A slide from exercise class 6:

Exercise 9.1.

Show that the relation $\leq$ on the set of decision problems is reflexive and transitive.

Give an example to show that it is not symmetric.

A slide from lecture 7

Theorem 9.9. The following five decision problems are undecidable.

4. *Equivalent*: Given two TMs T_1 and T_2, is $L(T_1) = L(T_2)$

Proof.

4. Prove that *Subset* $\leq$ *Equivalent* . . .

Subset: Given two TMs T_1 and T_2, is $L(T_1) \subseteq L(T_2)$?

Equivalent: Given two TMs T_1 and T_2, is $L(T_1) = L(T_2)$

Exercise 9.10.

a. Given two sets A and B, find two sets C and D, defined in terms of A and B, such that $A = B$ if and only if $C \subseteq D$.

b. Show that the problem *Equivalent* can be reduced to the problem *Subset*.

AcceptsEverything:

Given a TM T with input alphabet Σ, is $L(T) = \Sigma^*$?

Equivalent: Given two TMs T_1 and T_2, is $L(T_1) = L(T_2)$

Exercise 9.11. Construct a reduction from *AcceptsEverything* to the problem *Equivalent*.

Exercise 9.23. Show that the property "accepts its own encoding" is not a language property of TMs.

Part of a slide from lecture 4:

Definition 7.33. An Encoding Function (continued)

For each move m of T of the form $\delta(p, \sigma) = (q, \tau, D)$

$$e(m) = 1^{n(p)}01^{n(\sigma)}01^{n(q)}01^{n(\tau)}01^{n(D)}0$$

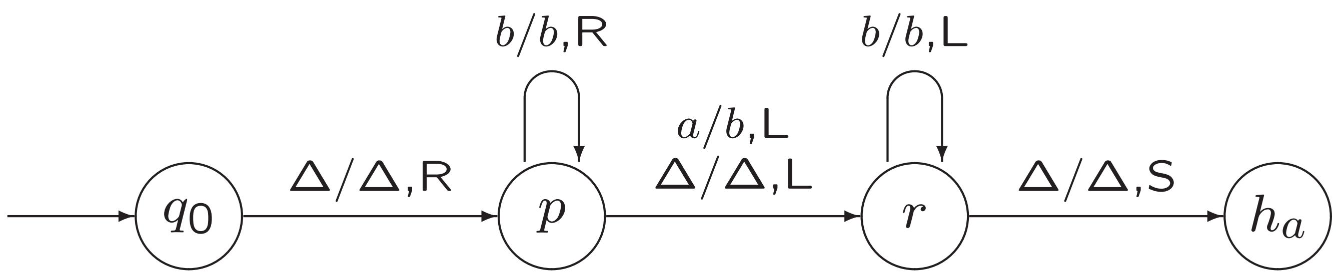
We list the moves of T in some order as $m_1, m_2, \ldots, m_k$, and we define

$$e(T) = e(m_1)0e(m_2)0\ldots0e(m_k)0$$

Exercise 9.23. Show that the property "accepts its own encoding" is not a language property of TMs.

A slide from lecture 4:

Example 7.34. A Sample Encoding of a TM



$\underline{1}$1101011101010 0 1110111011110111010 0
11110110111110110110 0 1111010111111010110 0
11111011101111101110110 0 1111101010101110 0

Exercise 9.12.
For each decision problem below, determine whether it is decidable or undecidable, and prove your answer.

a. Given a TM T, does it ever reach a nonhalting state other than its initial state if it starts with a blank tape?

Exercise 9.12.

For each decision problem below, determine whether it is decidable or undecidable, and prove your answer.

b. Given a TM T and a nonhalting state q of T, does T ever enter state q when it begins with a blank tape?

e. Given a TM T, is there a string it accepts in an even number of moves?

j. Given a TM T, does T halt within ten moves on every string?

l. Given a TM T, does T eventually enter every one of its nonhalting states if it begins with a blank tape?

Exercise 9.13.

In this problem TMs are assumed to have input alphabet $\{0, 1\}$. For a finite set $S \subseteq \{0, 1\}^*$, P_S denotes the decision problem: Given a TM T, is $S \subseteq L(T)$?

a. Show that if $x, y \in \{0, 1\}^*$, then $P_{\{x\}} \leq P_{\{y\}}$.

b. Show that if $x, y, z \in \{0, 1\}^*$, then $P_{\{x\}} \leq P_{\{y,z\}}$.

c. Show that if $x, y, z \in \{0, 1\}^*$, then $P_{\{x,y\}} \leq P_{\{z\}}$.

d. Is it true that for every two finite subsets S and U of $\{0, 1\}^*$, $P_S \leq P_U$.