FOCS

Lecture 4

QUICK SUMMARY

- BASIC NOTATION AND CONCEPTS OF SET THEORY
  - IMPORTANT SETS
  - BASIC SET RELATIONSHIPS
  - BASIC SET OPERATIONS
  - VENN DIAGRAMS
  - SOME EQUIVALENCES
  - PRINCIPLE OF EXCLUSION & INCLUSION
  - BASICS OF EXPRESSIONS / ALGEBRAIC PROPERTIES / LAWS
    - COMMUTATIVITY
    - ASSOCIATIVITY
    - DISTRIBUTIVITY   // BRACKETS MATTER!
  - LAWS LAWS LAWS (TH 6.5, SCHAUM, L3, PG.18)
  - DUALITY
  - POWERSETS
  - CARDINALITY OF POWERSETS
  - BINARY NUMBERS

Relations

Relations

Examples:

- “<” is a relation between 2 numbers
- “=“
- “⊆”

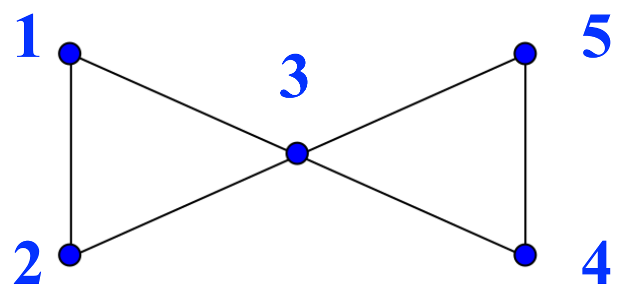
“a and b are in relation < if a<b”

<(a,b) is true

Relations

Examples:

Graphs:



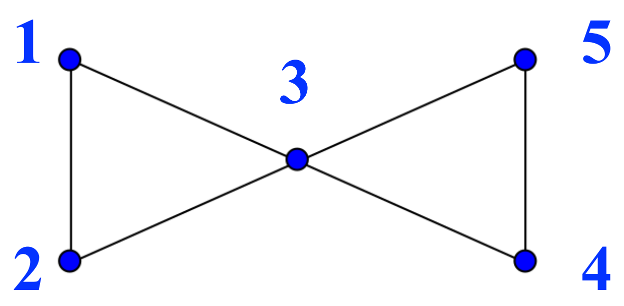
$$G = (V, E); V = \{1,2,3,4,5\}, E = \{\{1,2\}, \{1,3\}, \{2,3\}, \{3,5\}, \{5,4\}, \{3,4\}\}$$

"Edge in G" is a relation between the vertices of G

Relations

Examples:

Graphs:



