Homework 01: Number of Possible Alignments

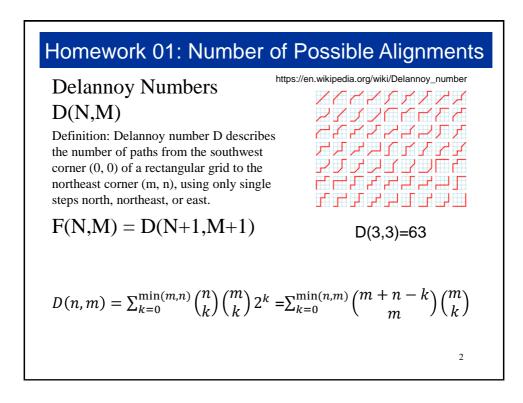
Let f(N,M) be equal to the number of possible alignments of sequences S and T, where |S| = N, and |T| = M. Then we have:

 $\begin{aligned} f(i,0) &= f(0,j) = 1 &, \text{for } 0 \leq i \leq N \text{ and } 0 \leq j \leq M \\ f(i,j) &= f(i-1,j) + f(i,j-1) + f(i-1,j-1) \end{aligned}$

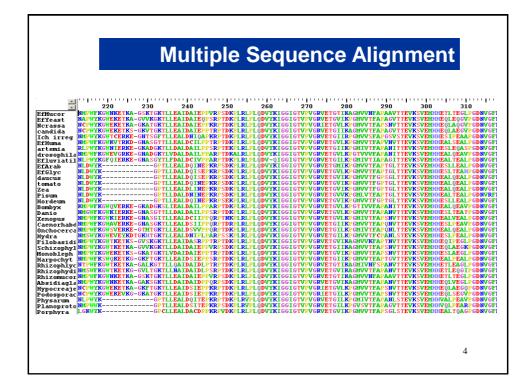
,for $1 \le i \le N$ and $1 \le j \le M$

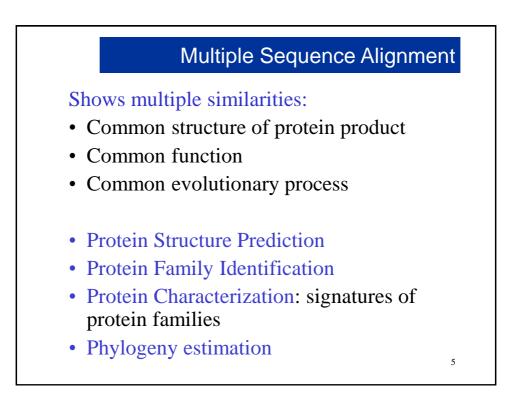
		T1	T2	T3	T4	T5	T6	T7	T8	T9
	1	1	1	1	1	1	1	1	1	1
S1	1	3	5	7	9	11	13	15	17	19
S2	1	5	13	25	41	61	85	113	145	181
S3	1	7	25	63	129	231	377	575	833	1159
S4	1	9	41	129	321	681	1289	2241	3649	5641
S5	1	11	61	231	681	1683	3653	7183	13073	22363
S6	1	13	85	377	1289	3653	8989	19825	40081	75517
S7	1	15	113	575	2241	7183	19825	48639	108545	224143
S8	1	17	145	833	3649	13073	40081	108545	265729	598417
S9	1	19	181	1159	5641	22363	75517	224143	598417	1462563
S10	1	21	221	1561	8361	36365	134245	433905	1256465	3317445

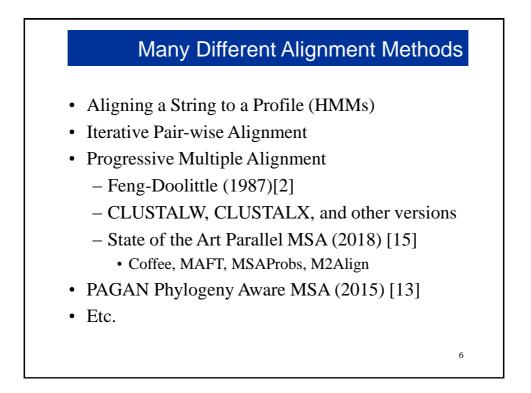
A. Torres, A. Cabada, J.J. Nieto, An Exact Formula for the Number of Alignments Between Two DNA Sequences. International Journal of Biomathematics, Vol. 09, No. 04, 1650053 (2016) Research Articles. (Also in DNA Sequence, 2003.)



Rank	Time	Language	
1	19.02	.cpp	←
2	34.00	.cpp	
3	63.82	.cpp	
4	70.16	.cpp	
5	83.00	.cpp	
6	86.00	.java	<
7	115.00	.py	Length 1000
8	118.02	.cc	
9	118.96	.cpp	
10	119.05	.java	
11	157.29	.java	
12	280.20	.java	
13	303.68	.cpp	
14	2700.00	.py	
15	3000.00	.cpp	
16	4129.23	.py	
17	13975.00	.py	
18	18761.96	.py	
19	19460.27	.py	3







Example: Signature Profiles

7

Helicases

- A protein to unwound DNA for further read for duplication, transcription, recombination or repair.
- Werner's syndrome an aging disease is believed to be due to a gene WRN that codes for a helicase protein.

