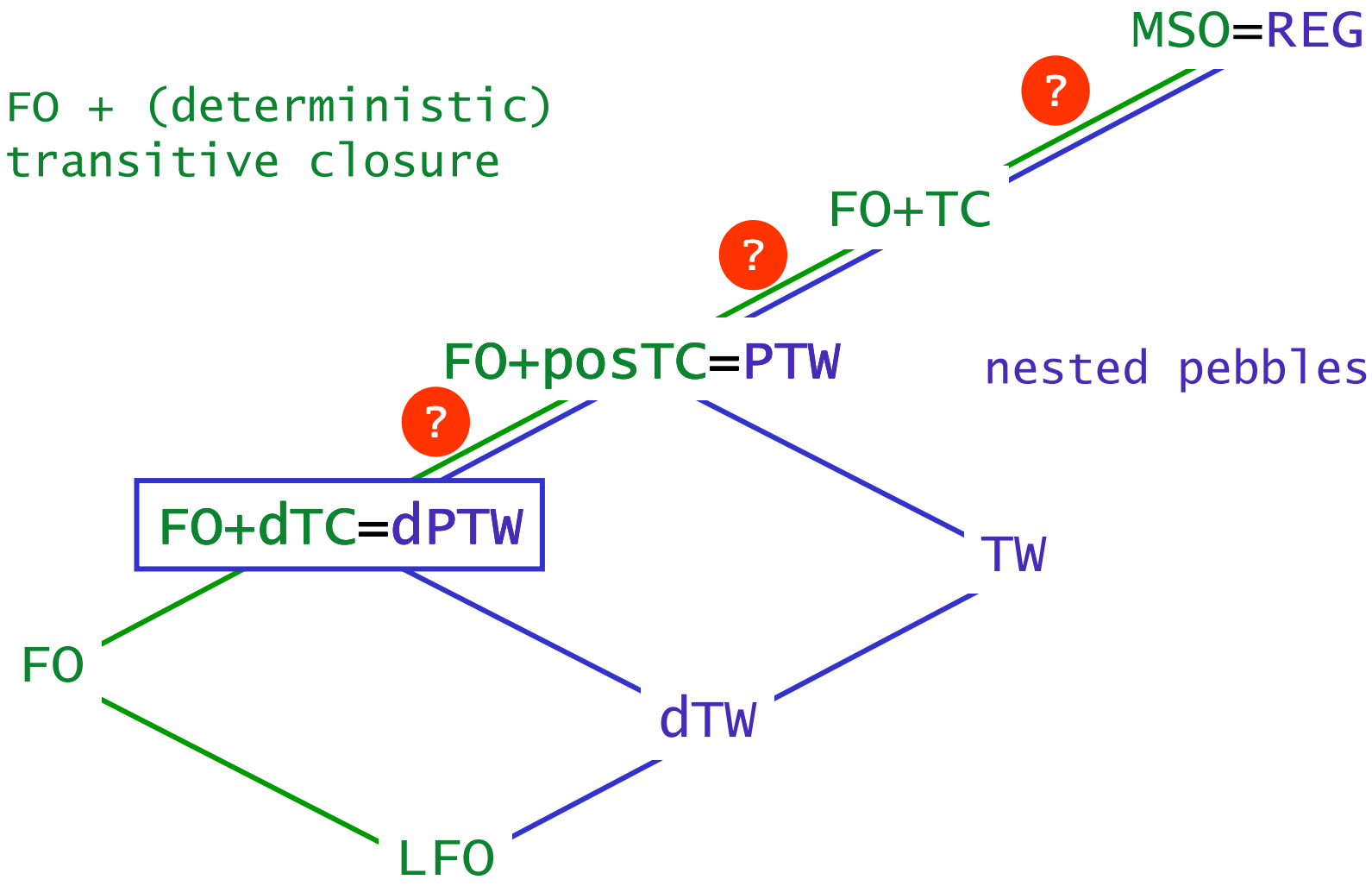


transition diagram

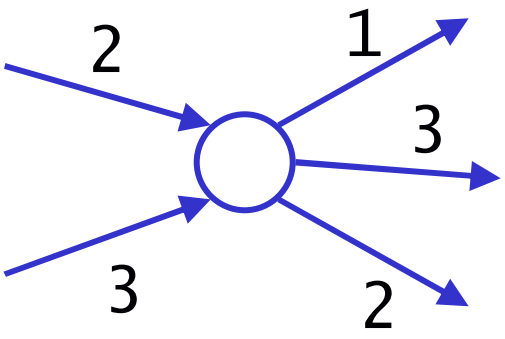
$$\varphi^{n}_{pq}(u,v) = \varphi^{(n-1)\#}_{p',q'}(u,v)$$

