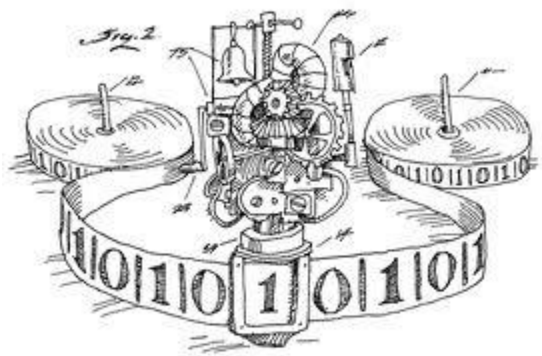
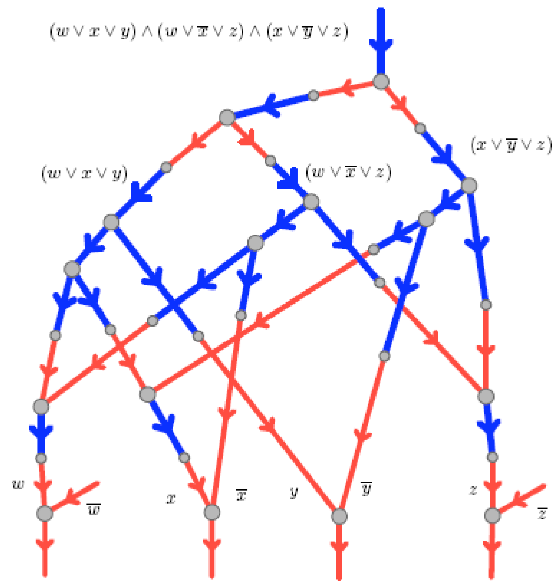
TipOver is NP-Complete

NP & TipOver



$(w \vee x \vee y) \wedge (w \vee \overline{x} \vee z) \wedge (x \vee \overline{y} \vee z)$



NP

3SAT

part I

constraint logic

‘graph games’