A Signature Profile for Helicases

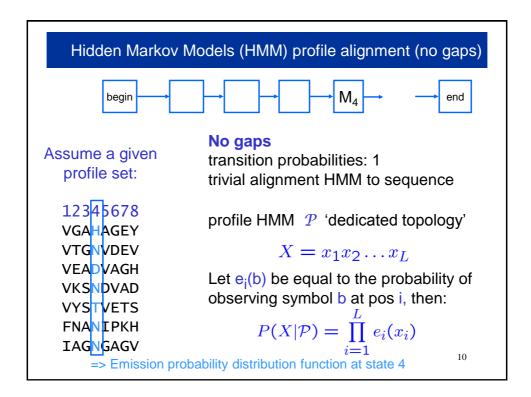
- Conserved sequence signatures or motifs
- Some of these motifs are unique identifiers for helicases
- Maybe functional units

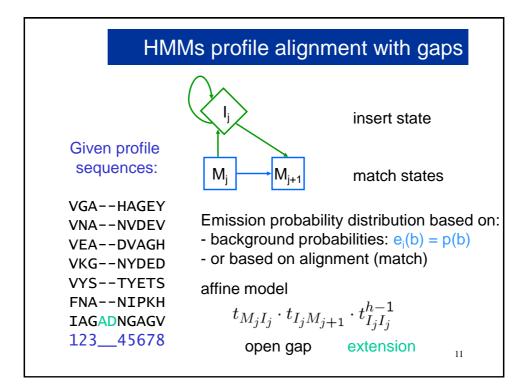
Multiple Alignment Profile Col 1 Co1 2 Col 3 aba 50% 25% 50% а аb b 0 % 75% 0 % b 25% с 0 % 0 % 25% 0 % 50% _ а С **Multiple Alignment Profile:** Character frequencies given per column • p_i(a) is the fraction of a's in column i p(a) is the fraction of a's overall • Log likelihood ratio $\log(p_i(a)/p(a))$ can be used. 8

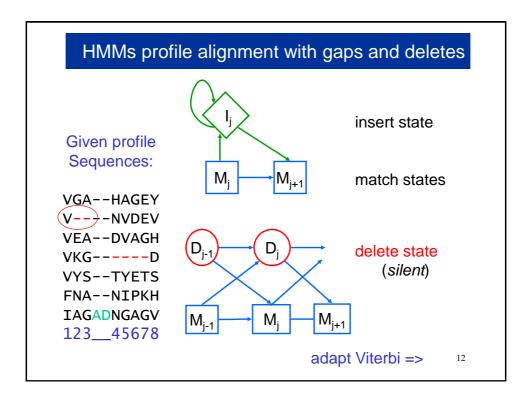
Aligning a String to a Profile

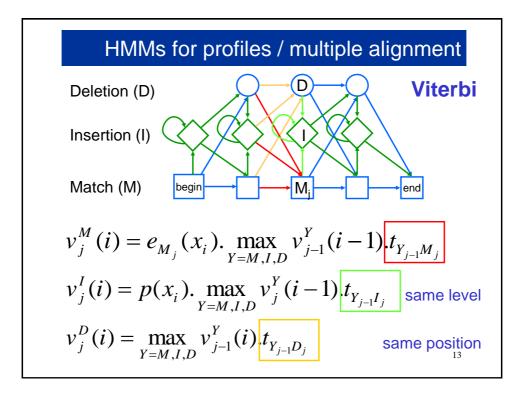
Definition 4.16 For an alignment S' of length l, a profile is an $l \times |\Sigma \cup \{-\}|$ matrix, whose columns are probability vectors denoting the frequencies of each symbol in the corresponding alignment column.

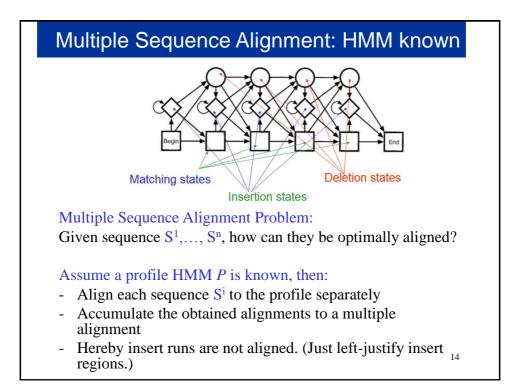
12	- 63		12	25	-	F			1	262		22				20	12	
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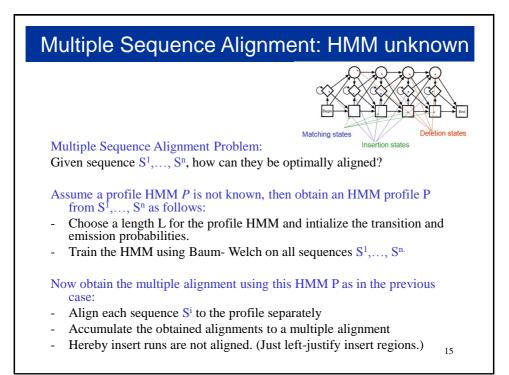


