Wrap Up

conclusion

conclusion: nice uniform family of graph games, suitable for the various game classes

not in this presentation:

deterministic classes are hard to prove complete: timing constraints
bounded det. ncl has no known planar normal form

2pers. games need two types of edges (apart from colours), for each of the players

for teams one needs hidden info, otherwise equivalent to 2p games

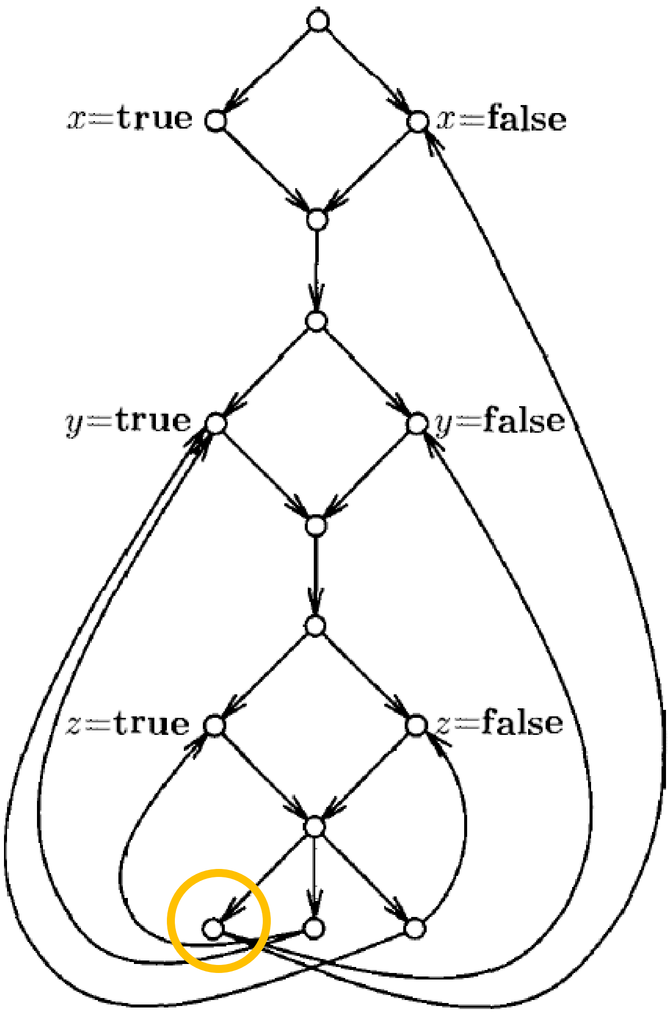
conclusion

roots can be found in the literature (see Geography)

take care: game of life (what is the ‘goal’?) is PSPACE, it also is undecidable ☺
(on infinite grid)

example of P complete:
the domino topling simulation

geography



[Schäfer, J.CSS, 1978]

THM. Geography is PSPACE-complete

two players on directed graph alternately pick next vertex, without repetition

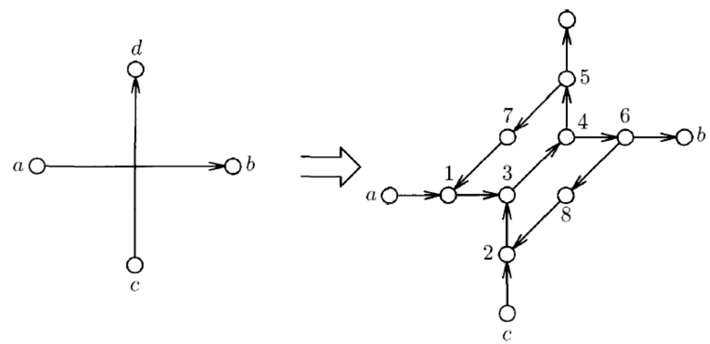
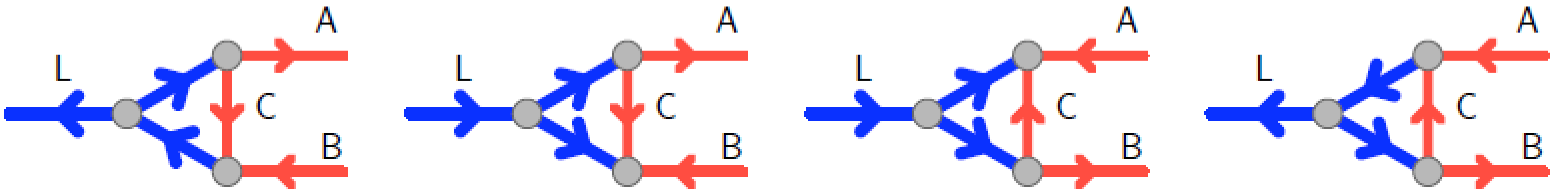