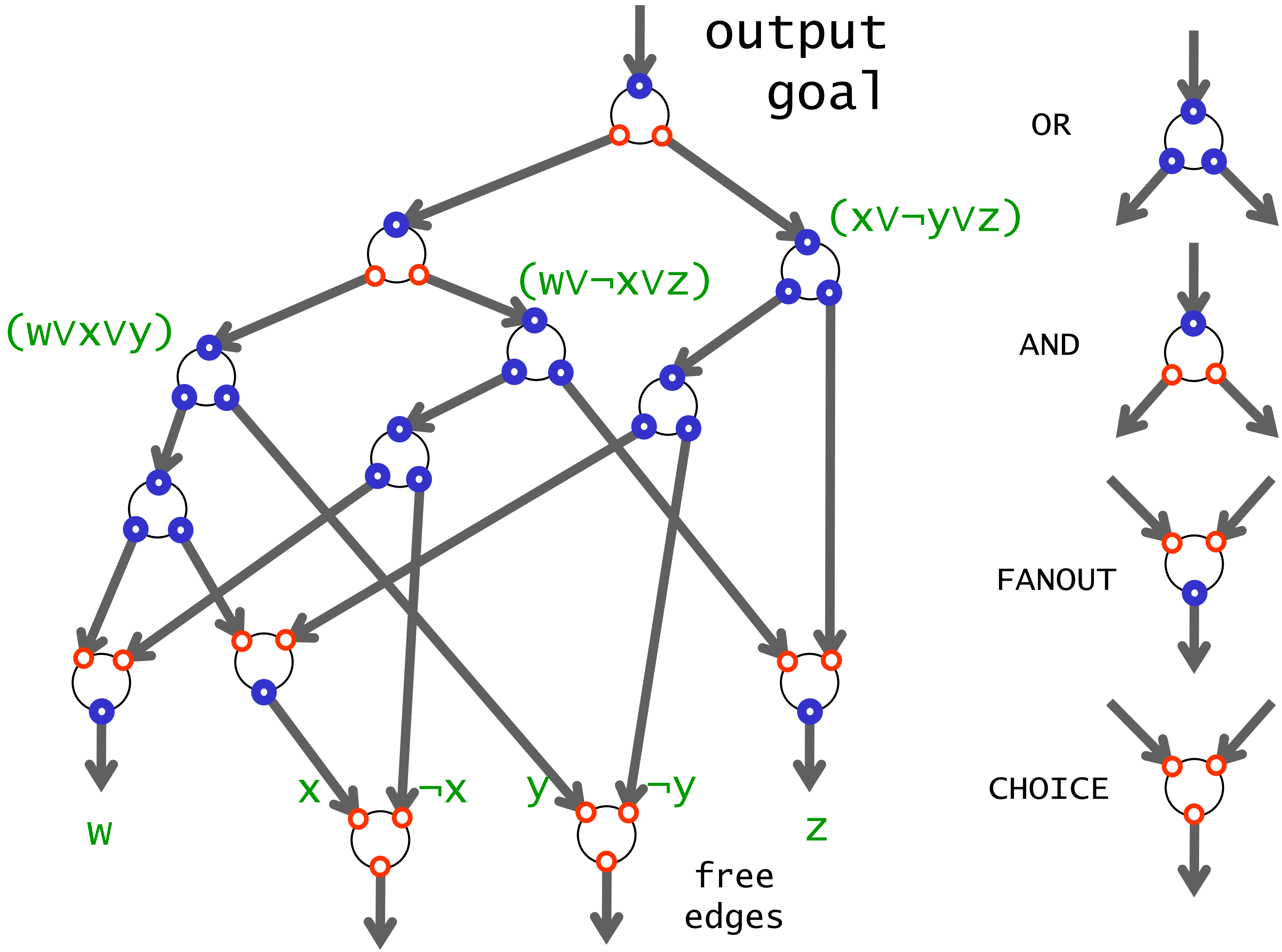
Bounded NCL

part II

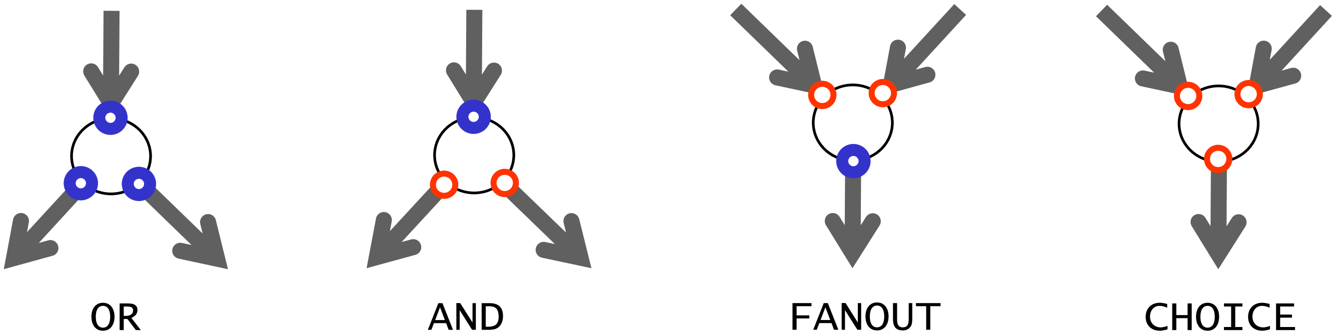
games in particular

TipOver

formula constraint graph



conclusion



BOUNDED NCL - nondet constraint logic

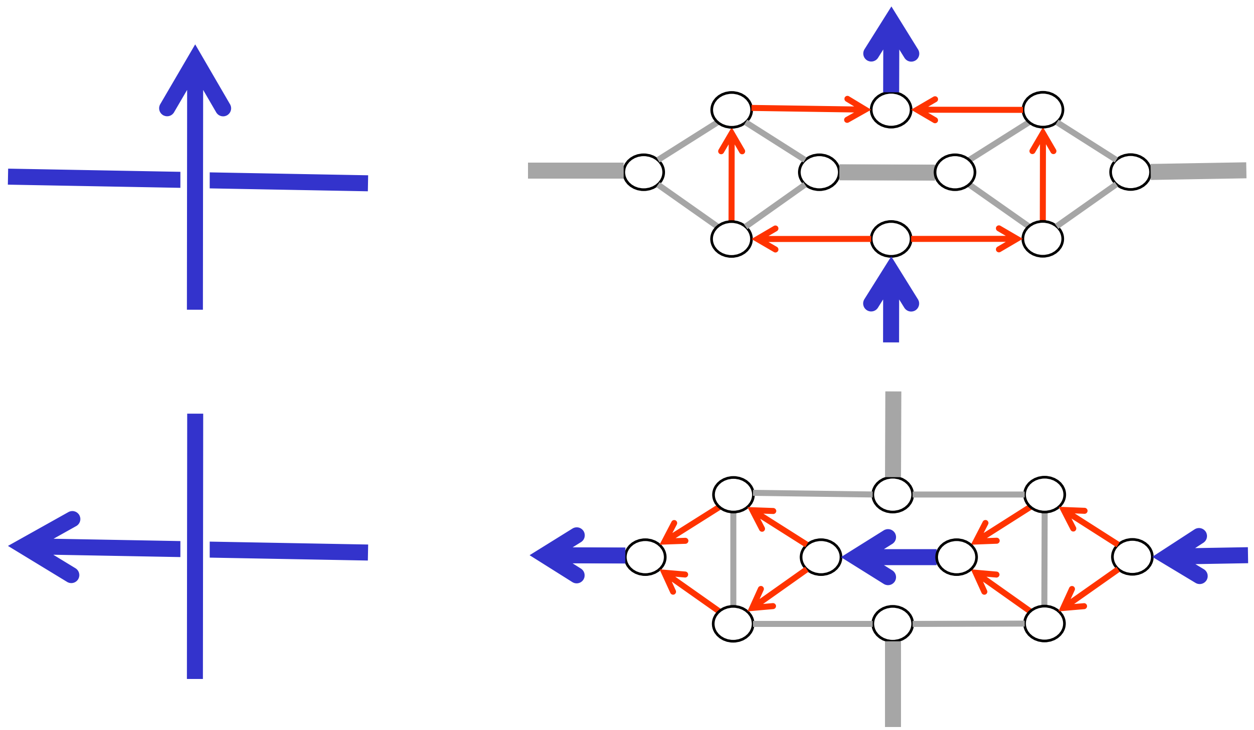
instance: constraint graph G, edge e

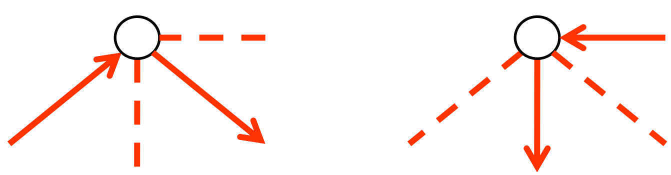
question: sequence which reverses *each edge at most once*, ending with e

Bounded NCL is NP-complete

however: topling domino's cannot cross

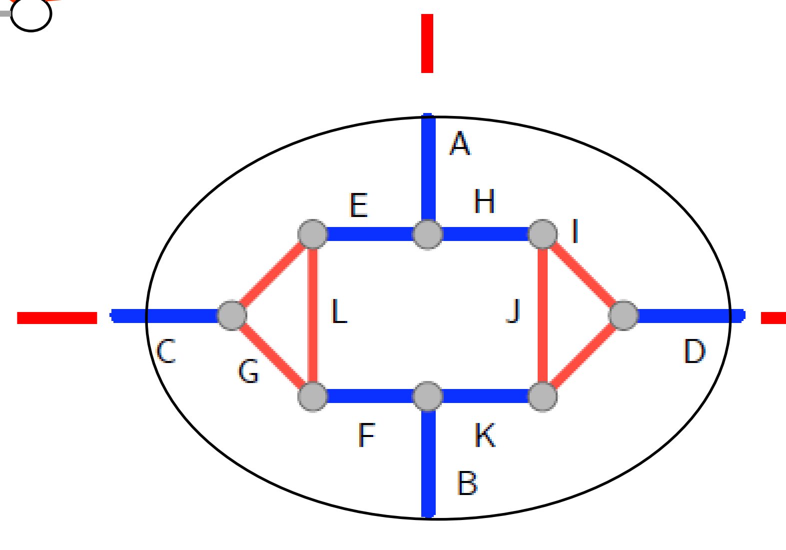
planar crossover gadget





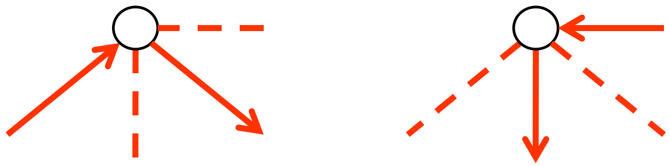
necessary behaviour

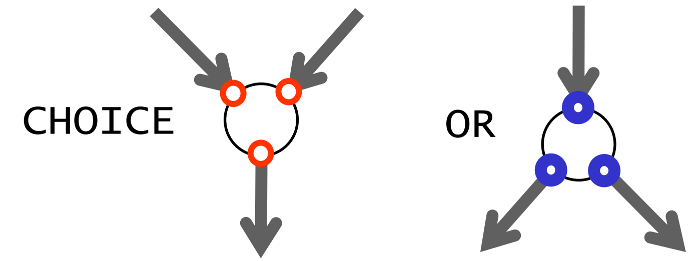
either of them or both!