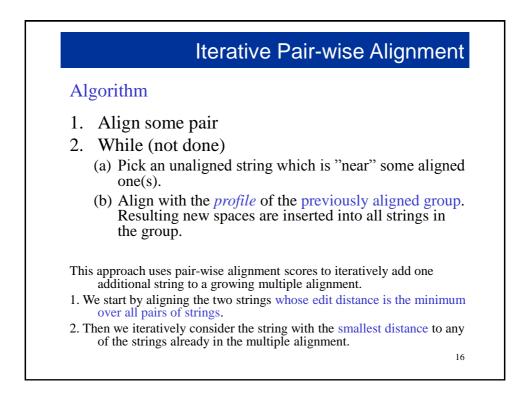


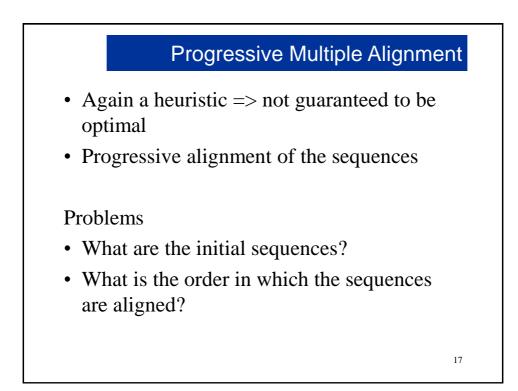


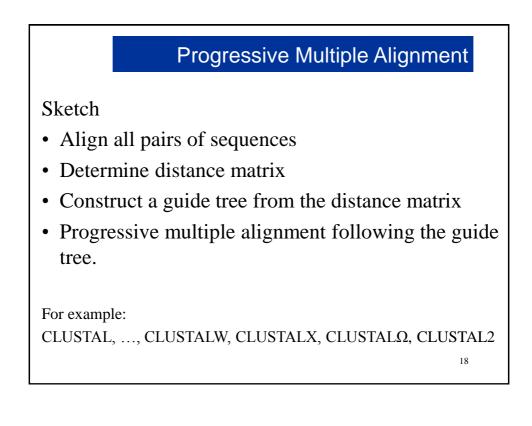


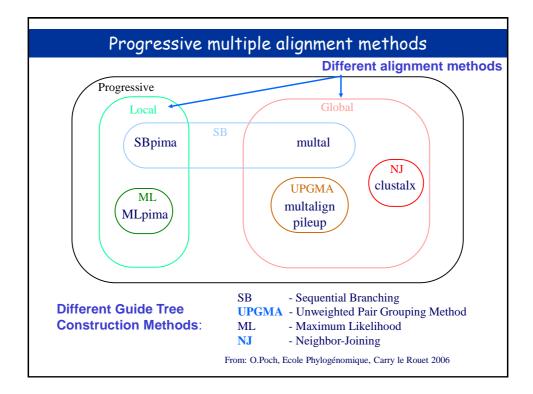


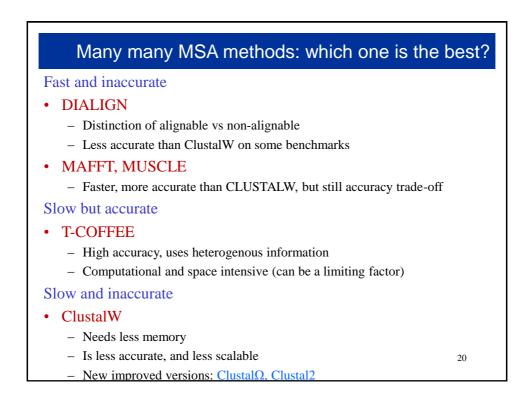


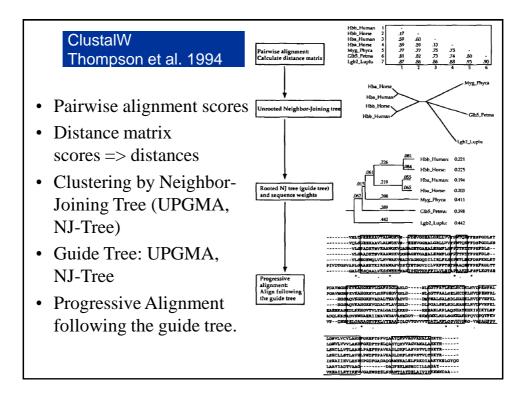


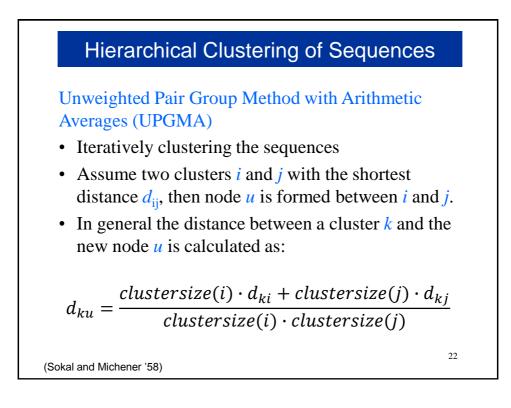


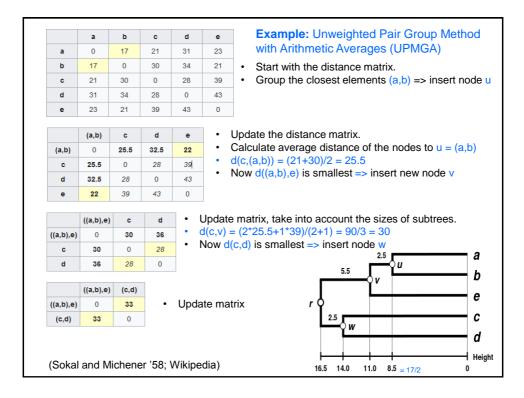


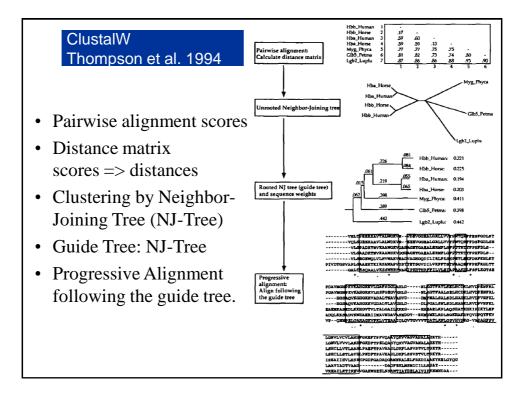


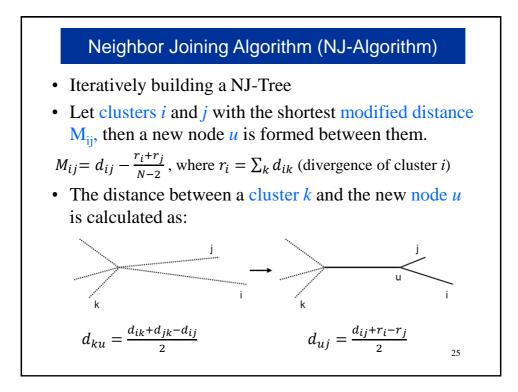


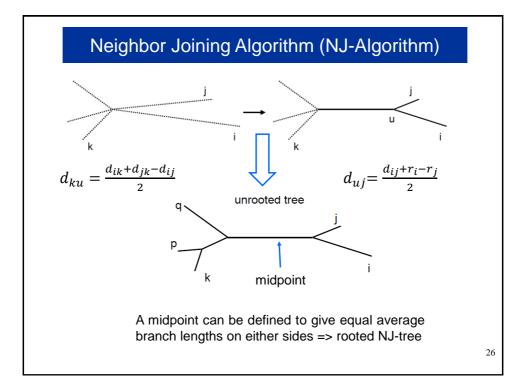


