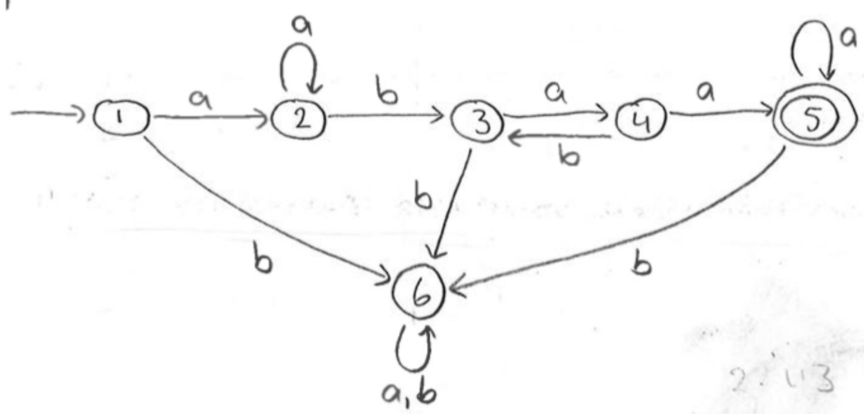
SOLUTION EXAM AUTOMATA THEORY

Thursday 23 December 2021, 10:15 - 13:15

1. [10 pts]

21.44

1(a)



21.48 / 21.51

(b) $Q = \{1,2,3,4,5,6\}$

$\Sigma = \{a,b\}$

$q_0 = 1$

$A = \{5\}$

$\delta^*(q_0, aaba) = 4$

2. [9 pts] Let m, n be arbitrary numbers ≥ 0 with $m \neq n$. Without loss of generality, assume $m < n$.

   - If $m = 0$, then of course $n \geq 1$.
     Then choose $z = a$.
     The string $a^m z = a^0 a = a \in L$, because 0 occurrences of aa and 0 occurrences of ba.
     The string $a^n z = a^n a = a^{n+1} \notin L$, because $n + 1 \geq 2$, so the string contains at least 1 occurrence of aa, while 0 occurrences of ba.
   - If $m \geq 1$, we choose $z = a(ba)^m$.
     The string $a^m z = a^m a(ba)^m = a^{m+1}(ba)^m \in L$, because it contains m occurrences of aa and m occurrences of ba.
     The string $a^n z = a^n a(ba)^m = a^{n+1}(ba)^m \notin L$, because it contains n occurrences of aa and m occurrences of ba, while $m < n$.

   In hindsight we could have taken $z = a(ba)^m$ in all cases, because for $m = 0$ this is equal to a.

---

3. [8 pts]

3) $L = aa + ba + aaa + baa + (aa+ba)(a+b)^*(aa+ba)$

---

4. [15 pts]

4)

$r^0(i,j)$	$j=1$	2	3
$i=1$	$\Lambda + a$	b	$\emptyset$
2	a	Λ	b
3	a	b	Λ

$r^1(i,j)$	$j=1$
$i=1$	$\Lambda + a + (\Lambda+a)(\Lambda+a)^*(\Lambda+a)$
2	$a(\Lambda+a)^*(\Lambda+a) + a$
3	$a + a(\Lambda+a)^*(\Lambda+a)$

	$j=2$	$j=3$
$i=1$	$b + (\Lambda+a)(\Lambda+a)^* b$	$\emptyset + (\Lambda+a)(\Lambda+a)^*\emptyset$
2	$\Lambda + a(\Lambda+a)^* b$	$b + a(\Lambda+a)^*\emptyset$
3	$b + a(\Lambda+a)^* b$	$\Lambda + a(\Lambda+a)^*\emptyset$

Simplified

$r^1(i,j)$	$j=1$	$j=2$	$j=3$
$i=1$	a^*	$b + a^*b$	$\emptyset$
2	$a + aa^*$	$\Lambda + aa^*b$	b
3	$a + aa^*$	$b + aa^*b$	$\Lambda +$

Further simplified

$r^1(i,j)$	$j=1$	$j=2$	$j=3$
$i=1$	a^*	a^*b	$\emptyset$
2	aa^*	$\Lambda + aa^*b$	b
3	aa^*	$b + aa^*b$	Λ

$$r^2(3,1) = aa^* + (b + aa^*b)(\Lambda + aa^*b)^* aa^*$$

---

5. [16 pts]

   a. i. No, $L(G_1) \not\subseteq L$, because for example $aba \in L(G_1)$ via $S \Rightarrow AB \Rightarrow aAbaB \Rightarrow abaB \Rightarrow aba$, but $aba \notin L$.

      ii. Yes, $L \subseteq L(G_1)$.
      If you don't want any a's on the left, you can start as follows: $S \Rightarrow AB \Rightarrow B$.
      If you do want a's on the left, you can start as follows: $S \Rightarrow aAB$.

   b. i. Yes, $L(G_2) \subseteq L$.

      ii. No, $L \not\subseteq L(G_2)$, because with $aaAbabaBa$ you ensure that $i \geq 2$. The case of $i = 1$ and $k \geq 1$ is not possible, e.g. $abaa \in L$, but $abaa \notin L(G_2)$.