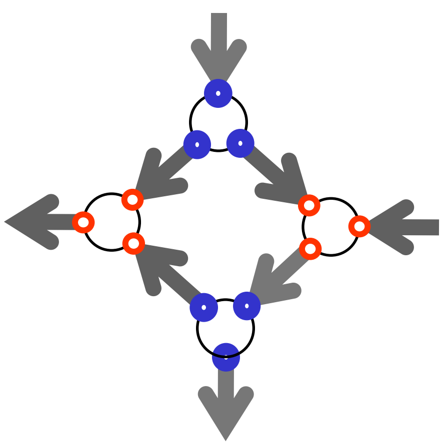


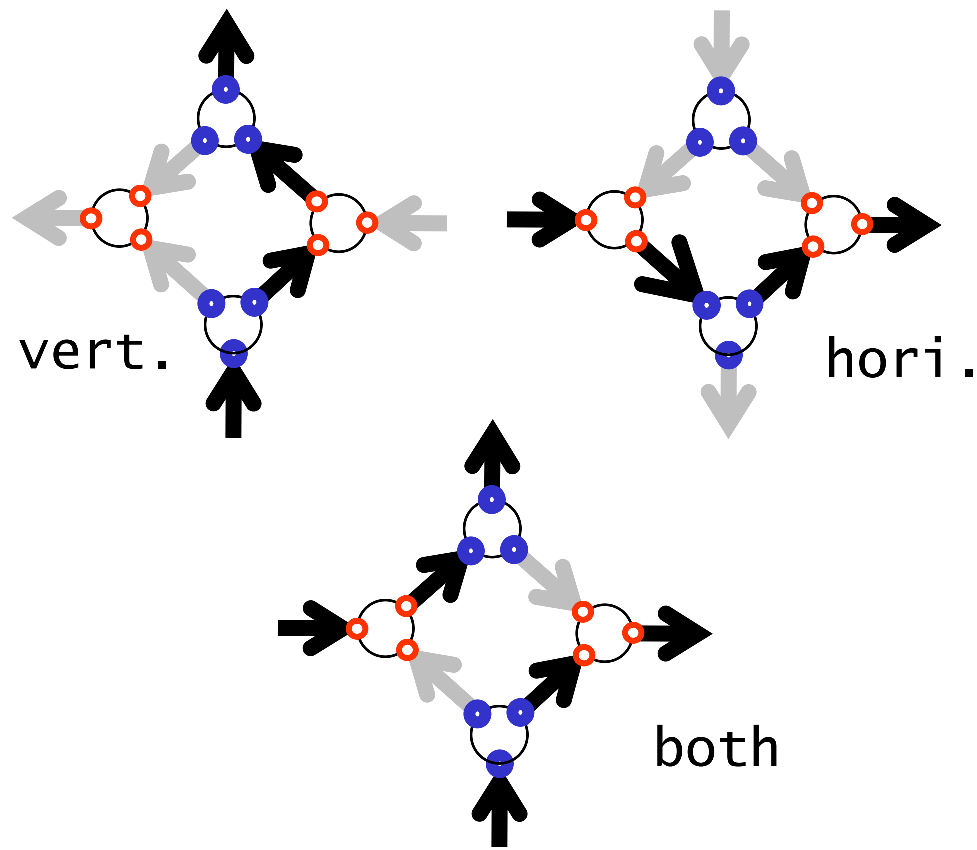
half-crossover ncl type

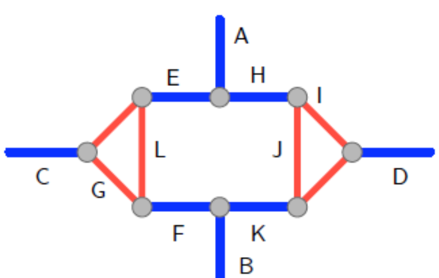
half-crossover







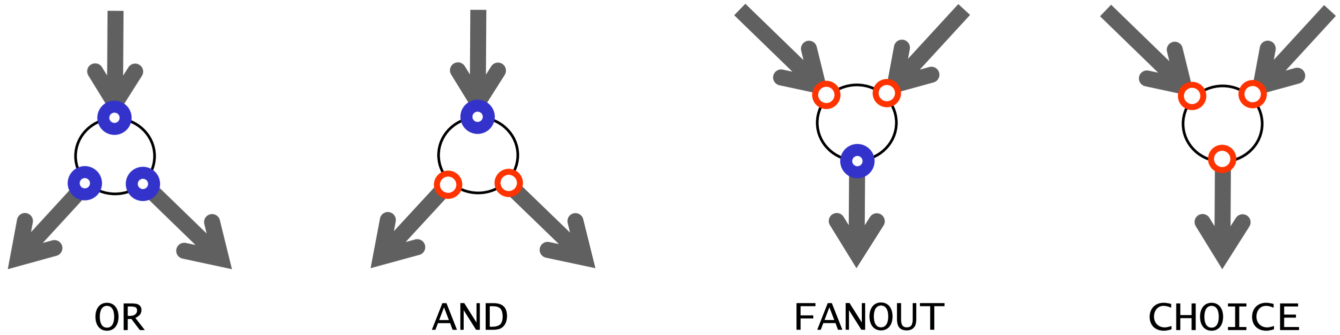




(b) Half-crossover

half-crossover *bounded* ncl type

conclusion(2)



BOUNDED NCL - nondet constraint logic

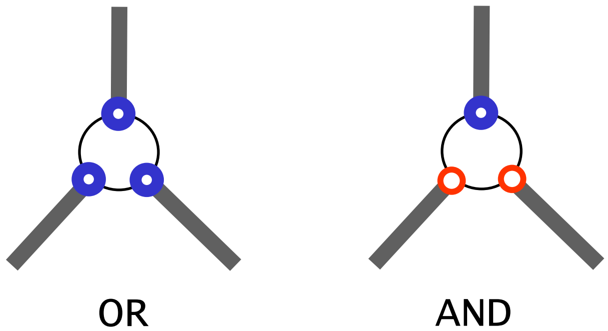
instance: constraint graph G, edge e

question: sequence which reverses *each edge at most once*, ending with e

Bounded NCL is NP-complete,

even for planar graphs,
with restricted vertices

conclusion (next hour)