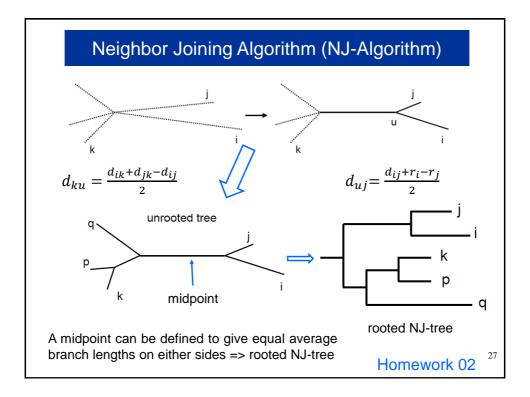


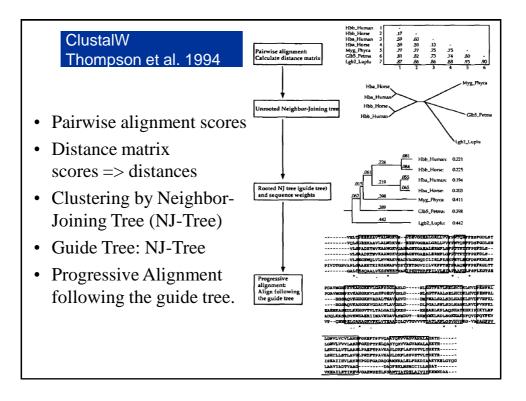


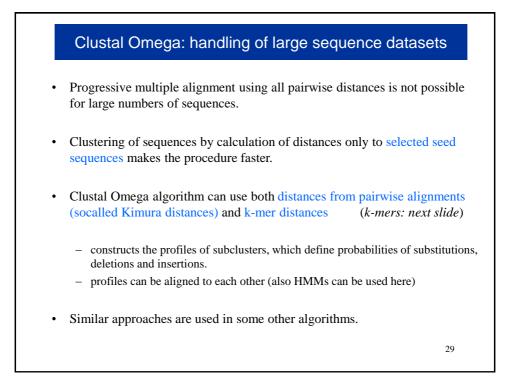


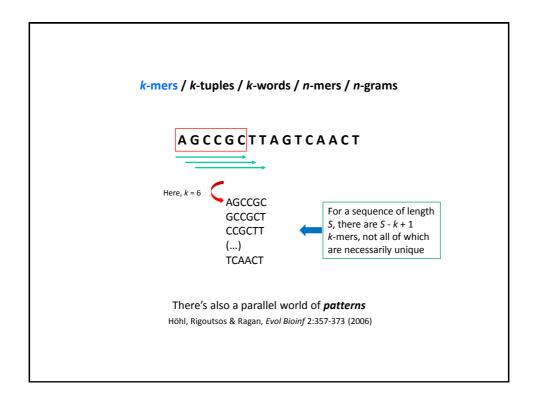


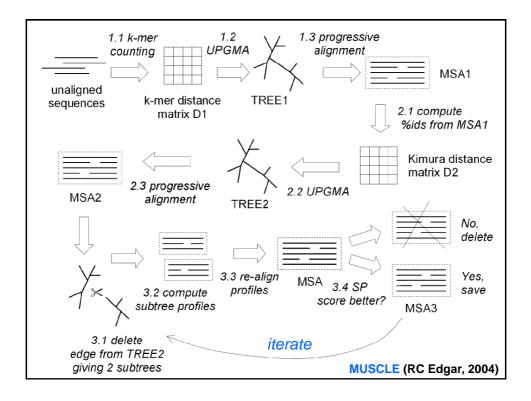


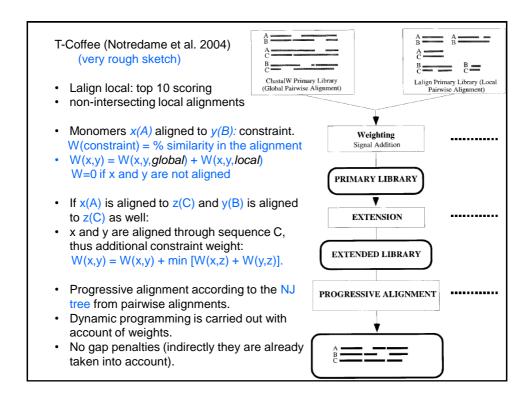


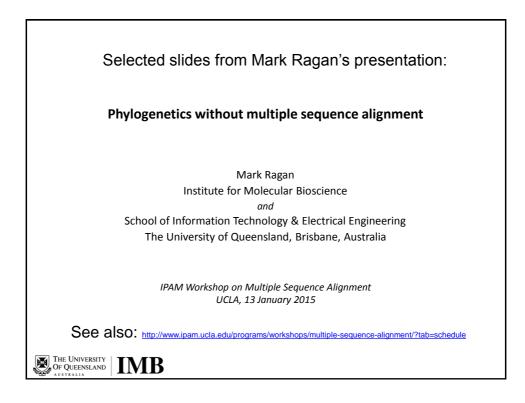


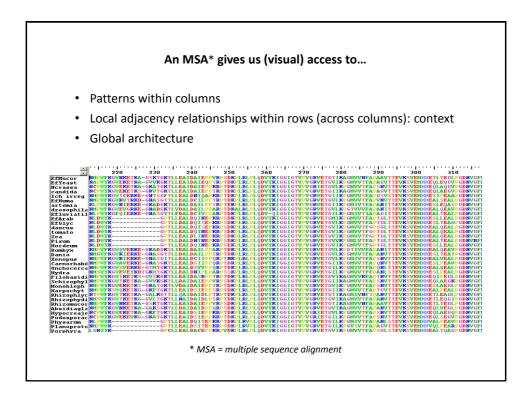


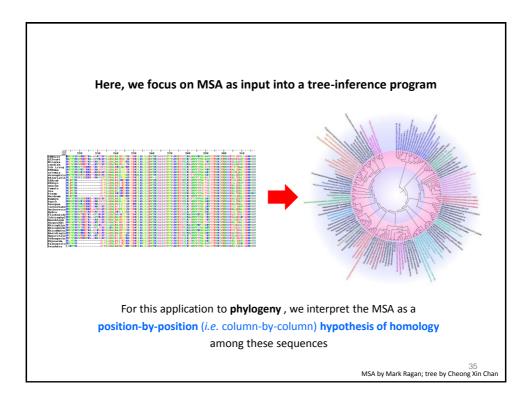


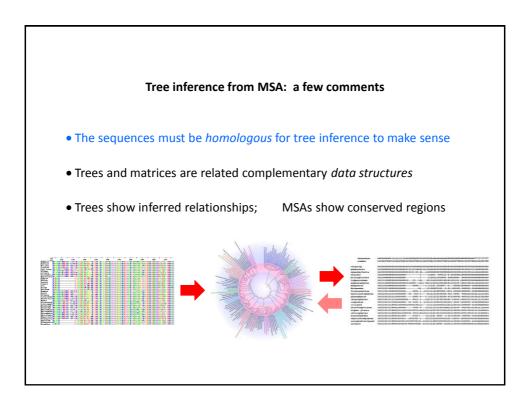


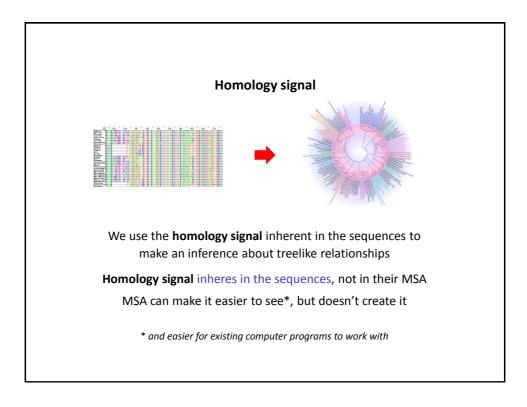


