Fundamentele Informatica 1 (I&E)

najaar 2015

http://www.liacs.leidenuniv.nl/~vlietrvan1/fi1ie/

Rudy van Vliet
kamer 124 Snellius, tel. 071-527 5777
rvvliet(at)liacs(dot)nl

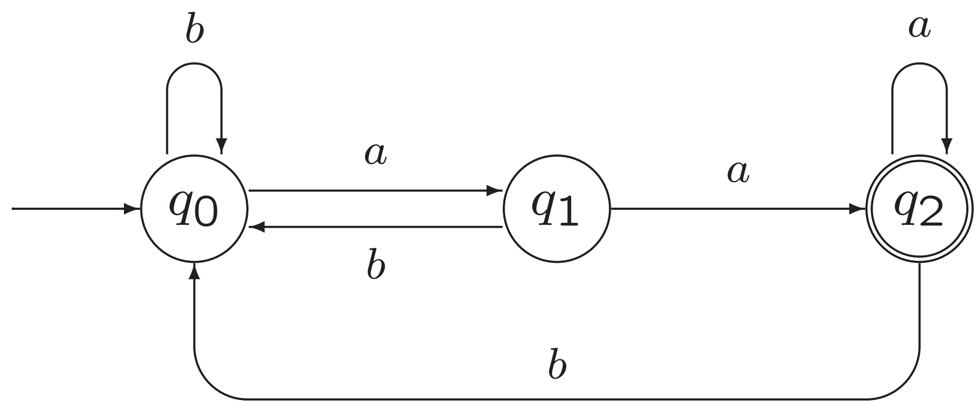
college 3, 3 november 2015

2.4 The Pumping Lemma

Example 2.1.

A finite automaton for accepting

$L_1 = \{x \in \{a, b\}^* \mid\ x$ ends with $aa\}$



2.4 The Pumping Lemma

Theorem 2.29.
The Pumping Lemma for Regular Languages.
Suppose L is a language over the alphabet Σ.
If L is accepted by a finite automaton $M = (Q, \Sigma, q_0, A, \delta)$,
and if n is the number of states of M,

then for every $x \in L$ satisfying $|x| \geq n$, there are three strings u, v, and w such that $x = uvw$ and the following three conditions are true:

1. $|uv| \leq n$.
2. $|v| > 0$ (i.e., $v \neq \Lambda$).
3. For every $i \geq 0$, the string uv^iw also belongs to L.

Application of pumping lemma:
mainly to prove that a language L cannot be accepted by a finite automaton.

How?

Suppose that there exists FA M with n states
that accepts L.

Apply pumping lemma,
and end up with contradiction.

Suppose that there exists FA M with n states that accepts L.

Pumping lemma:

for every $x \in L$ satisfying $|x| \geq n$,
there are three strings u, v, and w such that $x = uvw$
and the following three conditions are true:

1. $|uv| \leq n$.
2. $|v| > 0$ (i.e., $v \neq \Lambda$).
3. For every $i \geq 0$, the string uv^iw also belongs to L.

Suppose that there exists FA M with n states that accepts L.

We prove:

NOT

(

for every $x \in L$ satisfying $|x| \geq n$,
there are three strings u, v, and w such that $x = uvw$
and the following three conditions are true:

1. $|uv| \leq n$.
2. $|v| > 0$ (i.e., $v \neq \Lambda$).
3. For every $i \geq 0$, the string uv^iw also belongs to L.

)

Suppose that there exists FA M with n states that accepts L.

We prove:

There exists $x \in L$ satisfying $|x| \geq n$, such that

NOT
(

there are three strings u, v, and w such that $x = uvw$ and the following three conditions are true:

1. $|uv| \leq n$.
2. $|v| > 0$ (i.e., $v \neq \Lambda$).
3. For every $i \geq 0$, the string uv^iw also belongs to L.

)

Suppose that there exists FA M with n states that accepts L.

We prove:

There exists $x \in L$ satisfying $|x| \geq n$, such that for every three strings u, v, and w such that $x = uvw$

NOT
(

the following three conditions are true:

1. $|uv| \leq n$.
2. $|v| > 0$ (i.e., $v \neq \Lambda$).
3. For every $i \geq 0$, the string uv^iw also belongs to L.

)

Suppose that there exists FA M with n states that accepts L.

We prove:

There exists $x \in L$ satisfying $|x| \geq n$, such that for every three strings u, v, and w such that $x = uvw$

NOT all of the following three conditions are true:

1. $|uv| \leq n$.
2. $|v| > 0$ (i.e., $v \neq \Lambda$).
3. For every $i \geq 0$, the string uv^iw also belongs to L.

Suppose that there exists FA M with n states that accepts L.

We prove:

There exists $x \in L$ satisfying $|x| \geq n$, such that
for every three strings u, v, and w such that $x = uvw$

if
1. $|uv| \leq n$.
2. $|v| > 0$ (i.e., $v \neq \Lambda$).

then NOT
(

3. For every $i \geq 0$, the string uv^iw also belongs to L.

)

Suppose that there exists FA M with n states that accepts L.

We prove:

There exists $x \in L$ satisfying $|x| \geq n$, such that for every three strings u, v, and w such that $x = uvw$

if
1. $|uv| \leq n$.
2. $|v| > 0$ (i.e., $v \neq \Lambda$).

then

3. There exists $i \geq 0$, such that the string uv^iw does not belong to L.

Application of pumping lemma:
mainly to prove that a language L cannot be accepted by a finite automaton.

How?
Find a string $x \in L$ with $|x| \geq n$ that cannot be pumped up!

What is n?

What should x be?

What can u, v and w be?

Example 2.30. The language *AnBn*.

Let $L = \{a^i b^i \mid \ i \geq 0\}$.

Example 2.30. The language *AEqB*.

Let $L = \{x \in \{a,b\}^* \mid \; n_a(x) = n_b(x)\}$.

Example 2.31.

Let $L = \{x \in \{a, b\}^* \mid \; n_a(x) > n_b(x)\}$.

Example 2.32.

Let $L = \{a^{i^2} \mid\ i \geq 0\} = \{a^k \mid\ k \text{ is a square}\}$
$= \{\Lambda, a, aaaa, aaaaaaaaa, aaaaaaaaaaaaaaaa, \ldots\}$.

Example 2.33.

Let L be the set of legal C programs.

Example 2.34. Decision problems involving languages accepted by finite automata.

1. Given an FA $M = (Q, \Sigma, q_0, A, \delta)$, is $L(M)$ nonempty ?

1a. Determine reachable states.
1b. (Black box.) Use pumping lemma.

Exercise.
Let M be a finite automaton with n states and alphabet Σ. Prove the following claim:

$$L(M) \text{ is nonempty} \quad \Longleftrightarrow$$
$$L(M) \text{ contains a string } x \in \Sigma^* \text{ with } |x| < n$$

Example 2.34. Decision problems involving languages accepted by finite automata.

2. Given an FA $M = (Q, \Sigma, q_0, A, \delta)$, is $L(M)$ infinite ?

Use pumping lemma.

Exercise.
Let M be a finite automaton with n states and alphabet Σ. Prove the following claim:

$$\begin{aligned}&L(M) \text{ is infinite} \quad \iff \\ &L(M) \text{ contains a string } x \in \Sigma^* \text{ with } |x| \geq n \quad \iff \\ &L(M) \text{ contains a string } x \in \Sigma^* \text{ with } n \leq |x| < 2n\end{aligned}$$