$$G = (V, E); V = \{1,2,3,4,5\}, E = \{\{1,2\}, \{1,3\}, \{2,3\}, \{3,5\}, \{5,4\}, \{3,4\}\}$$

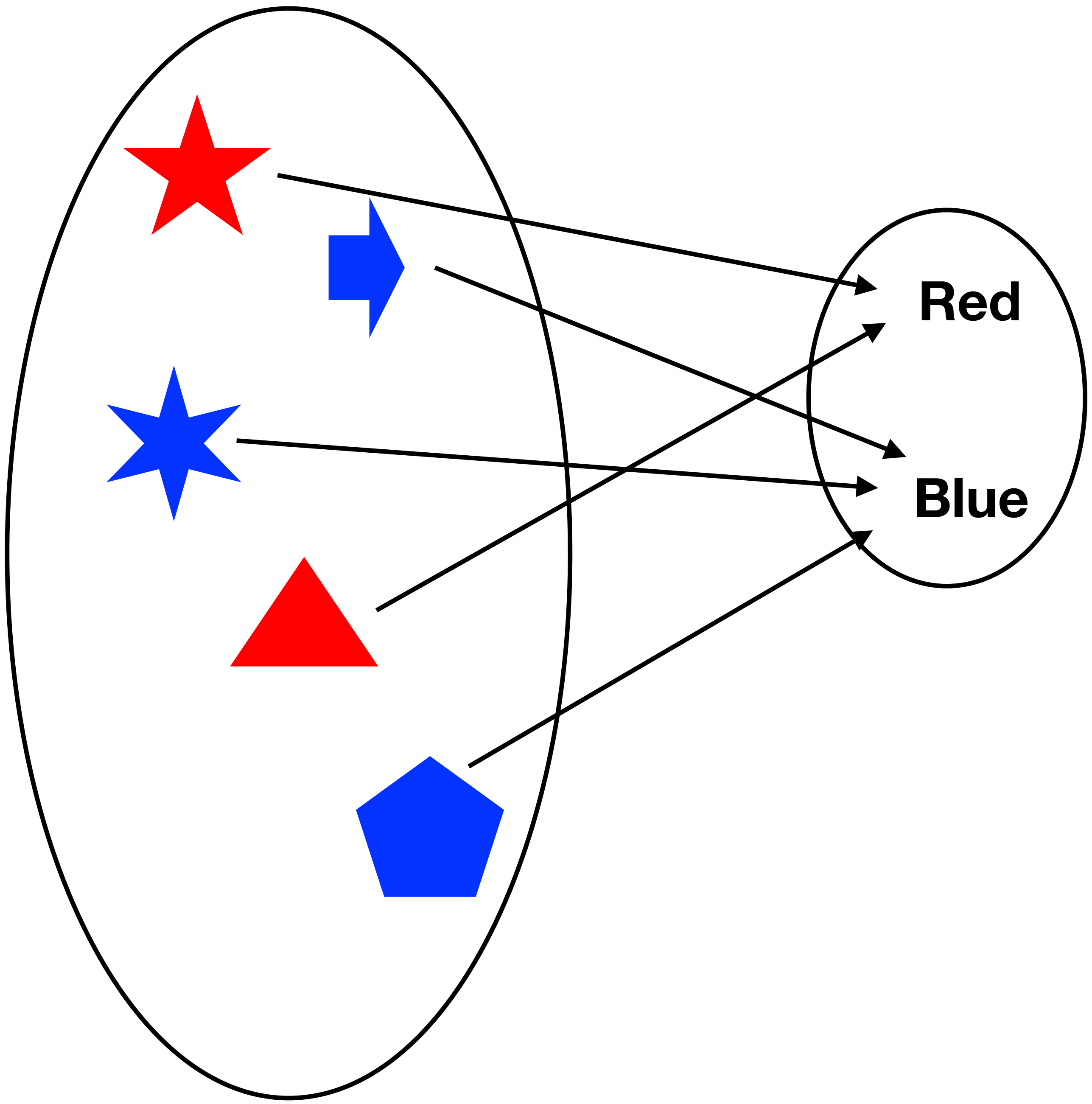
"Edge in G" is a relation between the vertices of G

vertices v_1 and v_2 are in relation "Edge" if…

Relations

Examples:

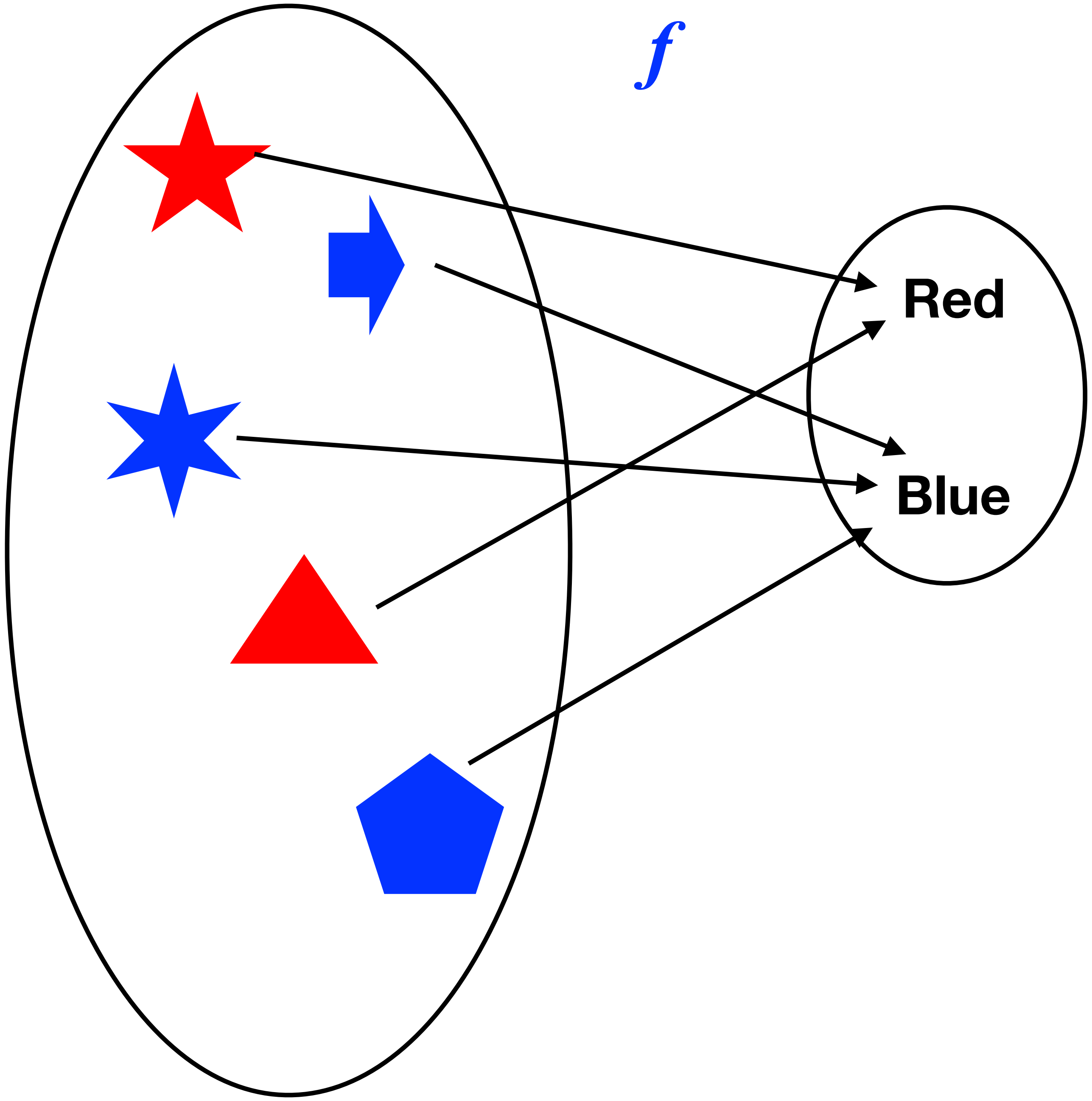
Functions:



Relations

Examples:

Functions:

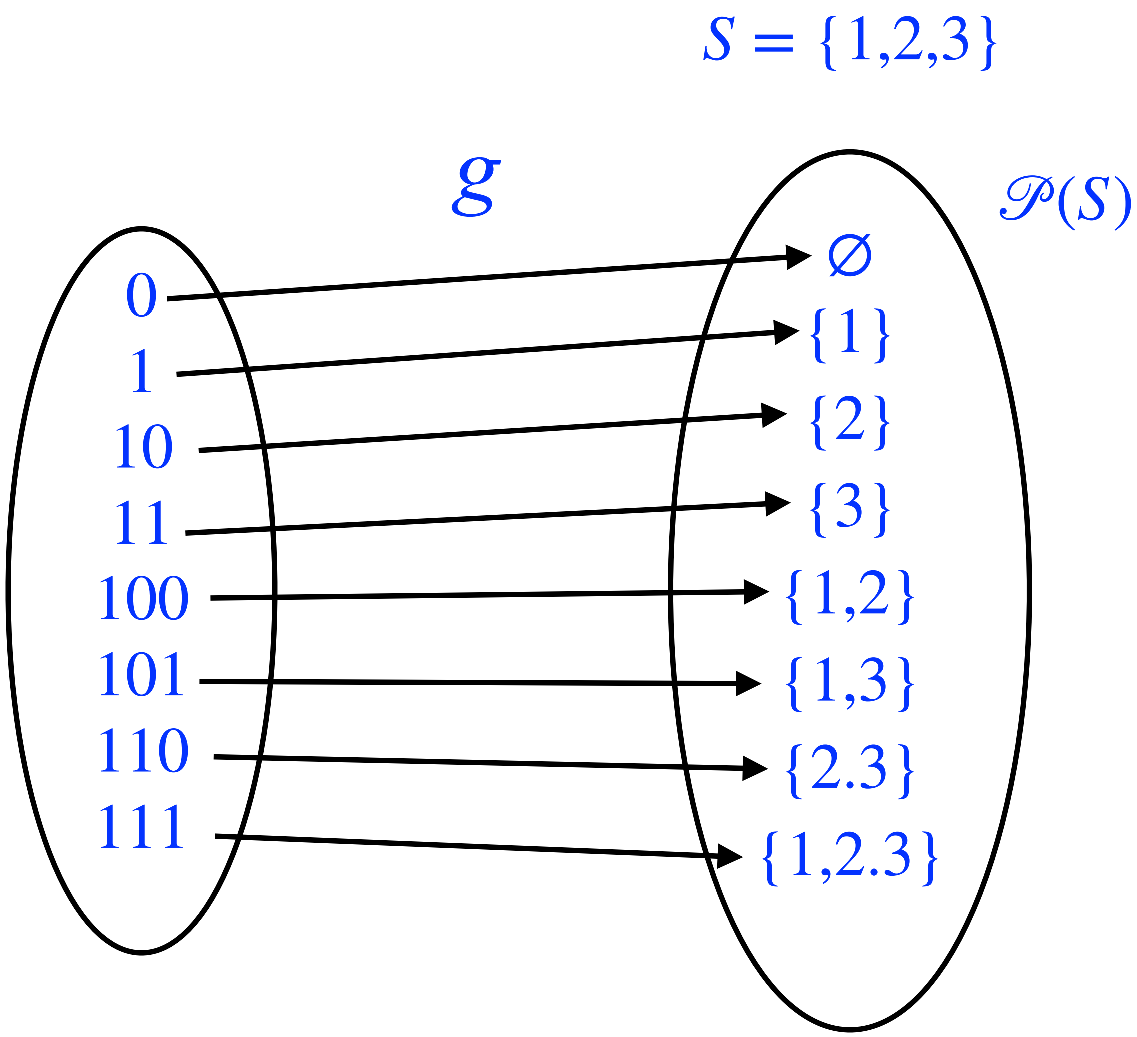


“star” and “red” are in relation f

Relations

Examples:

Functions:



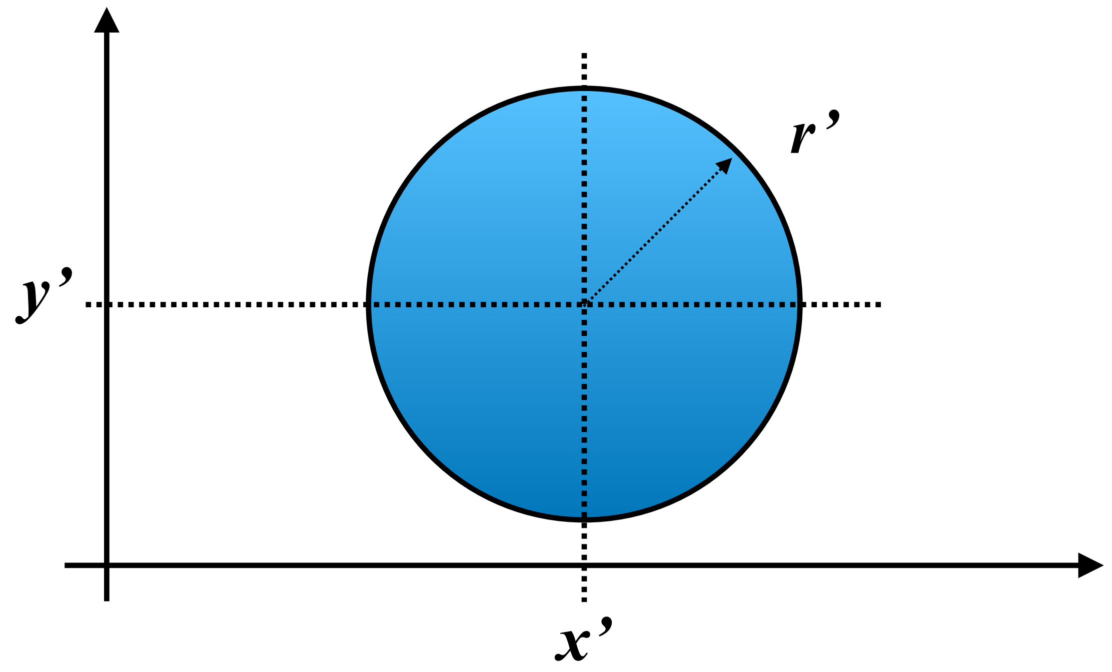
"100" and "{1,2}" are in relation g

"enumeration of the powerset of S"

Relations

Examples:

On continuous objects: plane



$$Disk = \{(x,y) \mid (x-x')^2 + (y-y')^2 \leq r^2\}$$

real values *x* and *y are in relation Disk if…*

week	Datum	Ma					Di					Wo					Do					Vr				
nr	Ma	1 2	3 4	5	6 7	8 9	1 2	3 4	5	6 7	8 9	1 2	3 4	5	6 7	8 9	1 2	3 4	5	6 7	8 9	1 2	3 4	5	6 7	8 9
36	2 sep	Intr. Inf.			Opening Acad.jr			WS								FDSD	CA	PM		wPM	vPM	AN1	CA			
37	9 sep	AN1	LA1		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	AN1	CA			
38	16 sep	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	Eerstejaarsweekend DLF				
39	23 sep	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	AN1	CA			
40	30 sep	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	Leidens Ontzet					Gesloten				
41	7 okt	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	AN1	CA			
42	14 okt	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	AN1	CA			
43	21 okt		t An1												t FDSD			t LA1								
44	28 okt	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	AN1	CA			
45	4 nov	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	AN1	CA			
46	11 nov	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	AN1	CA			
47	18 nov	AN1			FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM					
48	25 nov	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	AN1	CA			
49	2 dec	AN1	WS		FDSD	vPM	LA1	WS							LA1	FDSD	CA	PM		wPM	vPM	AN1	CA			
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51	16 dec	T LA1																				T CA				
52	23 dec											Gesloten					Gesloten					Gesloten				
1	30 dec	Gesloten					Gesloten					Gesloten														
2	6 jan	T PM										T AN1													T WS	
3	13 jan				T FDSD					H CA										Excursie DLF		Excursie DLF				
4	20 jan	Excursie DLF													H LA1											
5	27 jan				H AN1																				H WS	

The table is a relation… but a bit different
It is over three sets…

week	Datum	Ma					Di					Wo					Do					Vr				
nr	Ma	1 2	3 4	5	6 7	8 9	1 2	3 4	5	6 7	8 9	1 2	3 4	5	6 7	8 9	1 2	3 4	5	6 7	8 9	1 2	3 4	5	6 7	8 9
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The table is a relation… but a bit different

It is over three sets…

(44, Di-4, WS) are in relation “Table”

Relations. Properly.

