Fundamentele Informatica 1 & Foundations of Computer Science
Mid-term exam 21/10/2019, 11:00-13:00.
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The exam consists of 5 questions (1 page).
If you score x points, your grade equals $x/5$.
Mark clearly each question that you are answering.
Important: write your name, student id and the course name on your exam sheet.

Question 1. 10 P.

Draw the Venn diagrams for the following pairs of sets; determine the relationship of the pair: i.e. are they equal, is one a subset of the other, are they disjoint, or incomparable (none of the previous relationships).

1. $(A \oplus B)^c$ and $A^c \oplus B$
2. $(A \oplus B) \cup C$ and $(A \cup C) \oplus (B \cup C)$

Hint: Draw a separate Venn diagram for each of the four expressions, and compare the shaded regions.

Question 2. 10 P.

Use the rules (laws/axioms) of set algebra to simplify the following expression as much as possible:

$$((B^c \cup A)^c \cup B) \cup B.$$

Question 3. 6 P.

Consider the following binary relation R defined over the set $\{a, b, c, d\}$:

$$R = \{(a, a), (a, b), (a, d), (b, b), (b, d), (d, a), (d, b), (d, d), (c, c)\}.$$

Check if this relation is 1) reflexive; 2) symmetric; 3) antisymmetric; 4) transitive; 5) an equivalence relation; 6) a partial order. Motivate your answer, and in case the relation does not satisfy the property, provide a counterexample.

Question 4. 12 P.

How many numbers from $\{1, \ldots, 300\}$ are not divisible by 2, 3 nor 5 (so by none of them)? Find the answer by using the principle of inclusion and exclusion. Use a Venn diagram to illustrate the relevant sets and their relationships.

Hints: For $p \in \mathbb{N}^+$, let $D_p = \{k \in \{1, \ldots, 300\} \mid p \text{ divides } k\}$ be the set of numbers between 1 and 300 that are divisible by the number p. Then, the number of integers in $\{1, \ldots, 300\}$ that are divisible by either 2, 3 or 5 equals $|D_2 \cup D_3 \cup D_5|$ (where $|A|$ denotes the number of elements of A, i.e. the cardinality of A).
Note that $|D_p| = \frac{300}{p}$, if p divides 300, and $D_p \cap D_q = D_{p \times q}$, if p and q are relatively prime, that is have no common divisors. Specially, for $p, q \in \{2, 3, 5\}$ and $p \neq q$, a number is divisible by p and q if and only if it is a multiple of $p \times q$.

Question 5. 12 P.

Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$.

1. It is possible to find functions $f : A \to B$, for which there exists a subset $V \subseteq A$ such that $f^{-1}(f(V)) \neq V$. Specify one such function, by giving all the pairs $(x, f(x))$ (so the set $\{(x, f(x)) \mid x \in A\}$) and also give the corresponding set $V \subseteq A$.
2. It is possible to find functions $g : A \to B$, for which there exists a subset $W \subseteq B$ such that $g(g^{-1}(W)) \neq W$. Specify one such function, by giving all the pairs $(x, g(x))$ (so the set $\{(x, g(x)) \mid x \in A\}$) and also give the corresponding set $W \subseteq B$.
3. For the f and g you defined, determine if they are injective and or surjective.

Note that a function has to be total; it has a value for all the elements of its domain.