

## Exercise 7.14.

Draw a TM for the **component**  $Insert(\sigma)$ , which changes the tape contents from  $y\underline{z}$  to  $y\underline{\sigma}z$ .

Here  $y \in (\Gamma \cup \{\Delta\})^*$ ,  $\sigma \in \Gamma \cup \{\Delta\}$ , and  $z \in \Gamma^*$ .

You may assume that  $\Gamma = \{a, b\}$ .

### Exercise 7.13.

Suppose  $T$  is a TM that accepts every input. We might like to construct a TM  $R_T$  such that for every input string  $x$ ,  $R_T$  halts in the accepting state with exactly the same tape contents as when  $T$  halts on input  $x$ , but with the tape head positioned at the rightmost nonblank symbol on the tape.

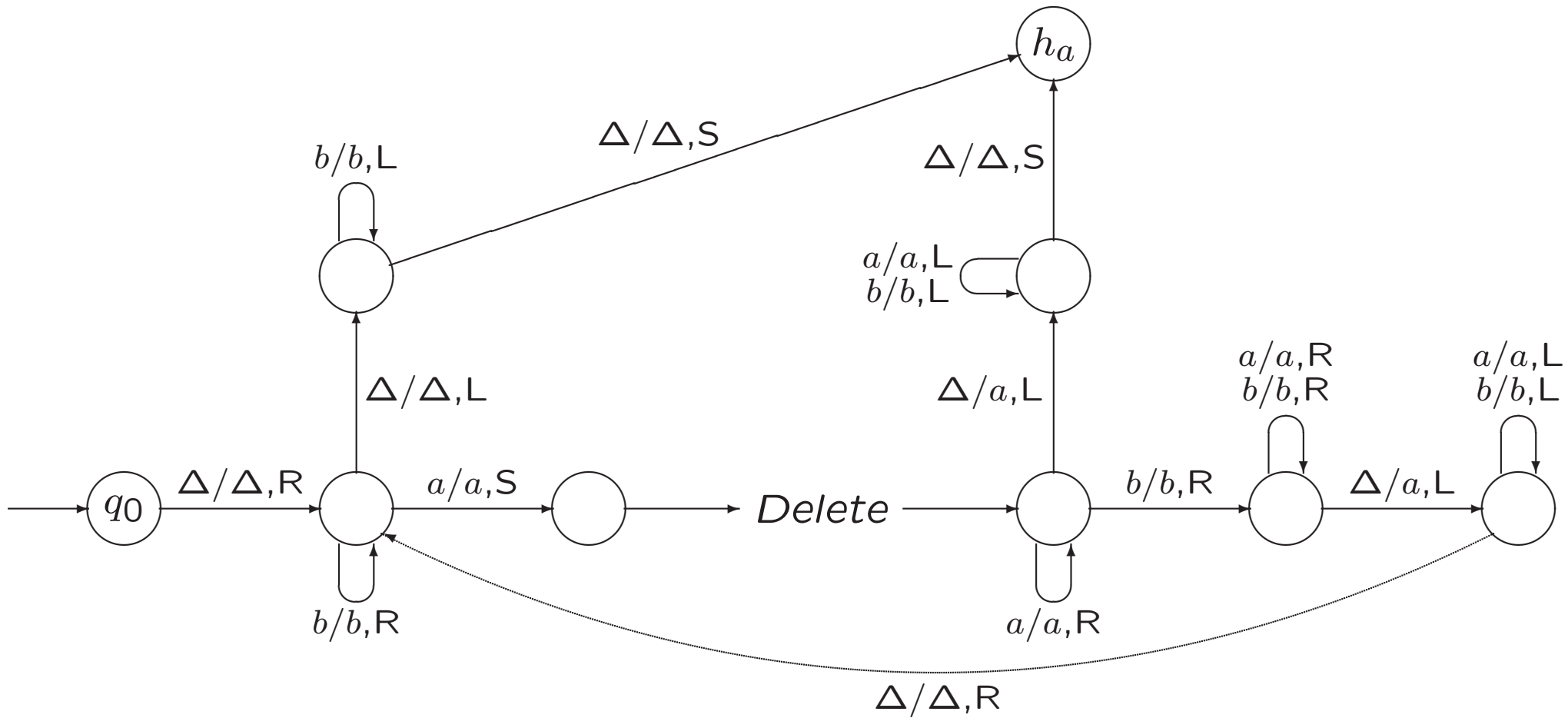
Show that there is no fixed TM  $T_0$  such that  $R_T = TT_0$  for every  $T$ . (In other words, there is no TM capable of executing the instruction “move the tape head to the rightmost nonblank tape symbol” in every possible situation.)

