*A slide from exercise class 6:*

**Exercise 9.1.**

Show that the relation $\leq$ on the set of decision problems is reflexive and transitive.

Give an example to show that it is not symmetric.

*A slide from lecture 7*

**Theorem 9.9.** The following five decision problems are undecidable.

4. *Equivalent*: Given two TMs $T_1$ and $T_2$, is $L(T_1) = L(T_2)$

**Proof.**

4. Prove that *Subset* $\leq$ *Equivalent* . . .

*Subset*: Given two TMs $T_1$ and $T_2$, is $L(T_1) \subseteq L(T_2)$ ?

*Equivalent*: Given two TMs $T_1$ and $T_2$, is $L(T_1) = L(T_2)$

**Exercise 9.10.**

**a.** Given two sets $A$ and $B$, find two sets $C$ and $D$, defined in terms of $A$ and $B$, such that $A = B$ if and only if $C \subseteq D$.

**b.** Show that the problem *Equivalent* can be reduced to the problem *Subset*.

*AcceptsEverything*:

Given a TM $T$ with input alphabet $\Sigma$, is $L(T) = \Sigma^*$ ?

*Equivalent*: Given two TMs $T_1$ and $T_2$, is $L(T_1) = L(T_2)$

**Exercise 9.11.** Construct a reduction from *AcceptsEverything* to the problem *Equivalent*.

**Exercise 9.23.** Show that the property "accepts its own encoding" is not a language property of TMs.

*Part of a slide from lecture 4:*

**Definition 7.33.** An Encoding Function (continued)

For each move $m$ of $T$ of the form $\delta(p, \sigma) = (q, \tau, D)$

$$e(m) = 1^{n(p)}01^{n(\sigma)}01^{n(q)}01^{n(\tau)}01^{n(D)}0$$

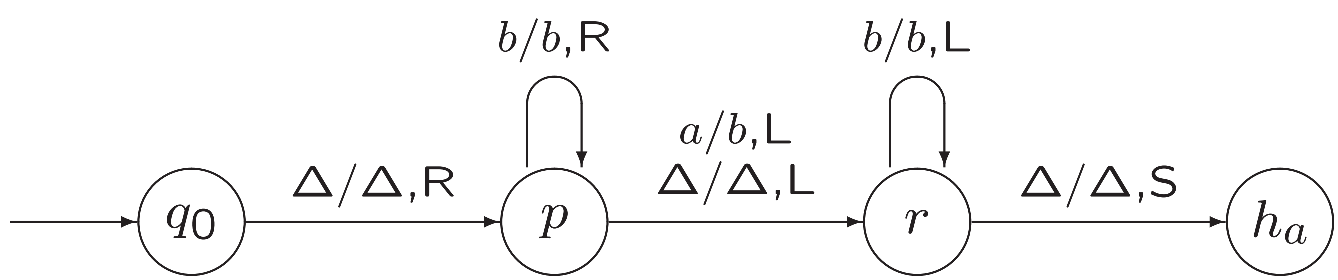
We list the moves of $T$ in some order as $m_1, m_2, \ldots, m_k$, and we define

$$e(T) = e(m_1)0e(m_2)0\ldots0e(m_k)0$$

**Exercise 9.23.** Show that the property "accepts its own encoding" is not a language property of TMs.

*A slide from lecture 4:*

**Example 7.34.** A Sample Encoding of a TM




$\underline{1}$11010111101010 0 11110111011110111010 0
111101101111101110110 0 111101011111010110 0
11111011101111101110110 0 1111101010101110 0

**Exercise 9.12.**
For each decision problem below, determine whether it is decidable or undecidable, and prove your answer.

**a.** Given a TM $T$, does it ever reach a nonhalting state other than its initial state if it starts with a blank tape?

**Exercise 9.12.**
For each decision problem below, determine whether it is decidable or undecidable, and prove your answer.

**b.** Given a TM $T$ and a nonhalting state $q$ of $T$, does $T$ ever enter state $q$ when it begins with a blank tape?

**e.** Given a TM $T$, is there a string it accepts in an even number of moves?

**j.** Given a TM $T$, does $T$ halt within ten moves on every string?

**l.** Given a TM $T$, does $T$ eventually enter every one of its nonhalting states if it begins with a blank tape?

**Exercise 9.13.**

In this problem TMs are assumed to have input alphabet $\{0,1\}$. For a finite set $S \subseteq \{0,1\}^*$, $P_S$ denotes the decision problem: Given a TM $T$, is $S \subseteq L(T)$ ?

**a.** Show that if $x, y \in \{0,1\}^*$, then $P_{\{x\}} \leq P_{\{y\}}$.

**b.** Show that if $x, y, z \in \{0,1\}^*$, then $P_{\{x\}} \leq P_{\{y,z\}}$.

**c.** Show that if $x, y, z \in \{0,1\}^*$, then $P_{\{x,y\}} \leq P_{\{z\}}$.

**d.** Is it true that for every two finite subsets $S$ and $U$ of $\{0,1\}^*$, $P_S \leq P_U$.