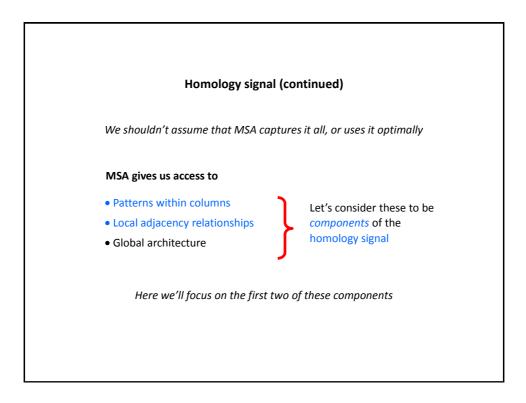












Pattern and adjacency

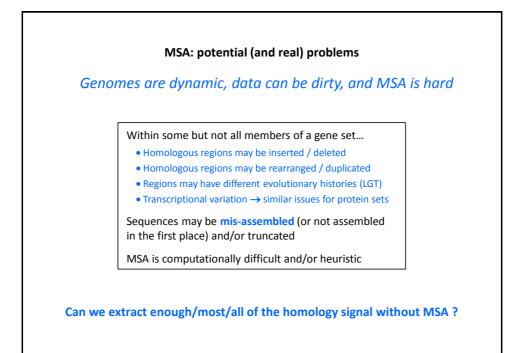
The column component needs to capture "sameness" of a character across sequences

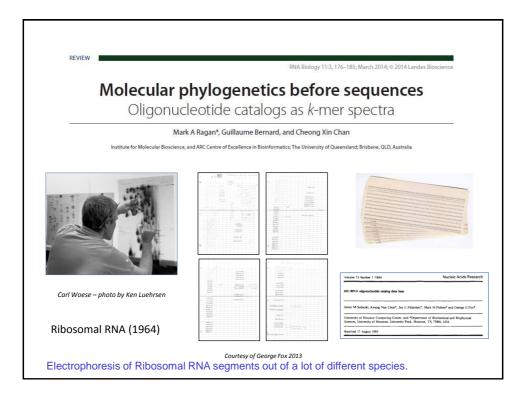
For application in phylogenetics, "sameness" has to mean *homology* (or *orthology*). It's difficult to build a statistical case that a particular single character in one sequence is homologous with a particular one in a second sequence.