replacing x_n by u

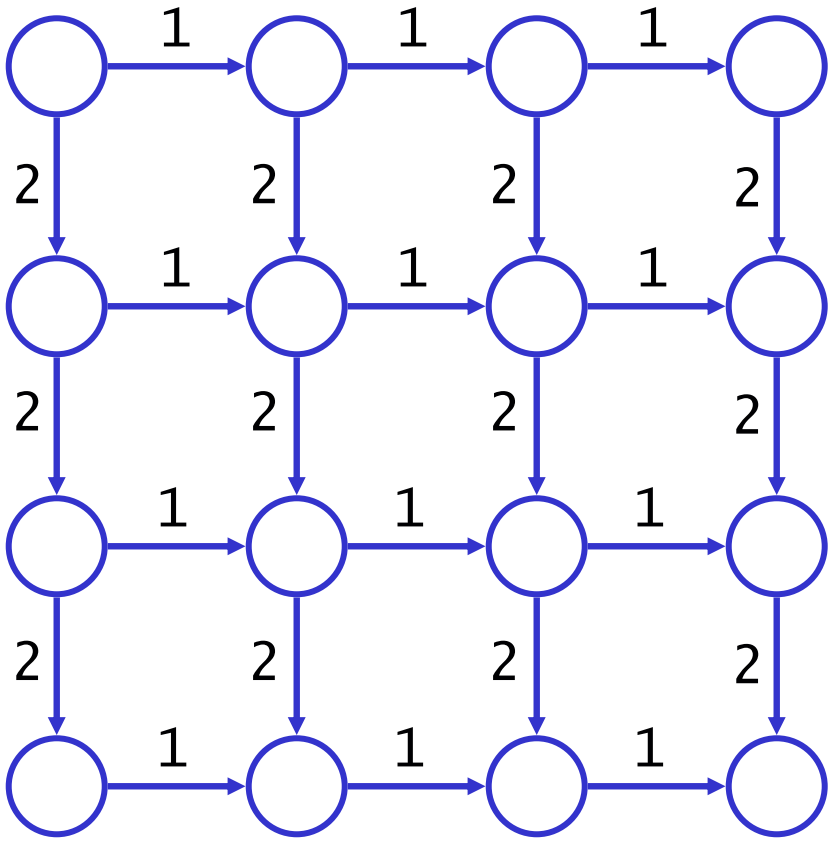
single head on trees



from trees to graphs



locally injective



grid, torus

