Fundamentele Informatica 1 (I&E)

najaar 2015

http://www.liacs.leidenuniv.nl/~vlietrvan1/fi1ie/

Rudy van Vliet
kamer 124 Snellius, tel. 071-527 5777
rvvliet(at)liacs(dot)nl

college 4, 6 november 2015

3.1 Regular Languages and Regular Expressions

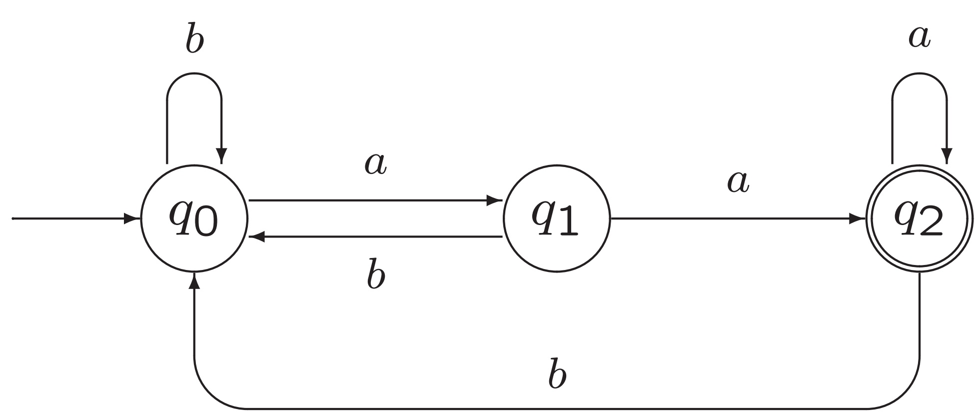
3.2 Nondeterministic Finite Automata

A slide from lecture 3:

Example 2.1.

A finite automaton for accepting

$L_1 = \{x \in \{a,b\}^* \mid \ x \text{ ends with } aa\}$



reg. languages	FA	reg. grammar	reg. expression
determ. cf. languages	DPDA		
cf. languages	PDA	cf. grammar	
re. languages	TM	unrestr. grammar	

3.1. Regular Languages and Regular Expressions

(Part of) two slides from lecture 1:

Combination of union, concatenation, Kleene star:

$L_1 \cup L_2 L_3^* = \ldots$

$(L_1 \cup L_2) L_3^*$

$L_1 \cup (L_2 L_3)^*$

$(L_1 \cup L_2 L_3)^*$

Description of languages:

by formula: $L_1 = \{ab, bab\}^* \cup \{b\}\{ba\}^*\{ab\}^*$

From exercise class 2:

Exercise 2.12. For each of the following languages, draw an FA accepting it.

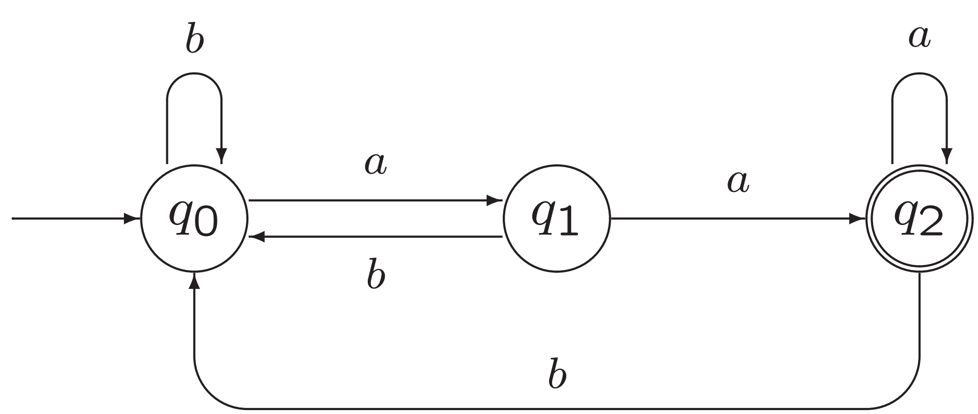
a. $\{a, b\}^*\{a\}$
b. $\{bb, ba\}^*$
c. $\{a, b\}^*\{b, aa\}\{a, b\}^*$
d. $\{bbb, baa\}^*\{a\}$
e. $\{a\} \cup \{b\}\{a\}^* \cup \{a\}\{b\}^*\{a\}$

A slide from lecture 3:

Example 2.1.

