1) **a.** Choose $p \hat{=}$ 'shines today', and $q \hat{=}$ 'shines tomorrow'. We get $p \rightarrow (\neg q)$.

c. Choose $p \hat{=}$ 'shines today', and $r \hat{=}$ 'rains today'. We explicily have to express *exclusive or*, $(p \vee r) \wedge \neg(p \wedge r)$.

d. Both options, or only one of them?

2) 'In full' suggests to add also the outer parentheses we usually omit.

$((\neg p) \rightarrow (p \wedge r))$
$((p \rightarrow q) \rightarrow (r \rightarrow (s \vee t)))$
$((p \vee q) \rightarrow ((\neg p) \vee r))$
$p \vee q \wedge r$ has no fixed meaning if we strictly follow the rules of the exercise. The binding order of $\vee$ and $\wedge$ is not specified, associativity seems to hold only for $\rightarrow$.

3) **b.** Sorry, I mean natural language 'and', to have two tables, to compare the two related formulas.

c. We see from the table that $(q \wedge (r \rightarrow q))$ is actually equivalent to q.

p	q	r	$p \vee$	$(\neg($	$q \wedge$	$(r \rightarrow q)))$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	T	T	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	T	F	F	F	T	T
F	F	T	T	T	F	F
F	F	F	T	T	F	T
			4	3	2	1

4) **a.** $(\forall x)A(m, P(x))$ is not correct as '$P(x)$' means the logical statement 'x is a professor' (true or false) and can not be written in place of a person 'professor x'.
$(\forall x)(P(x) \rightarrow A(m, x))$ For every x, when professor, then Mary admires x
This is *not* the same as $(\forall x)(P(x) \rightarrow A(m, x))$ which also states that all x are professors. We assume there are also students.

b. $(\exists x)(P(x) \wedge A(x, m))$ There is a person who is professor and admires Mary.

c. No student attended every lecture. Tricky. There is no single student that attended all lectures. $\neg(\exists x)(\forall y)(\ S(x) \wedge L(y) \wedge B(x, y)\ )$
Every student missed at least one lecture: $(\forall x)(\ S(x) \rightarrow (\exists y)(L(y) \wedge \neg B(x, y))\ )$

5) **a.** For every person there is a person that is the mother of the first. $(\forall x)(\exists y)M(y, x)$

b. $(\ (\forall x)(\exists y)M(y, x)\ ) \wedge (\ (\forall x)(\exists y)F(y, x)\ )$
or $(\forall x)(\exists y)(\exists z)(M(y, x) \wedge M(z, x))$

d. $(\exists x)(\exists y)(F(\mathrm{Ed}, x) \wedge F(x, y))$

e. Probably: $(\forall x)(\ (\exists y)F(x, y) \rightarrow (\exists y)(F(x, y) \vee M(x, y))\ )$

j. $H(\text{Ed}, \text{Patsy})$

k. Brother of husband of Monique, or husband of sister of Monique, assuming old-fashioned marriages.

6) In the first case ϕ is not true. If we choose $x = b$ and $y = a$ then $R(x, y)$ is true, as (b, a) is in the relation given. However we can not find $z \in A$ that makes $R(y, z)$ true, as there is no pair (b, z) in the relation. This makes $R(x, y) \to R(y, z)$ false (for this x, this y and all z). Consequently ϕ is false.

Second case. Assume x and y are chosen that make $R(x, y)$ true. We consider cases to find z.
$x = b$, $y = c$ then we choose $z = b$,
$x = a$, $y = b$ then we choose $z = c$,
$x = c$, $y = b$ then we choose $z = c$.
Hence we can alwyas find z, which makes the statement true.