NCL - nondet constraint logic

instance: constraint graph G, edge e

question: sequence which reverses e

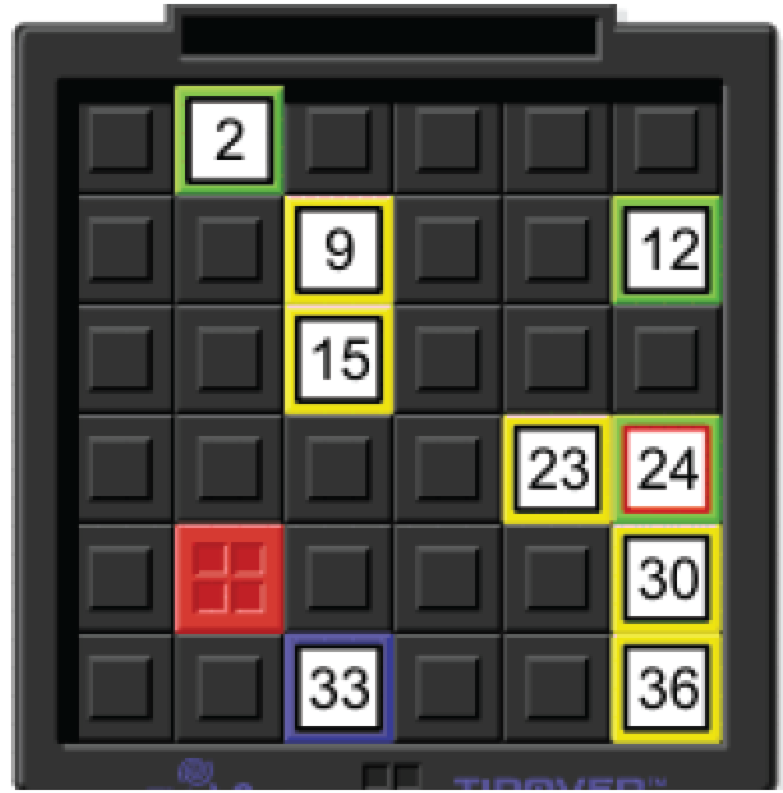
NCL is PSPACE-complete,

even for planar graphs,

with restricted vertices

application: TipOver

http://www.puzzles.com/products/tipover/PlayOnLine.htm



2

3

4

goal

24 initial position

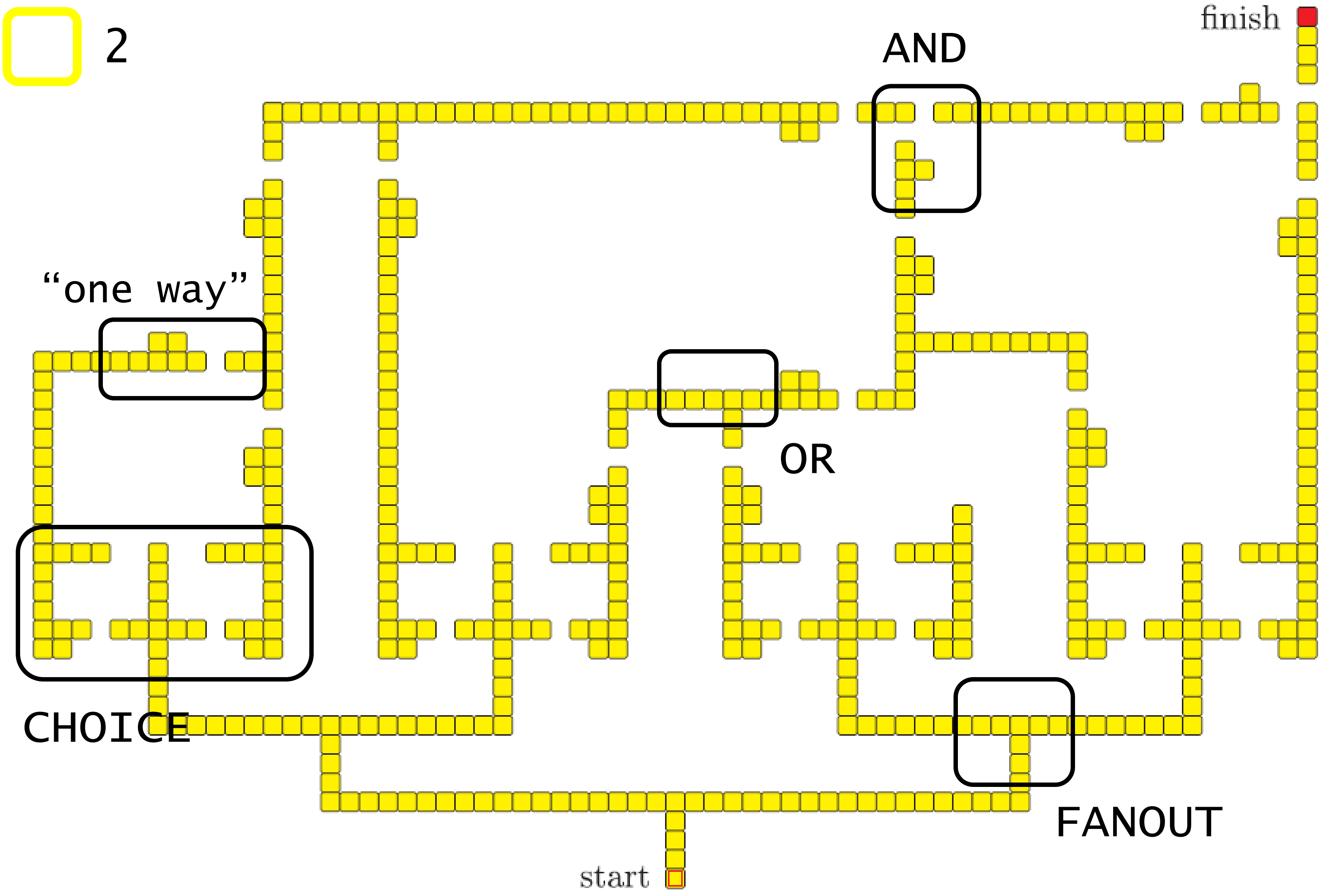


Figure 9-7: TipOver puzzle for a simple constraint graph.

gadgets: “one way”, OR

invariant:

- can be reached ⇔ can be inverted
- all visited positions remain connected

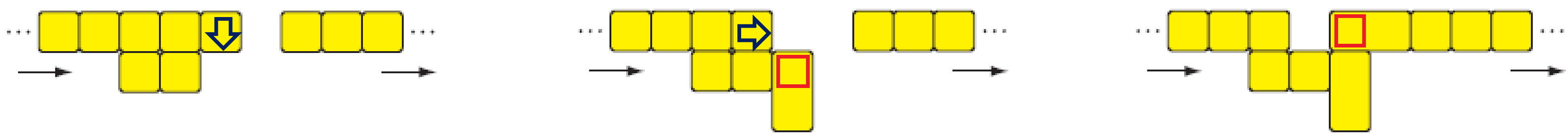
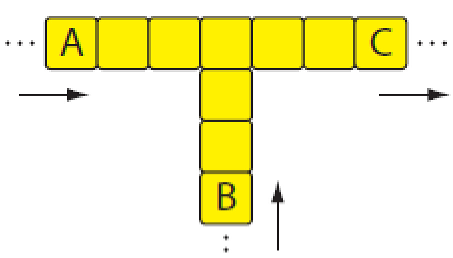
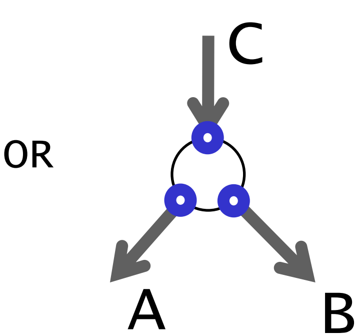


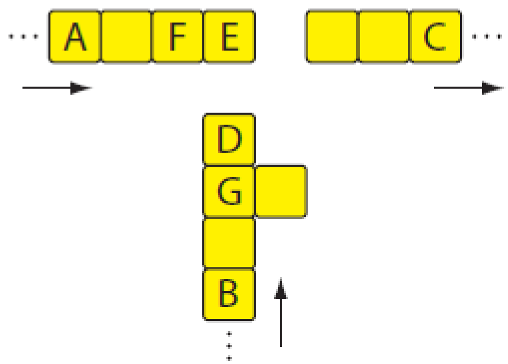
Figure 9-3: A wire that must be initially traversed from left to right. All crates are height two.



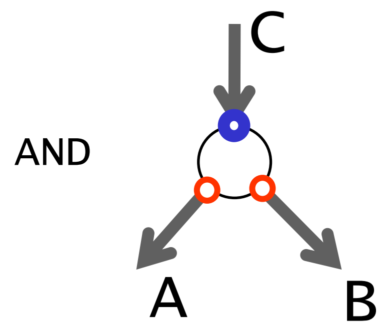
(a) OR gadget. If the tipper can reach either A or B, then it can reach C.



gadgets: AND



(b) AND gadget. If the tipper can reach both A and B, then it can reach C.



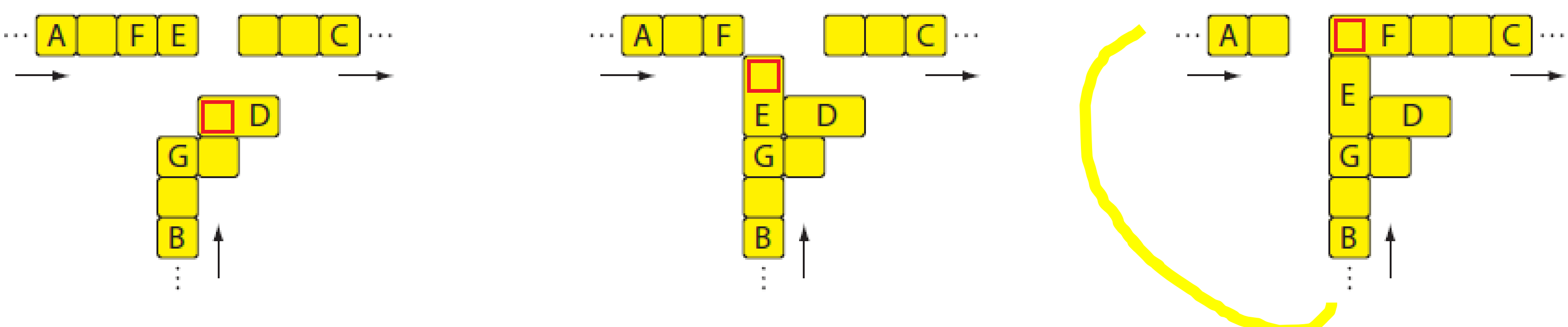
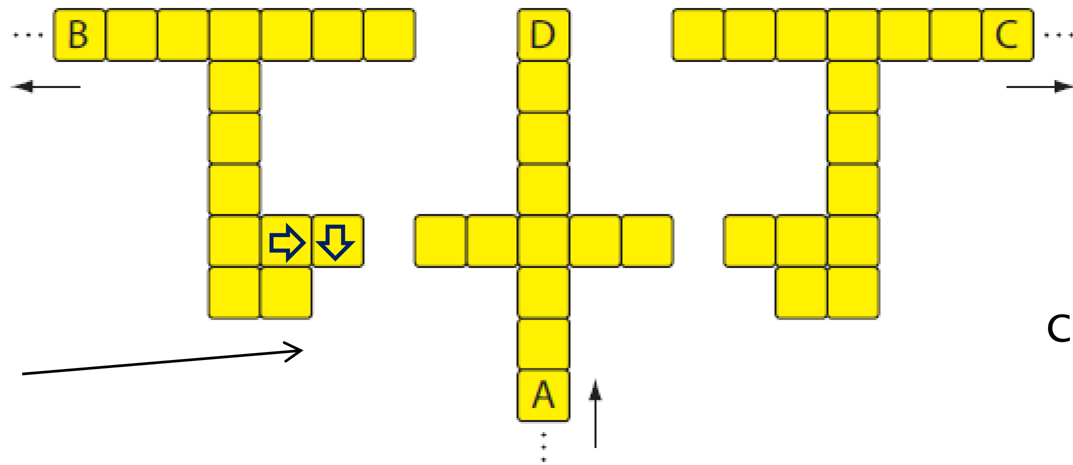


Figure 9-5: How to use the AND gadget.