nested pebbles to logic

$lab_a(x)$
$edg_i(x,y)$

~~$x \leq y$~~
$x = y$

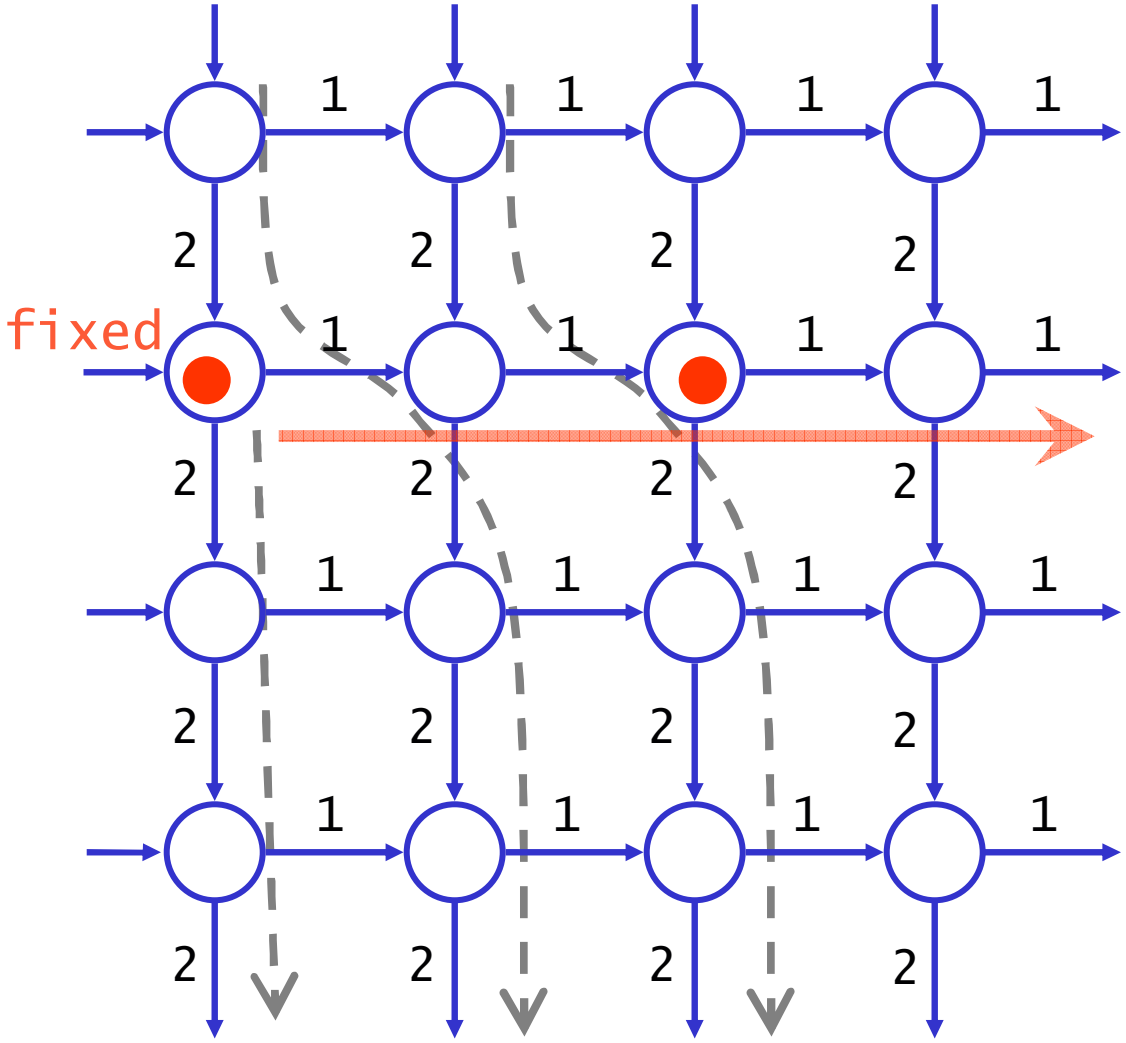
$\neg \ \wedge \ \vee$
$\forall x \ \exists x$