Tuples (finite ordered lists)

$$A_1, A_2, A_3 \ldots, A_n$$

$$(a_1, a_2, a_3, \ldots, a_n) \qquad a_i \in A_i$$

$$(a_1, a_2, a_3, \ldots, a_n) = (b_1, b_2, b_3, \ldots, b_n)$$

if and only if

for all $i \in \{1, \ldots, n\}, a_i = b_i$.

Relations. Properly.

2-tuples: ordered pairs

$$(a, b) = (c, d) \Leftrightarrow a = c \;\&\; b = d$$

$$(a, b) = (b, a) \Rightarrow a = b$$

In general $(a, b) \neq (b, a)$ **but** $\{a, b\} = \{b, a\}$

Relations. Properly.

Cartesian products

$$A_1, A_2, A_3 \ldots, A_n$$

$$A = A_1 \times A_2 \times A_3 \times \cdots \times A_n$$

Definition.
$A = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i\}$

Notation: $B^n := B^{\times n} := B \times B \times B \times \cdots \times B$

Relations. Properly.

Cartesian products

$$A = \{4,5\}; B = \{x, y\}$$

$$A \times B? \quad B \times A?$$

Let's work this out

$$A \times B = \{(4,x),(4,y),(5,x),(5,y)\}$$

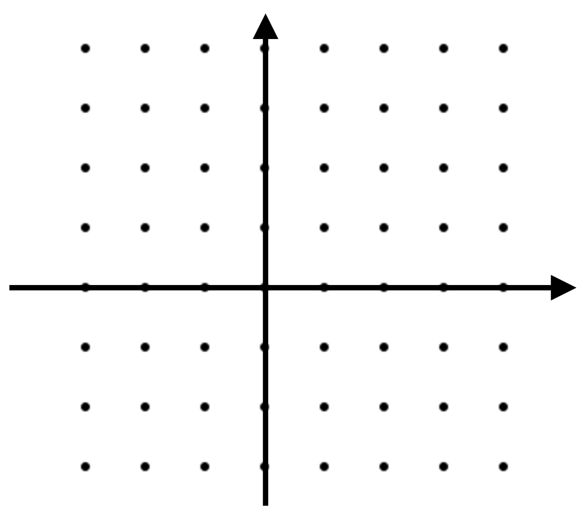
$\in A$ $\in B$

Relations. Properly.

Cartesian products:

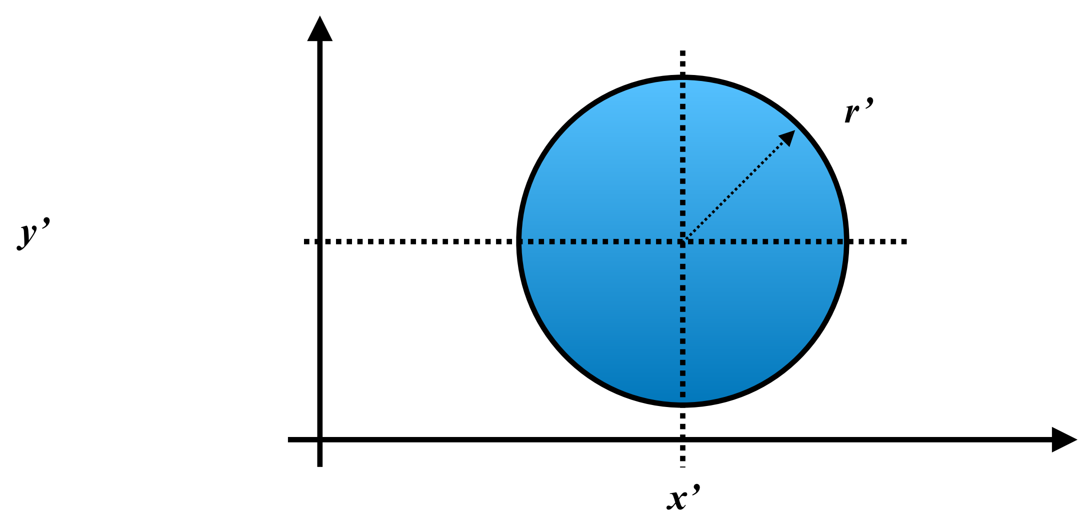
Grid

$$\mathbb{Z}^2 = \mathbb{Z} \times \mathbb{Z}$$



Plane

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$$



Space

$$\mathbb{R}^3$$

Relations. Properly.

Cartesian products.

Def. Relations are subsets of Cartesian products.

Def. Subsets of products of two sets are *binary relations*

$R \subseteq A \times B$ ***- binary relation***

$R \subseteq A_1 \times \ldots \times A_n$ ***-n-ary relation [“arity”]***

Relations. Properly.

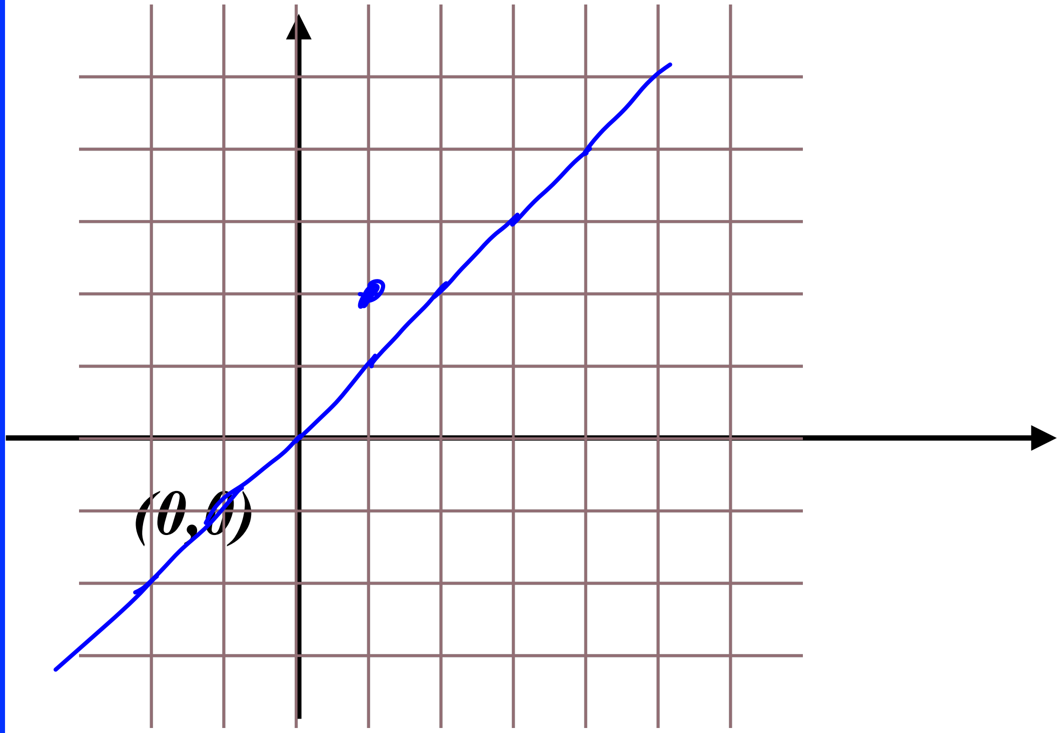
Cartesian products.

Relations are subsets of Cartesian products.

Subsets of products of two sets are *binary relations*

$< \in \mathbb{R}^2$

Let's work this out



Relations. Properly.