MSA uses adjacency (and sometimes global) information to build this support. Alternatively we might compare sets of adjacent characters (strings), which are less likely to occur by chance.

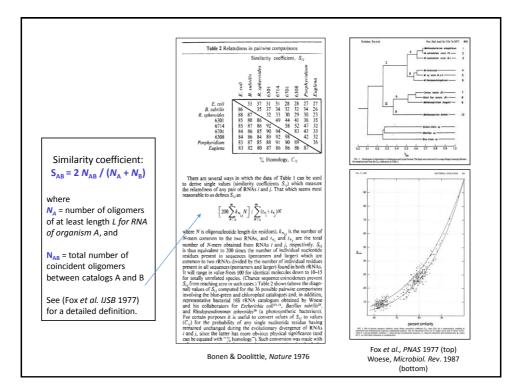
The adjacency component doesn't just provide statistical support for the column component

Because conserved function arises in part from chemical properties of adjacent residues (*e.g.* in making that part of the molecule an active site or α -helix), we expect homology signal to have an adjacency component in its own right.

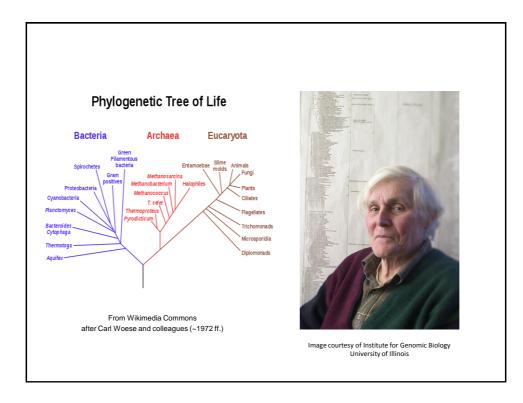


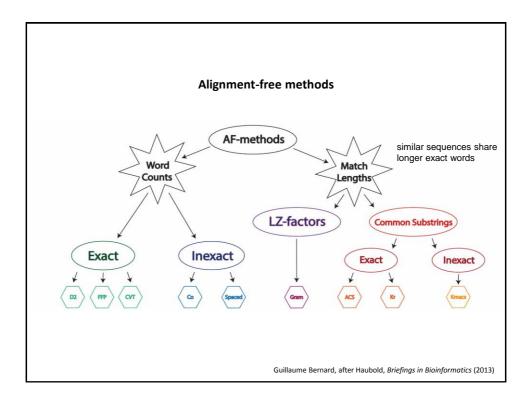


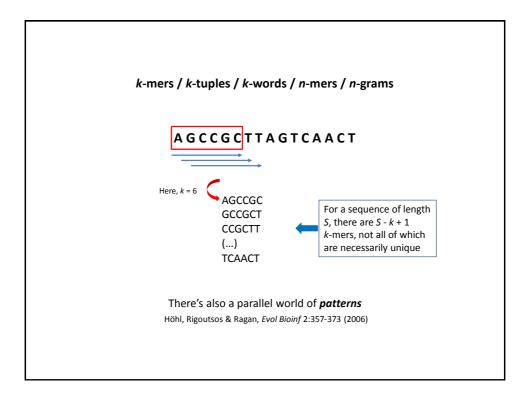
		Oligon									
4538 Evolutio	4508 Evolution: Fox et al.		Proc. Natl. Acad. Sci. USA 74 (1977)				Evolution: Fox et al.			Proc. Natl. Acad. Sci. USA 7	14 (2977)
			ags for 16S rRNA of 10						Table 1. (continued)		
Oligona- cleotide sequence	Present in organism number	Cligano- cleotide sequence	Present in organism number	Oligona- ciectide sequence	Present in organism number	Oligonu- cleutide	Present in organism	Oligonu- cleotide	Present in organism	Oligonu- clectide	Present organist number
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AG .	144	CUNCINES	6.0 H	ALTERCORG ALTRACTOG						strain M-1; 4. M. formicinum; 5. M.	