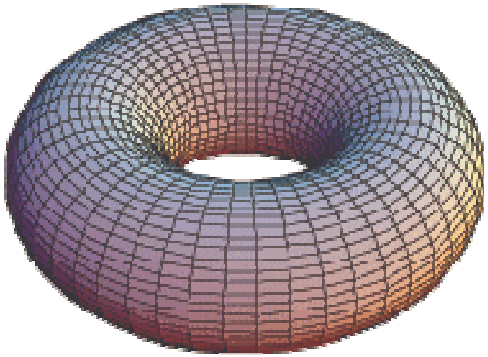
$\varphi^*(x,y)$

$$dPTW^k \subseteq FO{+}dTC^k$$

for families of graphs
(i.e. with fixed label alphabets)

walking the torus





two pebbles

(nested)

graphs with a guide

$$\mathrm{FO{+}dTC}^k = \mathrm{dPTW}^k$$

for families of *searchable* graphs
with a ‘guide’

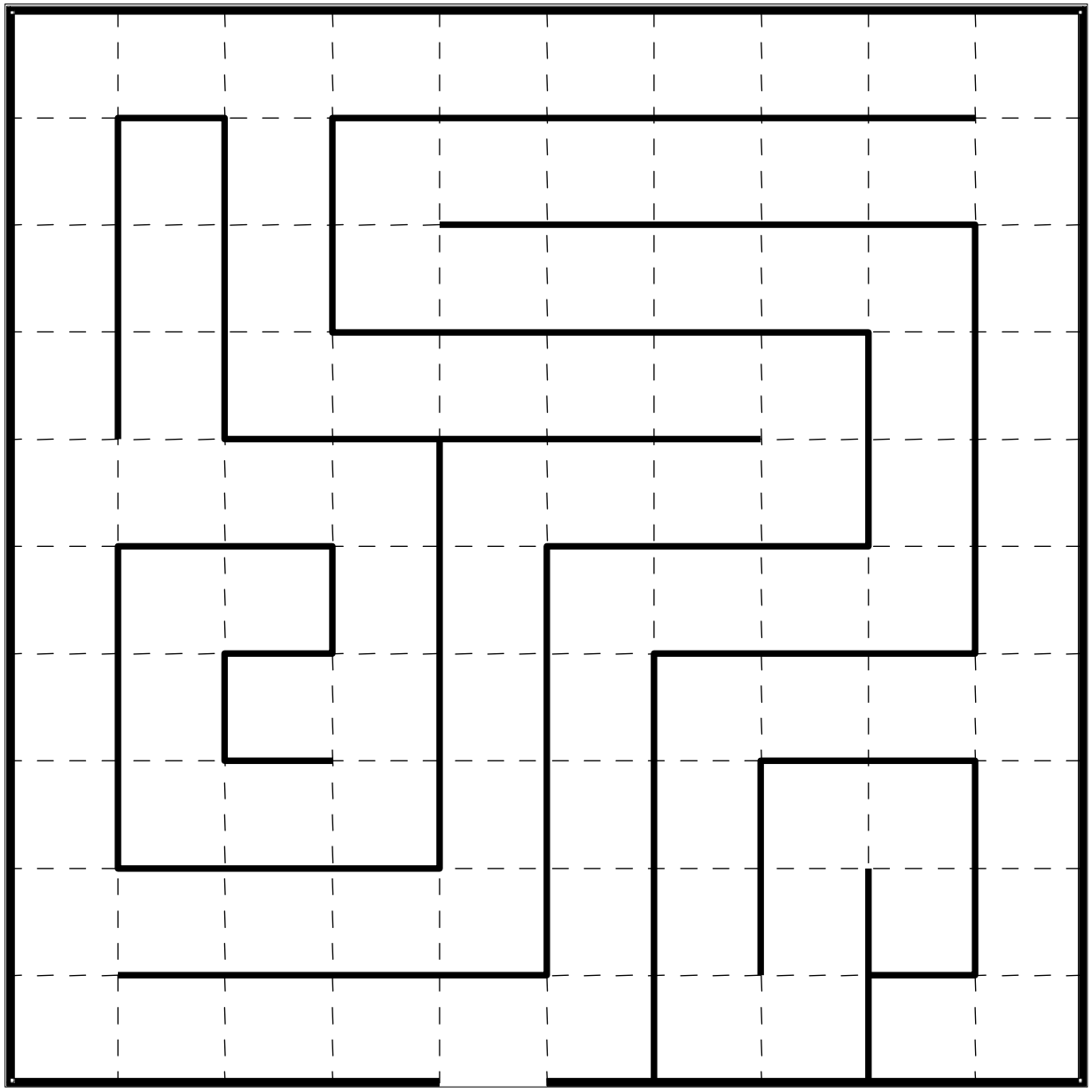
single head, deterministic
with pebbles
visits each node (at least) once
& halts