Suggestion: Assume there is such a TM  $T_0$ , and try to find two other TMs  $T_1$  and  $T_2$  such that if  $R_{T_1} = T_1T_0$  then  $R_{T_2}$  cannot be  $T_2T_0$ .

Assume that the tape contains at least one nonblank symbol, when  $T$  halts.

### Exercise 7.18.

The TM shown below computes a function  $f$  from  $\{a, b\}^*$  to  $\{a, b\}^*$ . For any string  $x \in \{a, b\}^*$ , describe the string  $f(x)$ .



### Exercise 7.19.

Suppose TMs  $T_1$  and  $T_2$  compute the functions  $f_1$  and  $f_2$  from  $\mathbb{N}$  to  $\mathbb{N}$ , respectively.

Describe how to construct a TM to compute the function  $f_1 + f_2$ .

Assume that both  $T_1$  and  $T_2$  use unary notation to represent natural numbers.

## Exercise 7.20.

Draw a transition diagram for a TM with input alphabet  $\{0, 1\}$  that interprets the input string as the binary representation of a nonnegative integer and adds 1 to it.

You may assume that the input string is not empty.

## Exercise.

Construct a 2-tape Turing machine  $T$  that has as input two strings  $w_1$  and  $w_2$  from  $\{a,b\}^*$  (both on the first tape, separated by a single blank, as usual), and that checks in linear time whether or not  $w_2$  is an anagram of  $w_1$  (a rearrangement of the letters). If so, then  $T$  should accept, otherwise, it should reject.

*Hint: in order to check if  $w_2$  is an anagram of  $w_1$ , you might look at the number of occurrences of letters in  $w_1$  and  $w_2$ .*

## Exercise 7.23.

Draw a transition diagram for a three-tape TM that works as follows:

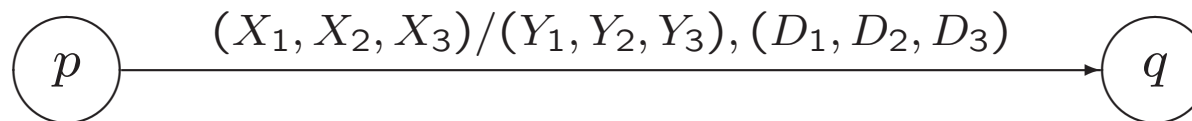
starting in the configuration  $(q_0, \underline{\Delta}x, \underline{\Delta}y, \underline{\Delta})$ ,

where  $x$  and  $y$  are **nonempty** strings of 0's and 1's of the same length,

it halts in the configuration  $(h_a, \underline{\Delta}x, \underline{\Delta}y, \underline{\Delta}z)$ ,

where  $z$  is the string obtained by interpreting  $x$  and  $y$  as binary representations and adding them.

Use transitions of the following form:



### Exercise 7.24.

In Example 7.5, a TM is given that accepts the language  $\{xx \mid x \in \{a, b\}^*\}$ .

Draw a TM with tape alphabet  $\Gamma = \{a, b\}$  that accepts this language.



## Exercise 7.25.

We can consider a TM with a *doubly infinite* tape, by allowing the numbers of the tape squares to be negative as well as positive. In most respects the rules for such a TM are the same as for an ordinary one, except that now when we refer to the configuration  $xq\sigma y$ , including the initial configuration corresponding to some input string, there is no assumption about exactly where on the tape the strings and the tape head are.

Draw a transition diagram for a TM with a doubly infinite tape that does the following: If it begins with the tape blank except for a single  $a$  somewhere on it, it halts in the accepting state with the head on the square with the  $a$ .