A finite automaton for accepting

$L_1 = \{x \in \{a, b\}^* \mid \ x \text{ ends with } aa\}$



Definition 3.1. Regular Languages over an Alphabet Σ.

If Σ is an alphabet,
the set $\mathcal{R}$ of regular languages over Σ is defined as follows.

1. The language $\emptyset$ is an element of $\mathcal{R}$,
   and for every $\sigma \in \Sigma$, the language $\{\sigma\}$ is in $\mathcal{R}$.

2. For any two languages L_1 and L_2 in $\mathcal{R}$,
   the three languages
   $L_1 \cup L_2$, $L_1 L_2$, and L_1^*
   are elements of $\mathcal{R}$.

(and nothing more)

$\{a, b\}^*\{aa\} \in \mathcal{R}$,
because $\{a, b\}^*\{aa\} = (\{a\} \cup \{b\})^*(\{a\}\{a\})$.

$\{\Lambda\} \in \mathcal{R}$, because ...

Regular Language	**Regular Expression**
$\emptyset$	$\emptyset$
$\{\Lambda\}$	Λ
$\{aab\}$	aab
$\{a, b\}^*$	$(a + b)^*$
$\{aab\}^*\{a, ab\}$	$(aab)^*(a + ab)$
$(\{aa, bb\} \cup \{ab, ba\}\{aa, bb\}^*\{ab, ba\})^*$	$\ldots$

$$(a^*b^*)^* = (a + b)^*$$

$$(a + b)^*ab(a + b)^* + b^*a^* = (a + b)^*$$

Example.

The Language of Strings Consisting of an Odd Number of a's

$\{a, aaa, aaaaa, aaaaaaa, \ldots\}$

Example 3.2. The Language of Strings in $\{a, b\}^*$ with an Odd Number of a's

Example 3.3. The Language of Strings in $\{a, b\}^*$ Ending with b and Not Containing aa

Exercise.

Find a regular expression corresponding to the language of all strings over $\{a, b\}$ of even length.

Example 3.4. Strings in $\{a, b\}^*$ in Which Both the Number of a's and the Number of b's are Even

Example 3.4. Strings in $\{a, b\}^*$ in Which Both the Number of a's and the Number of b's are Even

$$(aa + bb + (ab + ba)(aa + bb)^*(ab + ba))^*$$

Exercise.

Explain why

$$((a + ba)^*a + bba)^* \quad \neq \quad ((a + ba)^*(a + bba))^*$$

N.B.

$+$ is not concatenation

Example 3.5. Regular Expressions and Programming Languages

identifiers
numeric ‘literals’ (constants)

Regular Expressions in Unix

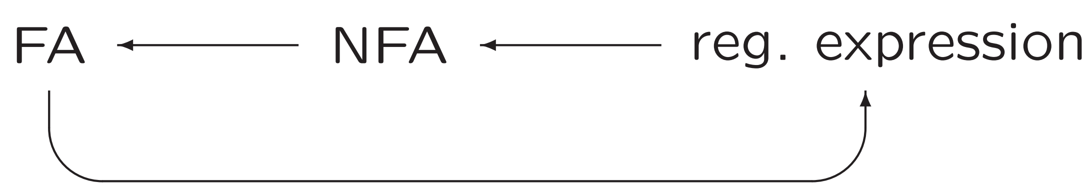
Lex

Vi

grep

3.2 Nondeterministic Finite Automata

reg. languages	FA	reg. grammar	reg. expression
determ. cf. languages	DPDA		
cf. languages	PDA	cf. grammar	
re. languages	TM	unrestr. grammar	



Example 3.6. Accepting the Language $\{aa, aab\}^*\{b\}$

Computation tree...

Example 3.9. Accepting the Language $\{aab\}^*\{a, aba\}^*$

Computation tree. . .