$(\forall x)\ \mathrm{lab}_0(x)$

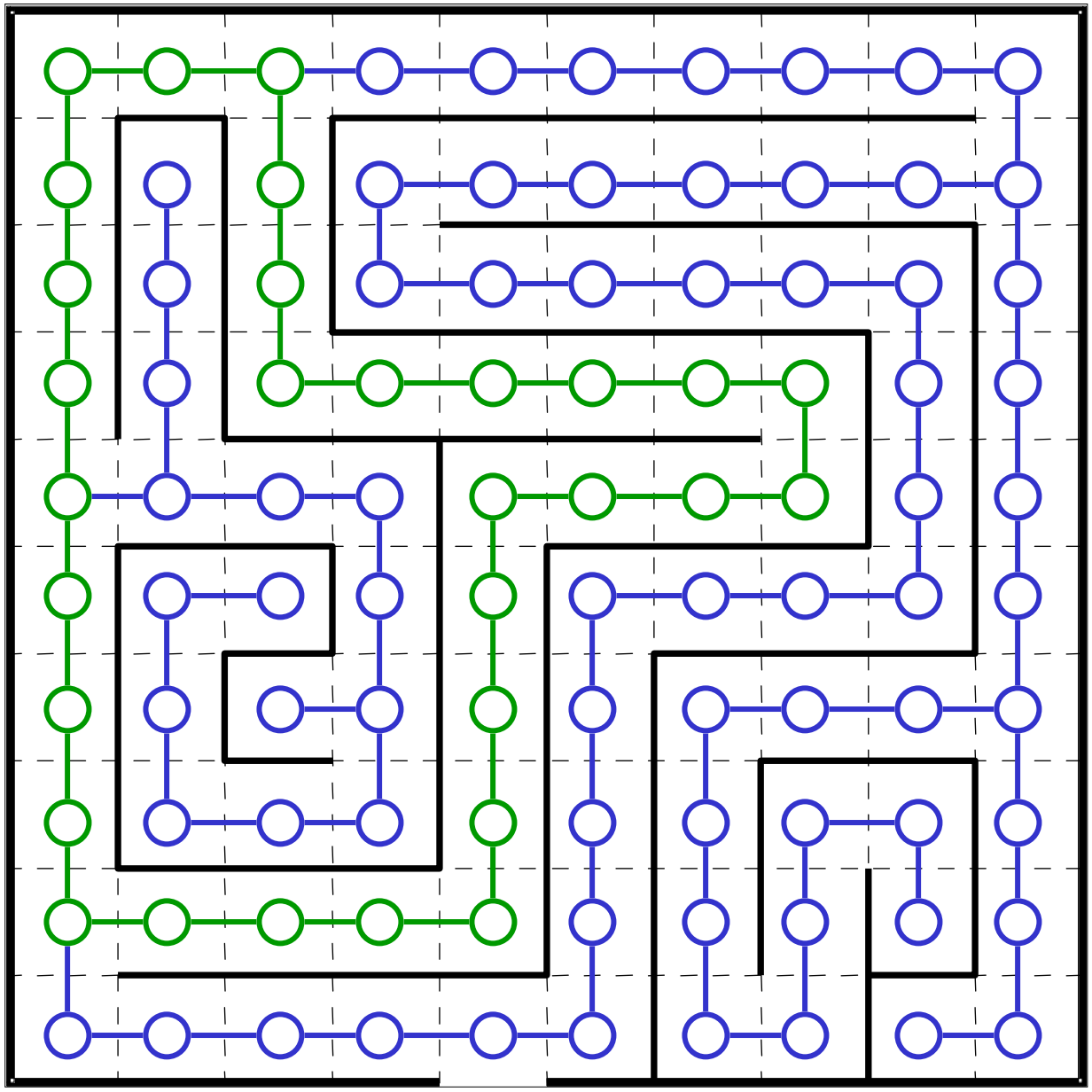
unranked trees, grids, toruses, …

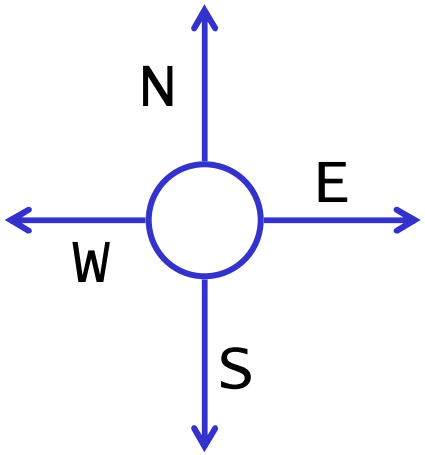
2 pebbles

mazes



mazes





Blum & Kozen

two heads!

(*not* nested)

searching with many heads