remains connected

gadgets: CHOICE, FANOUT



reconnect

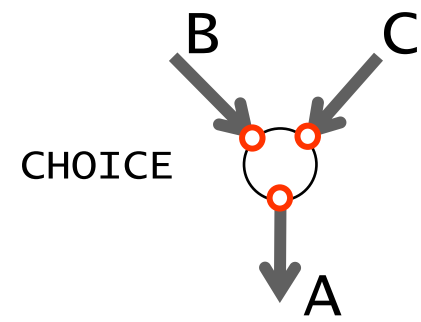
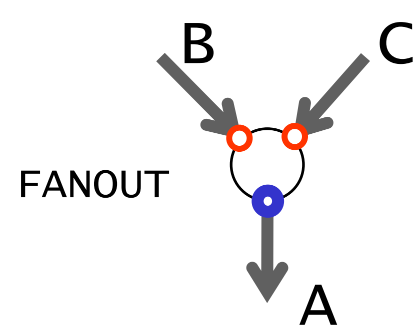


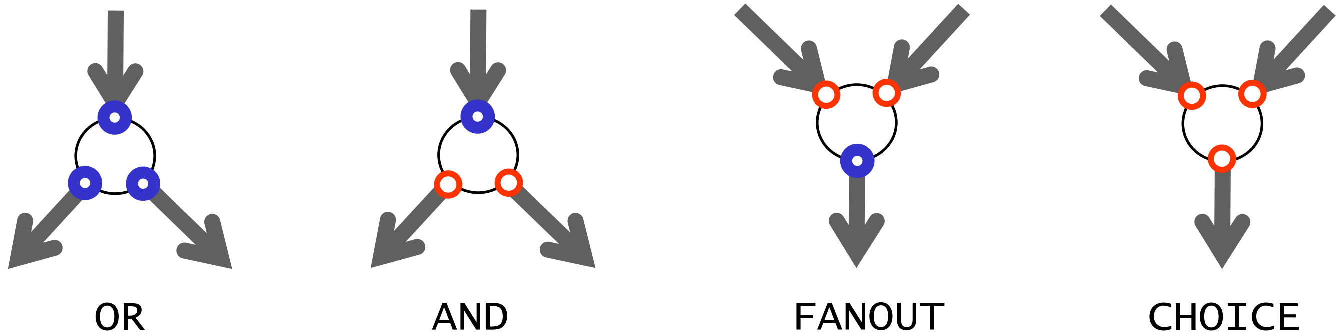
Figure 9-6: TipOver CHOICE gadget. If the tipper can reach A, then it can reach B or C, but not both.

use one-way gadgets at B and C

(control information flow)



conclusion



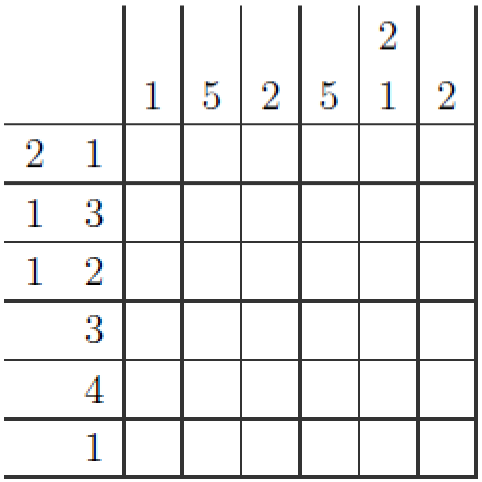
Bounded NCL is NP-complete,
even for planar graphs,
with restricted vertices

thm. TipOver is NP-complete

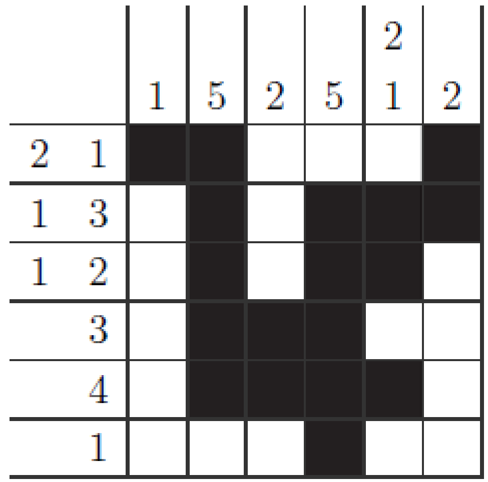
NP complete bounded games

Jan van Rijn: Playing Games:
The complexity of Klondike, Mahjong, Nonograms
and Animal Chess
(Master Thesis, 2013, Leiden)

