From exercise class 12:

Exercise 6.2.

In each case below, show using the pumping lemma that the given language is not a CFL.

a. $L = \{a^i b^j c^k \mid i < j < k\}$

b. $L = \{a^{2^i} \mid i \geq 0\}$

d. $L = \{a^i b^{2i} a^i \mid i \geq 0\}$

e. $L = \{s \in \{a, b, c\}^* \mid n_a(s) = \max\{n_b(s), n_c(s)\}\ \}$

g. $L = \{a^i b^j a^i b^{i+j} \mid i, j \geq 0\}$

From exercise class 12:

Exercise 6.5.

For each case below, decide whether the given language is a CFL, and prove your answer.

a. $L = \{a^i b^j a^j b^i \mid i, j \geq 0\}$

c. $L = \{scs \mid s \in \{a, b\}^*\}$

d. $L = \{sts \mid s, t \in \{a, b\}^* \text{ en } |s| \geq 1\}$

g. $L =$ the set of non-balanced strings of parentheses

Exercise.
(to demonstrate that in the application of the pumping lemma for context-free languages, one can often safely assume that $n \geq 2^2 = 4$)

Let $G = (V, \Sigma, S, P)$ be a context-free grammar in Chomsky normal form.

Suppose that $V = \{S\}$, i.e., that G has only one variable. What could $L(G)$ be in this case?

Exercise.

Give a context-free grammar for the language

$$\{a^i x\, y\, b^j \quad | \quad i, j \geq 0,\ x, y \in \{a, b\}^*,\ |x| = j \text{ and } |y| = i\}$$

Exercise 6.6.

If L is a CFL, does it follow that $r(L) = \{x^r \mid x \in L\}$ is a CFL? Give a proof or a counterexample.

Exercise 6.9.

In each case below, show that the given language is a CFL but that its complement is not.

b. $\{a^i b^j c^k \quad | \quad i \neq j \text{ or } i \neq k\}$

a. $\{a^i b^j c^k \quad | \quad i \geq j \text{ or } i \geq k\}$

Exercise 6.12.

a. Show that if L is a CFL and F is finite, $L - F$ is a CFL.

b. Show that if L is not a CFL and F is finite, $L - F$ is not a CFL.

c. Show that if L is not a CFL and F is finite, $L \cup F$ is not a CFL.

Exercise 6.13.

For each part below, say whether the statement is true or false, and give reasons for your answer.

a. If L is a CFL and F is regular, then $L - F$ is a CFL.

b. If L is not a CFL and F is regular, then $L - F$ is not a CFL.

c. If L is not a CFL and F is regular, then $L \cup F$ is not a CFL.

Theorem 6.13.

If L_1 is a context-free language and L_2 is a regular language, then $L_1 \cap L_2$ is a CFL.

Exercise 6.8.

Show that if L_1 is a DCFL and L_2 is regular, then $L_1 \cap L_2$ is a DCFL.