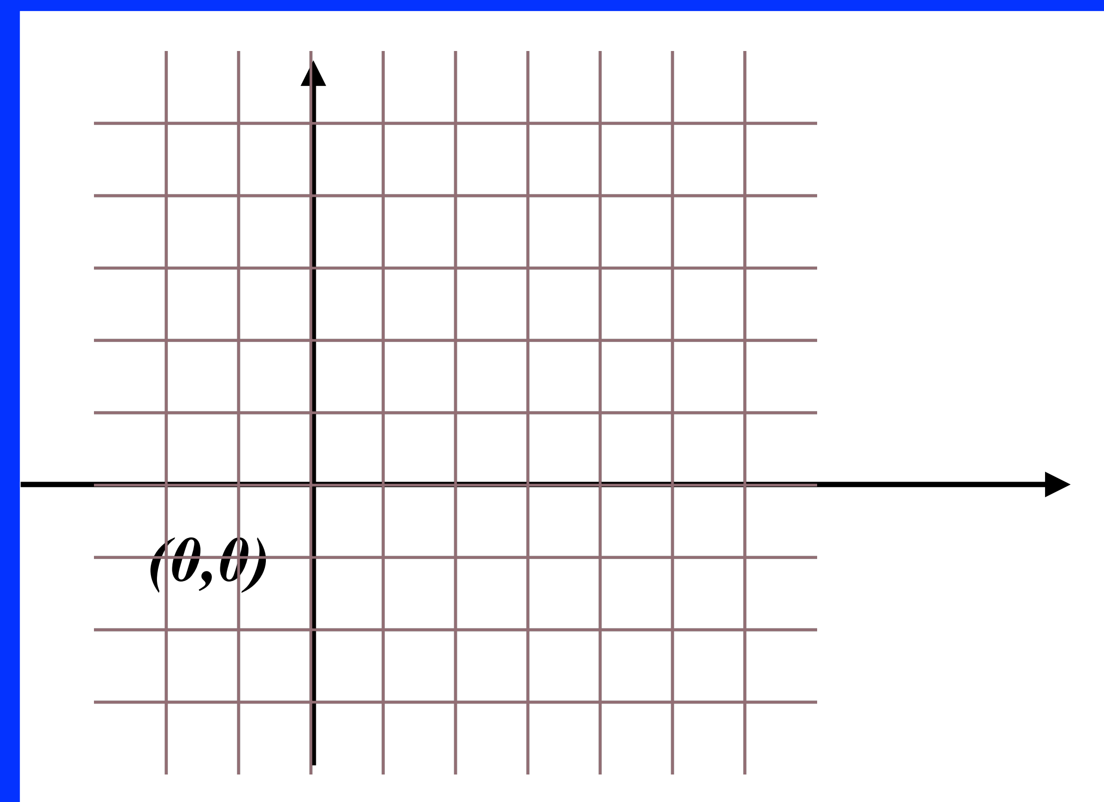
Cartesian products.

Relations are subsets of Cartesian products.

Subsets of products of two sets are *binary relations*

$$< \;=\; \{(a,b) \mid a, b \in \mathbb{R} \;\&\; b - a \in \mathbb{R}^{+}\}$$

Let’s work this out



Relations. Properly.

Confession: not *entirely formal…*
what is an ordered pair? an element of cartesian product…
but what is a cartestian product? set of ordered pairs…

circular!
axiomatic in a metalanguage…

Usually:

$(x, y) := \{x, \{x, y\}\}$ ***[Kuratowski definition]***

$(x_1, x_2, x_3) = (x_1, (x_2, x_3))$

Relations. Properly.

$(x, y) := \{x, \{x, y\}\}$ ***[Kuratowski definition]***

$(x_1, x_2, x_3) = (x_1, (x_2, x_3))$**...*and so on***

Not the only definition. There are other, see exercises.