Figure 19-4. Crossing edges in GEOGRAPHY.

application to GO
[Lichtenstein&Sipser, J.ACM, 1980]

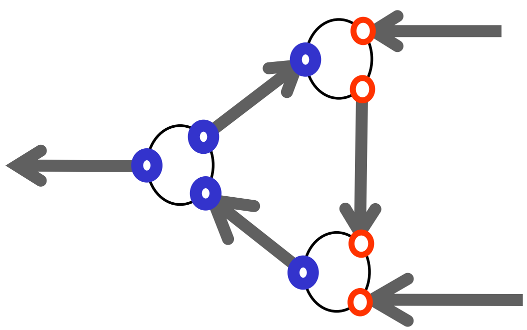
$$\exists x \forall y \exists z [(\neg x \vee \neg y) \wedge (y \vee z) \wedge (y \vee \neg z)].$$

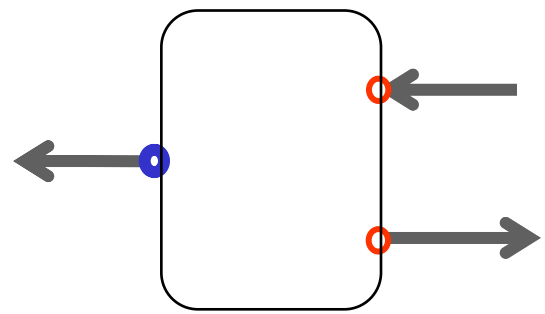
latch / protected OR



(a) Locked, A active (b) Unlocked, A active (c) Unlocked, B active (d) Locked, B active