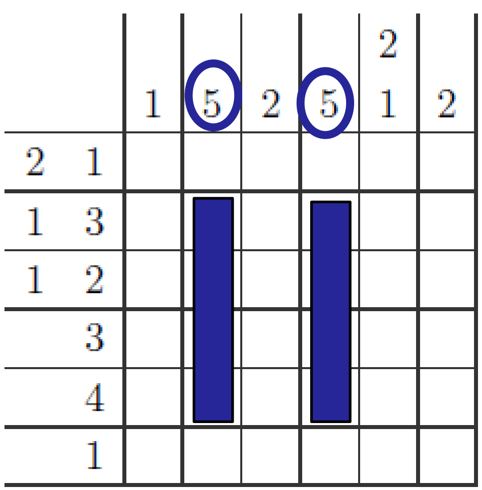
(ctd.) nonograms

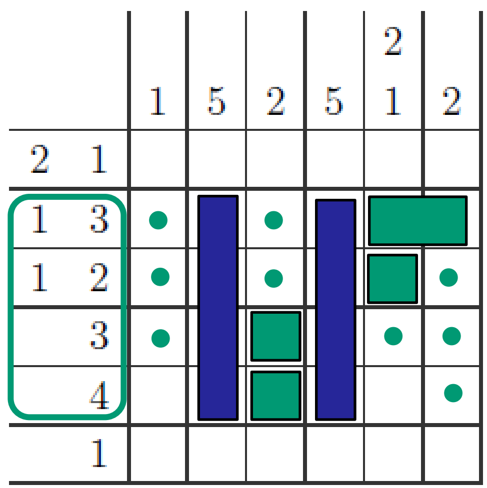


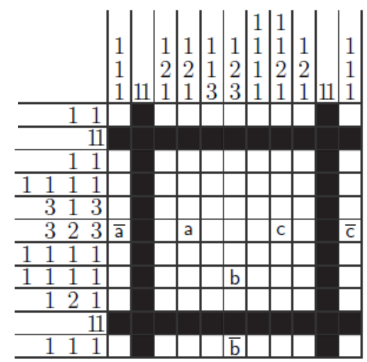
(a) 6 × 6 Nonogram



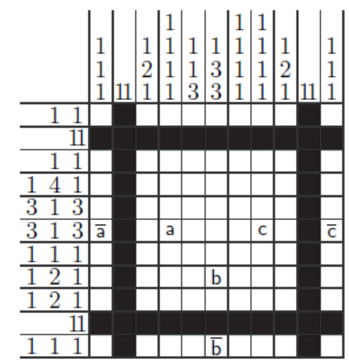
(b) Solved Nonogram



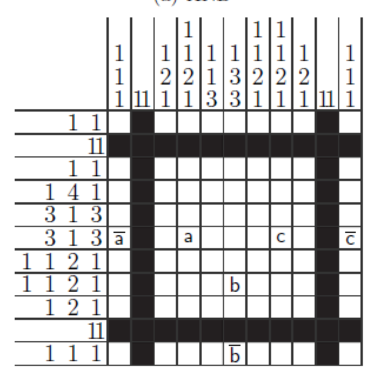




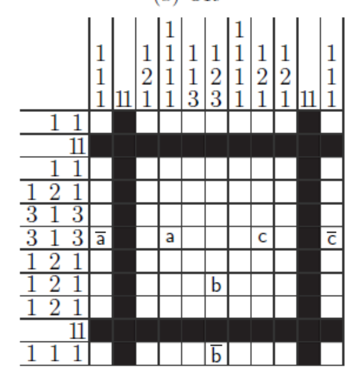
(a) AND



(b) OR



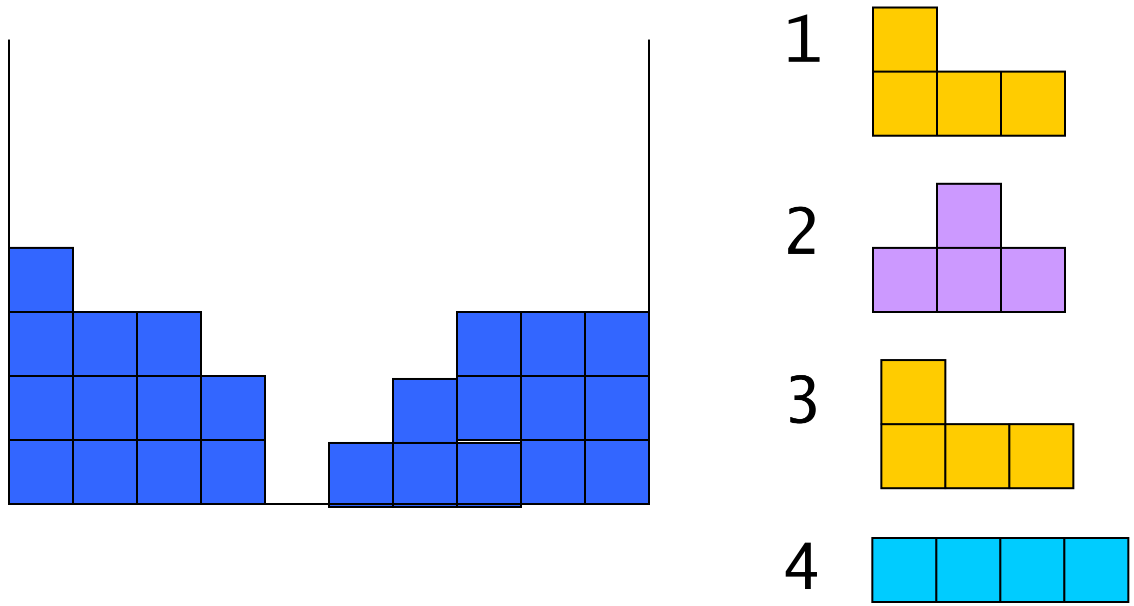
(c) FANOUT



(d) CHOICE

Tetris is NP complete

"Given an initial game board and a sequence of pieces, can the board be cleared?"



Breukelaar, Demaine, Hohenberger, Hoogeboom, Kosters, Liben-Nowell. Tetris is Hard, Even to Approximate. Selected Papers from the Ninth Int. Computing and Combinatorics Conf. (COCOON 2003). *Int. J. of Computational Geometry and Applications* 14 (2004) 41-68.

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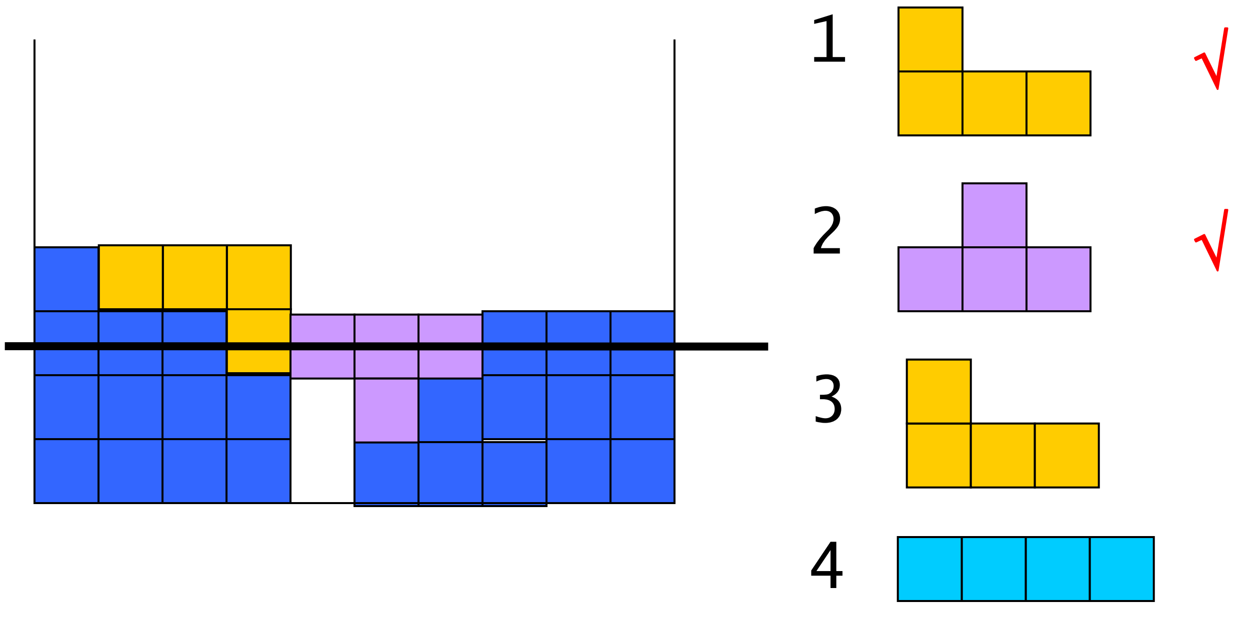
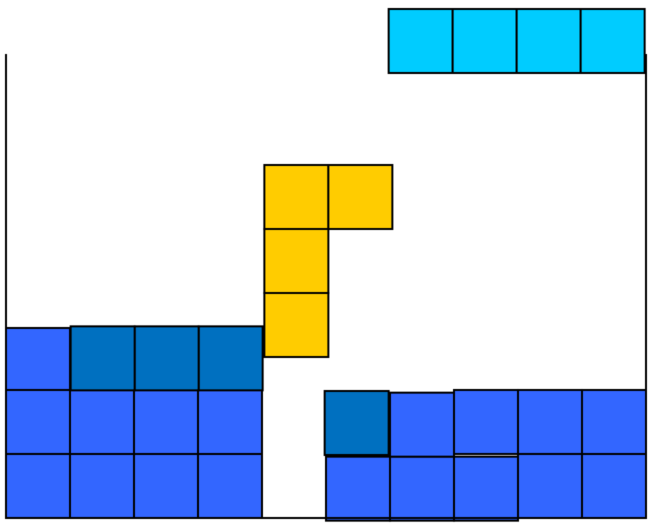
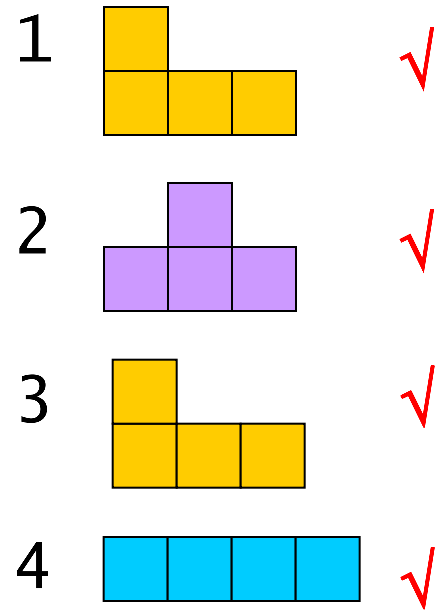