$$\mathrm{FO{+}dTC}^k = \mathrm{dPTW}^k$$

for families of *k-searchable* graphs

k heads, deterministic
with pebbles
visits each node (at least) once
& halts

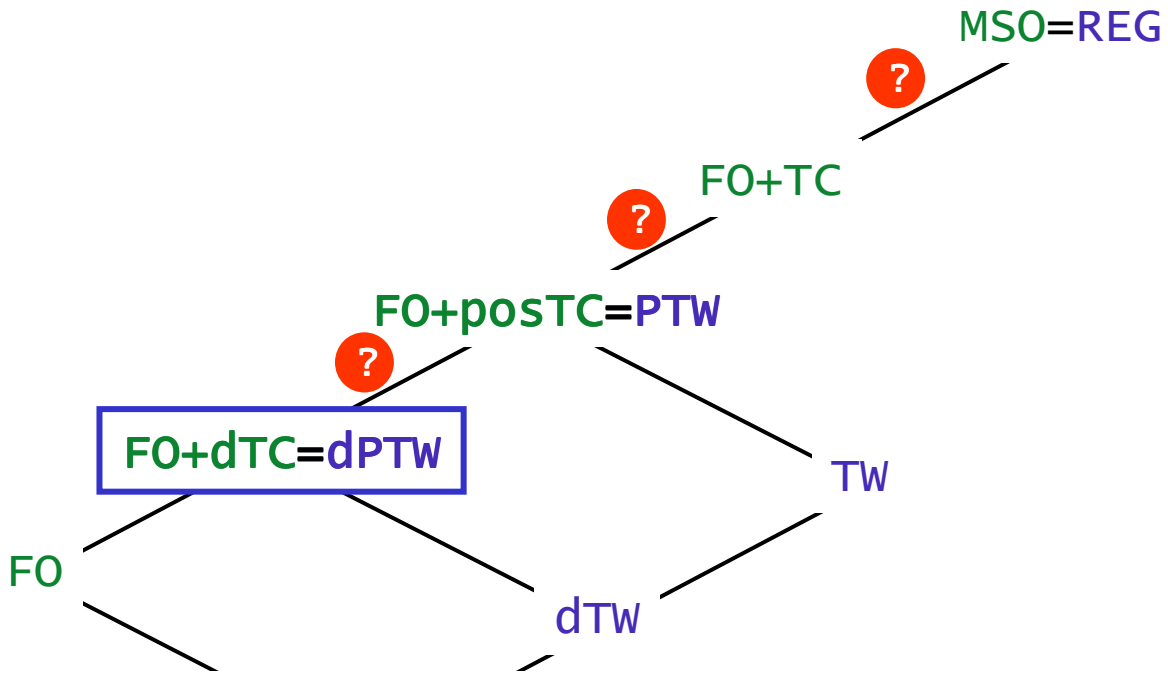
additional instruction
move head to pebble