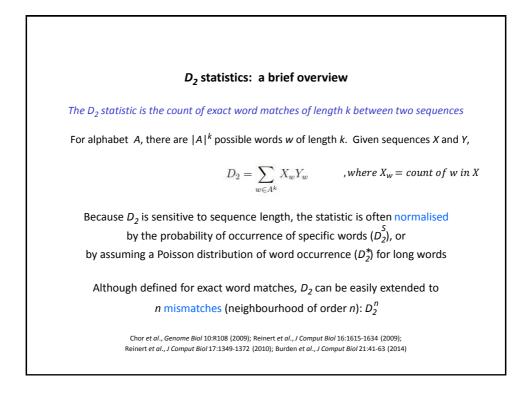


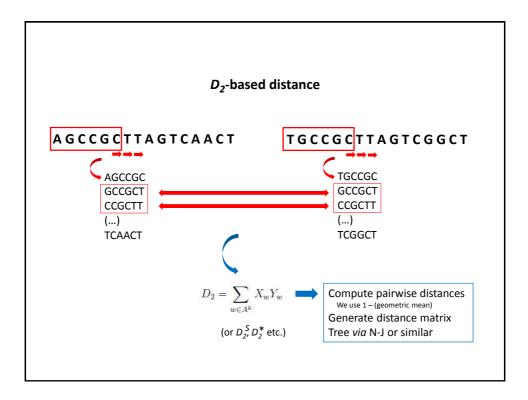
	The three kingdom	s (domains) of life
Proc. Natl. Acad. Sci. USA Vol. 74, No. 11, pp. 5088–5090, November 1977 Evolution		
Phylogenetic structure of the prok kingdoms	aryotic domain: The primary	Evolution: Wome and Fox Proc. Natl. Acad. Sci. USA 74 (1977) 5
(archaebacteria/eubacteria/urkaryote/16S ribosomal RNA/mol	ecular phylogeny)	Evolution: Wolse and Pox Proc. Natl. Acad. Sci. USA 14 (1917) 5
CARL R. WOESE AND GEORGE E. FOX*		Table 1. Association coefficients (S_{AB}) between representative members of the three primary kingdoms
Department of Genetics and Development, University of Illinois, Urbana, Illinois	63601	1 2 3 4 5 6 7 8 9 10 11 12 13
Communicated by T. M. Sonneborn, August 18, 1977		1. Sarcharsmyees errevisies, 185 — 0.29 0.33 0.05 0.06 0.08 0.09 0.11 0.08 0.11 0.11 0.08 0.0 2. Lenna miner, 185 0.29 — 0.36 0.10 0.05 0.06 0.10 0.09 0.11 0.10 0.10 0.13 0.0 3. Letta 185 0.33 0.65 — 0.06 0.06 0.07 0.09 0.06 0.01 0.01 0.01 0.00 0.05 0.05 0.05 0.05
ABSTRACT A phylogenetic analysis has of upon ribosomal RNA expanse characterization reveals that like of decreme [1] the representation of the set of the	to output physicatic classification between domains through the homogeneous end comparable to compare the comparison. This should be recognized by an appropriate transing. The diplet physication of the physicatic classification is a classification of the physication of the physication classification of the physicatic classification is a classification of the physication of th	4. Exclusioniz cuii 0.6 0.5 0.6 0.7 0.4 0.5 0.8 0.8 0.1 0.1 0.7
gotae nas served, it anyvning, to obscure the proteem of what extant groupings represent the various primeval branches from the common line of descent. The reason is that eukaryote/ prokaryote is not primarily a phylogenetic distinction, although	igin. Phylogenetic relationships cannot be reliably established in terms of noncomparable properties (7). A comparative ap- proach that can measure degree of difference in comparable	