Tetris is NP complete

"Given an initial game board and a sequence of pieces, can the board be cleared?"



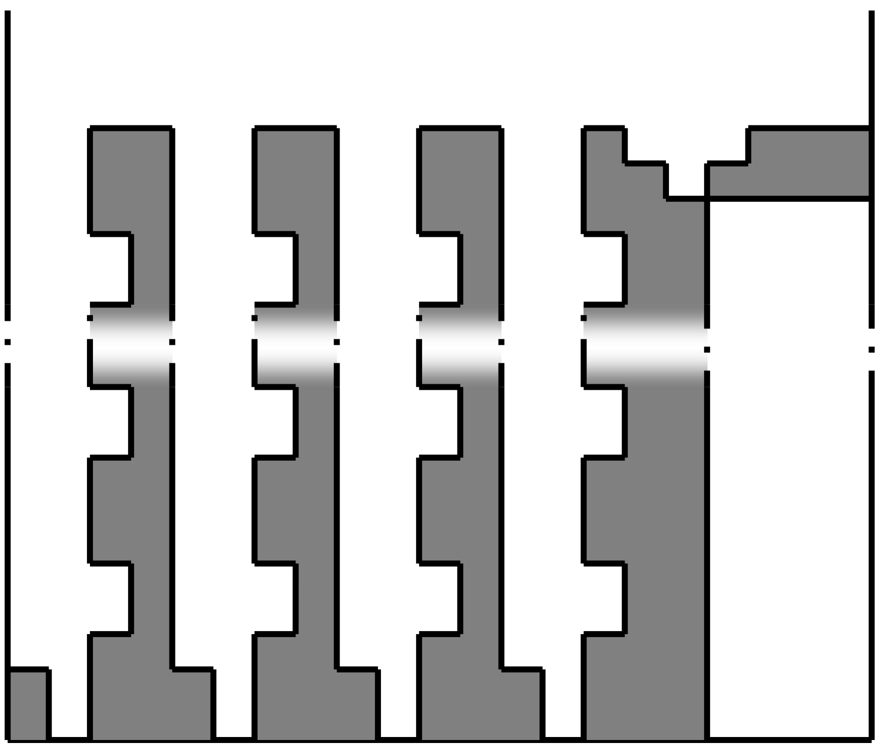


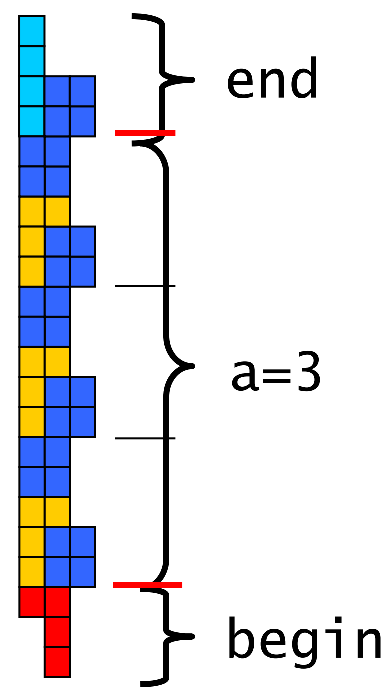
yes!

Tetris is NP complete

"Given an initial game board and a sequence of pieces, can the board be cleared?"

reduction from 3-partitioning problem
(can we divide set of numbers into triples?)



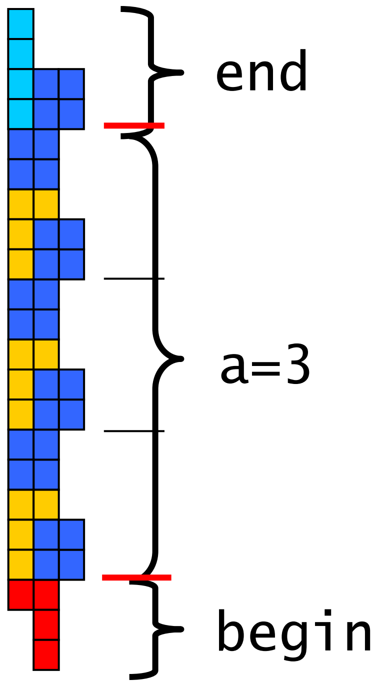


Tetris is NP complete

reduction from 3-partitioning problem
(can we divide set of numbers into triples?)

OPEN: directly with Bounded NCL ?

find OR, AND, FANOUT, CHOICE



Games, Puzzles & Computation

IPA Advanced Course on Algorithmics and Complexity

Eindhoven, 8 July 2016

Walter Kosters
Hendrik Jan Hoogeboom

LIACS, Universiteit Leiden