Logica (I&E)

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http://liacs.leidenuniv.nl/~vlietrvan1/logica/

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2. Predicate logic
2.3. Proof theory of predicate logic

De hoogste opgave van het menselijk kennen is: te begrijpen dat hij niet begrijpen kan.

A slide from lecture 10:

Analogy $\wedge$ and $\forall$

elimination		introduction
$\dfrac{\phi\wedge\psi}{\phi}\ \wedge\mathrm{e}_R$	$\dfrac{\phi\wedge\psi}{\psi}\ \wedge\mathrm{e}_L$	$\dfrac{\phi \qquad \psi}{\phi\wedge\psi}\ \wedge\mathrm{i}$
$\dfrac{\forall x\phi}{\phi[t/x]}\ \forall x\ \mathrm{e}$		$\dfrac{\boxed{\begin{array}{l} x_0 \\ \quad\vdots \\ \quad \phi[x_0/x] \end{array}}}{\forall x\phi}\ \forall x\ \mathrm{i}$

A slide from lecture 10:

Analogy $\vee$ and $\exists$

elimination | introduction

$$\frac{\phi\vee\psi \qquad \boxed{\begin{array}{c}\phi\\ \vdots\\ \chi\end{array}} \qquad \boxed{\begin{array}{c}\psi\\ \vdots\\ \chi\end{array}}}{\chi}\ \vee\mathrm{e} \qquad\qquad \frac{\phi}{\phi\vee\psi}\ \vee\mathrm{i}_R \qquad\qquad \frac{\psi}{\phi\vee\psi}\ \vee\mathrm{i}_L$$

$$\frac{\exists x\phi \qquad \boxed{\begin{array}{ll}x_0 & \phi[x_0/x]\\ & \quad\vdots\\ & \quad\chi\end{array}}}{\chi}\ \exists x\ \mathrm{e} \qquad\qquad \frac{\phi[t/x]}{\exists x\phi}\ \exists x\ \mathrm{i}$$

Example.

$$\exists x P(x),\ \forall x \forall y (P(x) \to Q(y)) \ \vdash \ \ldots$$

Example.

$$\exists x P(x),\ \forall x \forall y (P(x) \to Q(y)) \ \vdash\ \forall y Q(y)$$

Proof. . .

Why fresh variables

Example.

$$\exists x P(x),\ \forall x(P(x) \to Q(x)) \ \vdash \ \ldots$$

Why fresh variables

Example.

$$\exists x P(x),\ \forall x (P(x) \to Q(x)) \ \vdash\ \forall y Q(y)$$

'Proof'. . .

2.3.2. Quantifier equivalences

Is

$$\forall x \forall y \phi \ \dashv\vdash \ \forall y \forall x \phi$$

valid?

Is

$$(\forall x \phi) \wedge (\forall x \psi) \ \dashv\vdash \ \forall x (\phi \wedge \psi)$$

valid?

Is

$$(\forall x \phi) \wedge \psi \ \dashv\vdash \ \forall x (\phi \wedge \psi)$$

valid?

Is

$$\exists x(\phi \to \psi) \quad \dashv\vdash \quad \forall x \phi \to \psi$$

valid?

Example 2.12.

Not all birds can fly.

$$\neg\forall x(B(x) \rightarrow F(x))$$

$$\exists x(B(x) \land \neg F(x))$$

Theorem 2.13. Let ϕ and ψ be formulas of predicate logic.

1.(a) $\neg\forall x\phi \;\dashv\vdash\; \exists x\neg\phi$

(b) $\neg\exists x\phi \;\dashv\vdash\; \forall x\neg\phi$

Proof 1(a)...

Left-to-right:

$$\begin{aligned} \neg(p_1 \wedge p_2) &\vdash \neg p_1 \vee \neg p_2 \\ \neg\forall x P(x) &\vdash \exists x \neg P(x) \\ \neg\forall x\phi &\vdash \exists x\neg\phi \end{aligned}$$

Right-to-left...

Theorem 2.13. Let ϕ and ψ be formulas of predicate logic.

2. Assuming that x is not free in ψ:

   (a) $\forall x \phi \wedge \psi \ \dashv\vdash \ \forall x(\phi \wedge \psi)$

   (b) $\forall x \phi \vee \psi \ \dashv\vdash \ \forall x(\phi \vee \psi)$

   (c) ... (h)

Proof 2(a)...

Theorem 2.13. Let ϕ and ψ be formulas of predicate logic.

2. Assuming that x is not free in ψ:

   (a) $\forall x(\phi \land \psi) \ \vdash \ \forall x \phi \land \psi$

Proof. . .

Theorem 2.13. Let ϕ and ψ be formulas of predicate logic.

3.(a) $\forall x\phi \wedge \forall x\psi \quad \dashv\vdash \quad \forall x(\phi \wedge \psi)$

(b) $\exists x\phi \vee \exists x\psi \quad \dashv\vdash \quad \exists x(\phi \vee \psi)$

Proof 3(b). . .

Theorem 2.13. Let ϕ and ψ be formulas of predicate logic.

4.(a) $\forall x \forall y \phi \dashv\vdash \forall y \forall x \phi$

(b) $\exists x \exists y \phi \dashv\vdash \exists y \exists x \phi$

Proof 4(b). . .

Study this proof yourself.