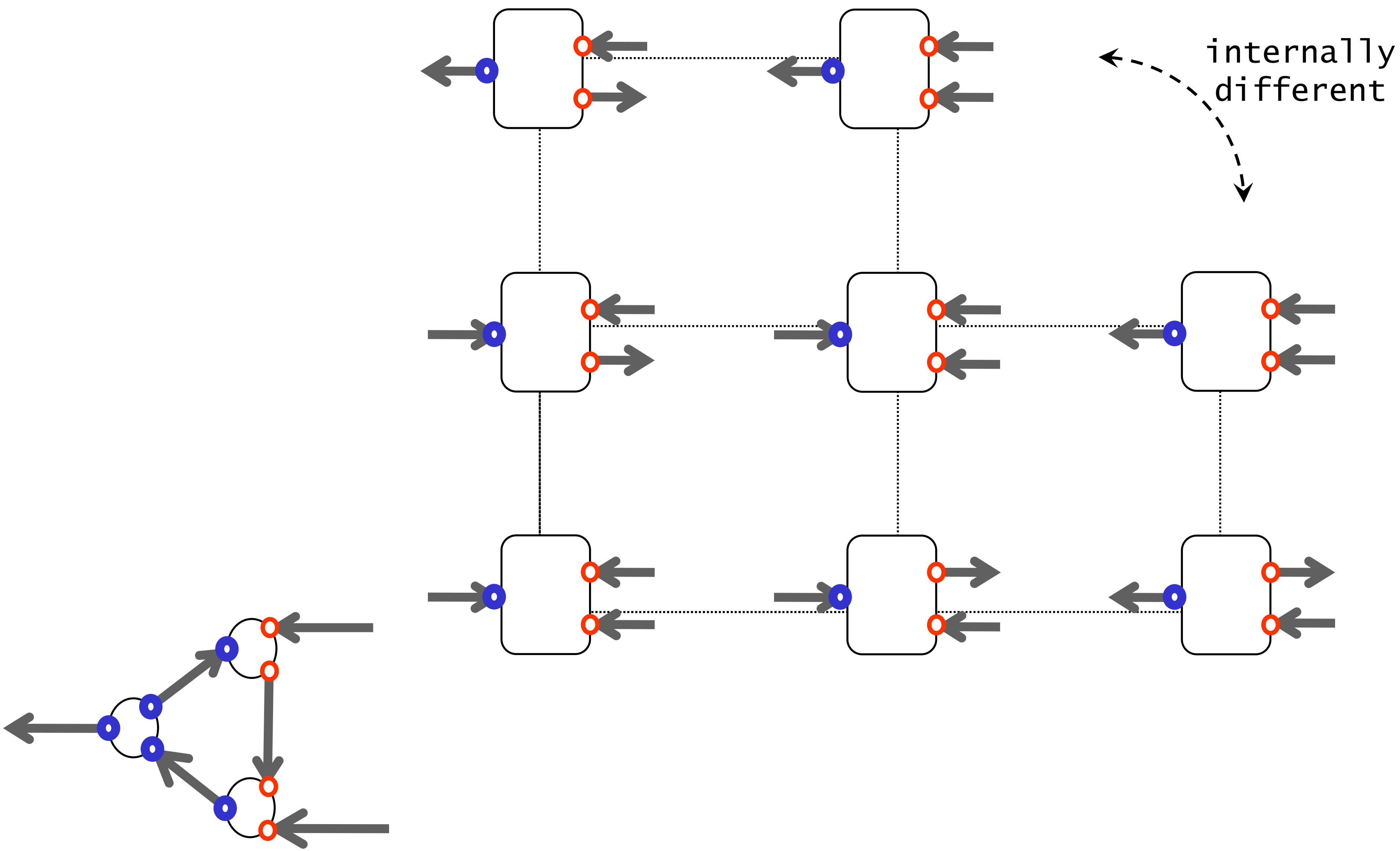
Figure 5-6: Latch gadget, transitioning from state A to state B.





we are not (really) interested in the internal states
only in the external connections:
input-output behaviour
synchronization, silent actions & simulation

latch behaviour



formula games – complete problems

NL **2SAT**

$(x_1 \vee x_3) \wedge (\neg x_5 \vee \neg x_3) \wedge (x_5 \vee x_1)$

P **HORN-SAT**

$(\neg x_3 \vee \neg x_2 \vee \neg x_5 \vee x_1)$ i.e. $(x_3 \wedge x_2 \wedge x_5 \rightarrow x_1)$

NP **SAT** satisfiablity

$\exists x_1 \; \exists x_3 \; \exists x_5 \; (x_1 \vee x_3 \vee \neg x_5) \wedge (\neg x_1 \vee \neg x_3) \wedge (x_5 \vee x_1)$

PSPACE **QBF** aka QSAT

$\exists x_1 \; \forall x_3 \; \exists x_5 \; (x_1 \vee x_3 \vee \neg x_5) \wedge (\neg x_1 \vee \neg x_3) \wedge (x_5 \vee x_1)$