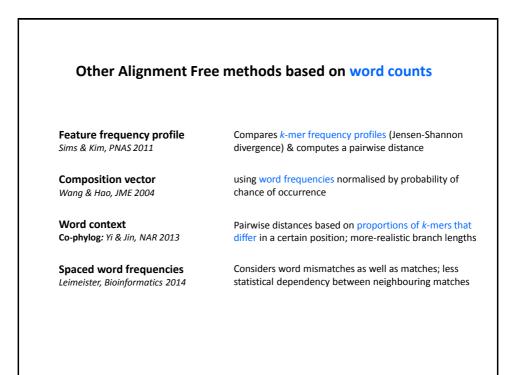




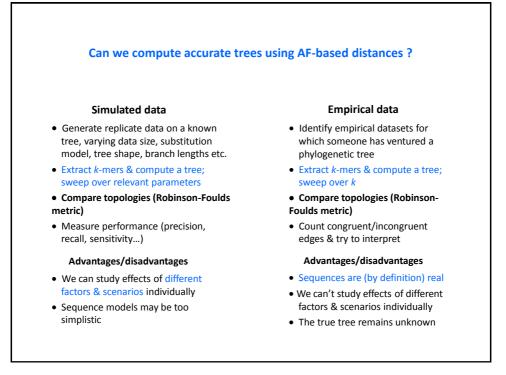


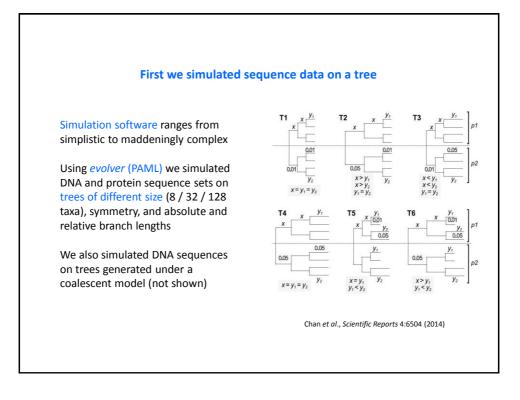


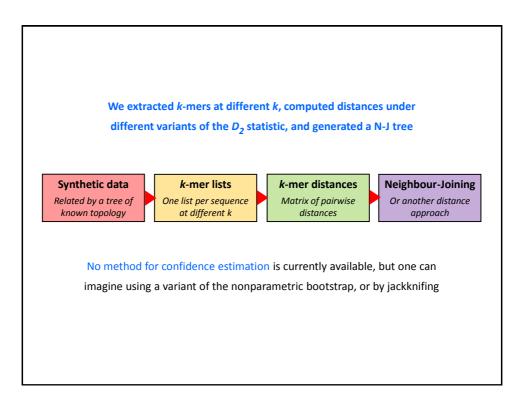


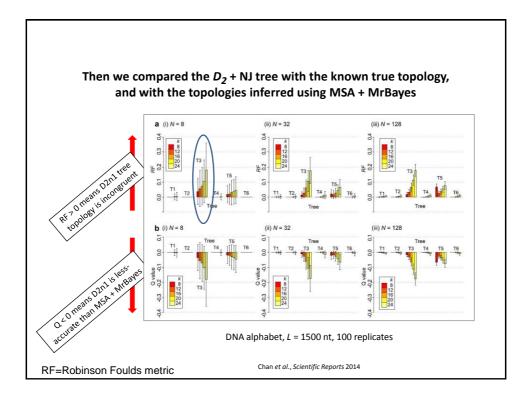


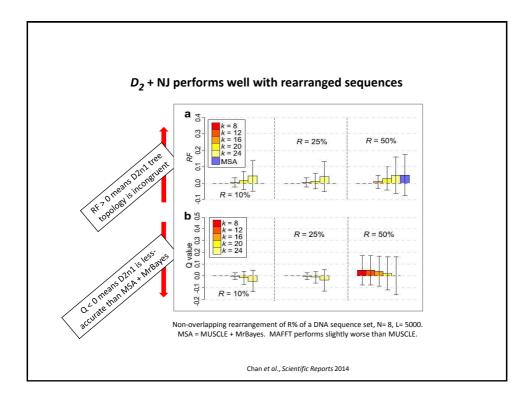
Alignment Free	(AF) methods based on match length
In general, simi	ilar sequences share longer exact words
Grammar-based distance d-gram: Russell, BMC Bioinf 2010	The concatenate of two sequences is more compressible (<i>e.g.</i> by Lempel-Ziv) if the sequences are similar
Average common substring Ulitsky, J Comp Biol 2006	Mean of longest matches between sequences, starting from each position; unlike L-Z, word overlap is allowed
Shortest unique substring Haubold, J Comp Biol 2009	Longest common substring + 1, corrected for random matches: "AF version of Jukes-Cantor distance"
Underlying subwords Comin, Algorith Mol Biol 2012	Like ACS, but discards common subwords that are covered by longer (more-significant) ones
<i>k</i> -Mismatch ACS (kmacs) Leimester, <i>Bioinformatics</i> 2014	ACS with k (in our notation, n) mismatches

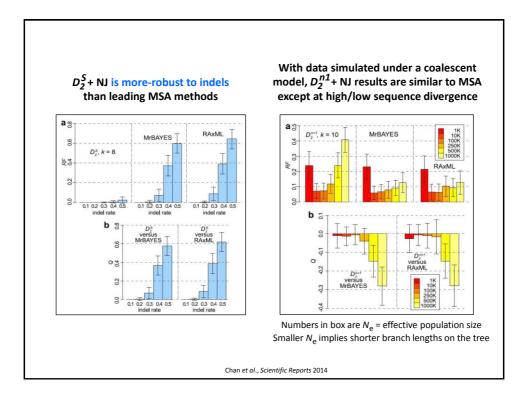




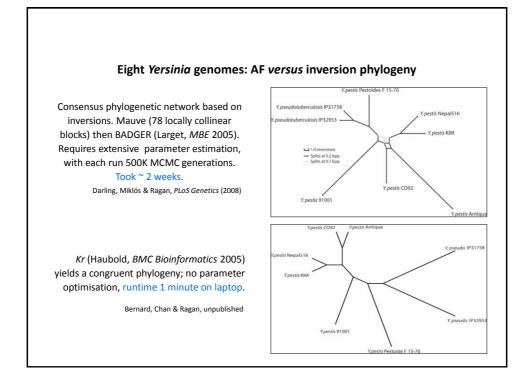


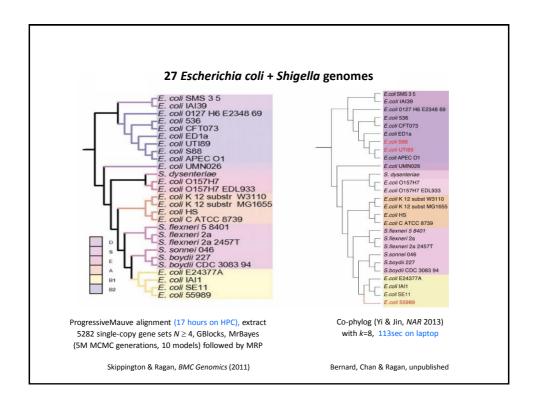






s computed	d from <i>k</i> -mer distances					
	Accuracy of D_2 methods increases with L					
D ₂	D ₂ methods are more robust to ancient					
MSA	sequence divergence, to rearrangement and to indel frequency					
D ₂						
D ₂ or MSA	D ₂ methods are more sensitive to recent sequence divergence and to the presence of incomplete (truncated) data					
D ₂ or MSA						
D ₂	Optimal k is negatively correlated with					
MSA	alphabet size, and is not greatly affected					
D ₂	by N or L in a biologically relevant range					
D ₂	D_2 methods are more scalable to large					
MSA	data than are MSA-based approaches, but usually require more memory					
	D ₂ MSA D ₂ or MSA D ₂ or MSA D ₂ or MSA D ₂ MSA D ₂					





Conclusions & outlook

AF methods hold considerable potential in phylogenetics & phylogenomics But MSA-based approaches have a six-decade head start

With synthetic data, AF methods perform better than MSA-based approaches under some evolutionarily relevant scenarios, but worse under others

With empirical data, the jury is still out

(Some) AF methods could likely be subsumed under a rigorous model, although probably at the cost of speed & scalability *i.e.* what makes them attractive in the first place

Efficient data structures & precomputation have much to offer

Other application areas include LGT analysis, and trees directly from NGS data Song et al., J Comp Biol 2013; Yi & Jin, NAR 2013