Cook & Rackoff
'Jumping Automata'

mazes
not all graphs

finally: work to do …

open for single head on trees:

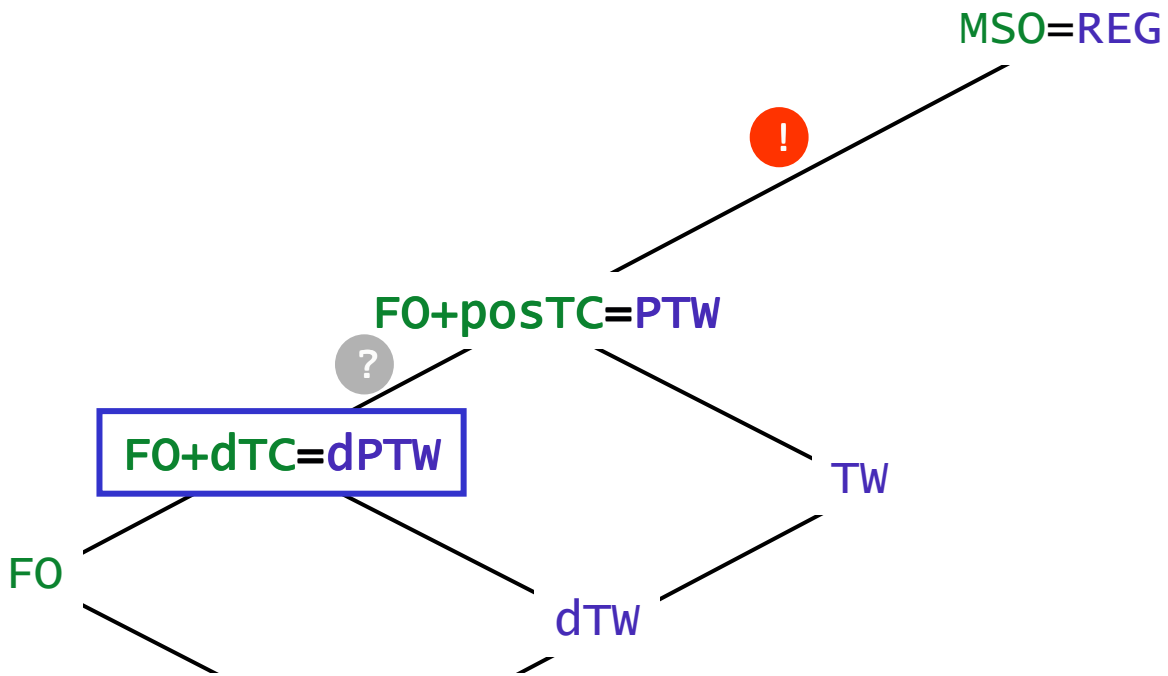


- ? dPTW ⊂ PTW ⊂ REG
- ? FO+dTC ⊂ FO+posTC ⊂ FO+TC ⊂ MSO
- ? pebble hierarchy
- ? type of pebbles physical vs. abstract
- ? alternation

work to do ...

until last week

Bojańczyk, Samuelides, Schwentick, Segoufin



- ! dPTW ⊆ PTW ⊂ REG
- ? FO+dTC ⊆ FO+posTC ⊆ FO+TC ⊆ MSO
- ! pebble hierarchy
- ! type of pebbles physical ⇔ abstract
- ? alternation

many heads? graphs?


‘tossing Pebbles’
Joost Engelfriet
Hendrik Jan Hoogeboom
Leiden NL
thank you …