formula games – complete problems

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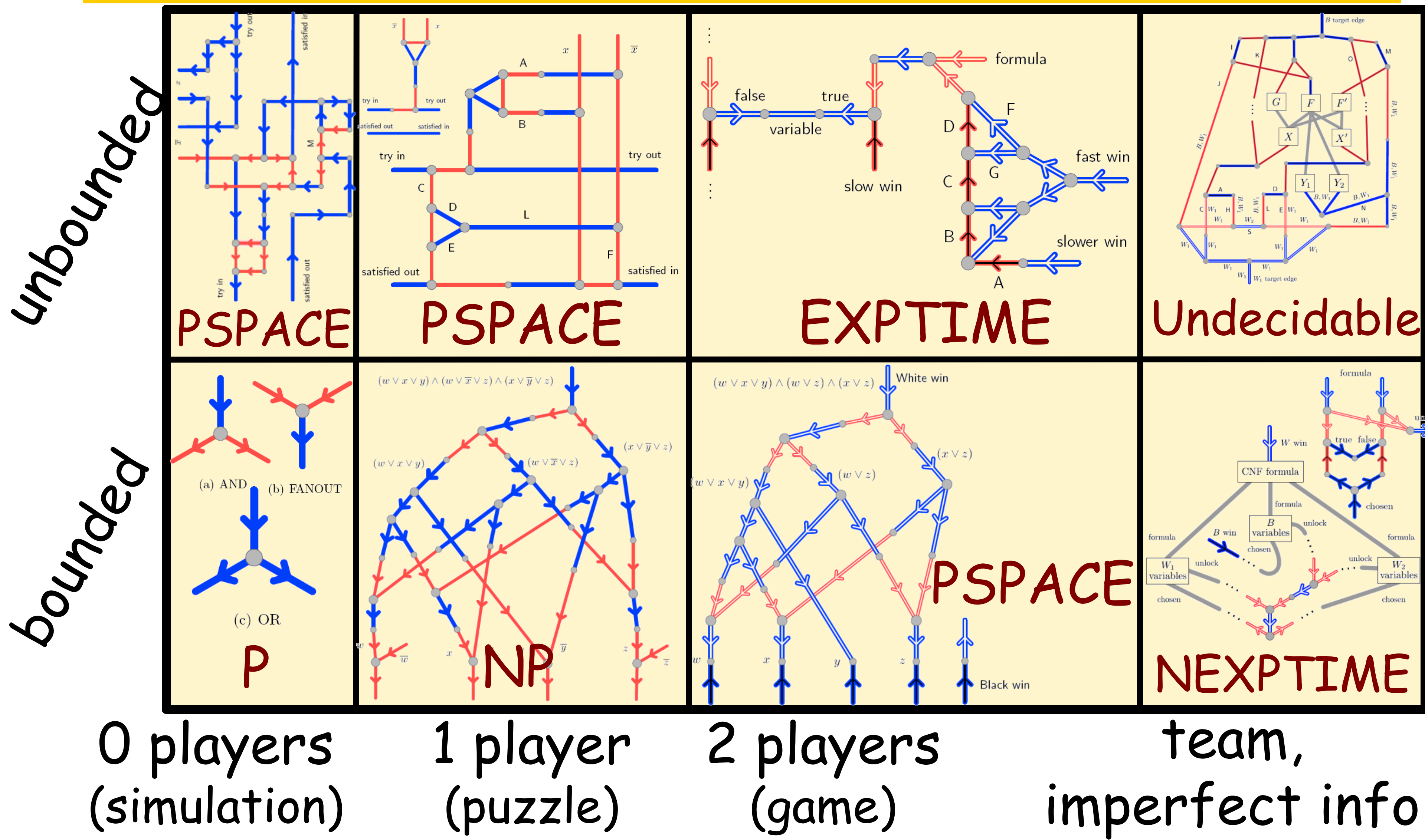
PSPACE **QBF** aka QSAT

$\exists x_1\ \forall x_3\ \exists x_5\ (x_1 \vee x_3 \vee \neg x_5) \wedge (\neg x_1 \vee \neg x_3) \wedge (x_5 \vee x_1)$

EXPTIME **G_6** [Stockmeyer & Chandra]

given: CNF formula F variables X Y
initial assignment
player I (and II) change single variable in X (Y)
taking turns, passing allowed
player I wins if F becomes true
question: does I have a forced win?

Constraint Logic
[Hearn & Demaine 2009]



Games, Puzzles & Computation

IPA Advanced Course on Algorithmics and Complexity

Eindhoven, 8 July 2016

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