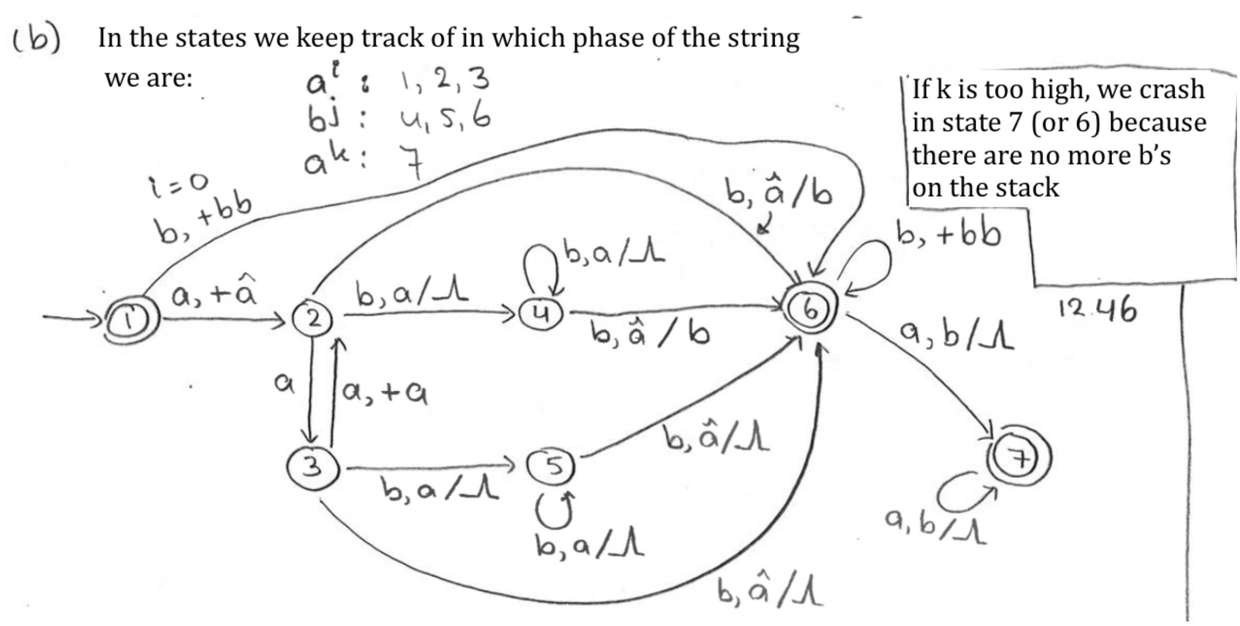
---

6. [17 pts]

   a. The first six elements in the canonical order of L are

$$\Lambda, b, ab, ba, bb, aab$$

   b. .



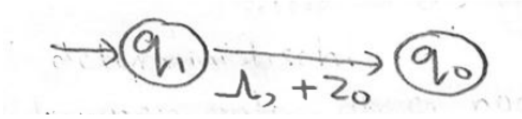
With the a's on the stack we keep track of how many a's of the a^i we have read, divided by 2, rounded up. We start with $\hat{a}$, so that we can recognize the bottom a on the stack.
In state 2 i is odd, and in state 3 i is even. When we start to read b's, for every b we remove an a from the stack (corresponding with two a's, because the b counts for two in $2j$). In state 4 for i odd, in state 5 for i even. When we remove $\hat{a}$ from the stack, we know that all a's of a^i have been compensated by enough b's. Then we go to accepting state 6, and for every b that we read we place 2 b's on the stack (b counts for two in $2j$), so that later in state 7 we can cross them off against the a's of a^k.

---

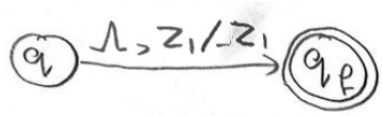
7. [12 pts]

   a. We say that x can be accepted by M by empty stack if $(q_0, x, Z_0) \vdash_M^* (q, \Lambda, \Lambda)$ for some $q \in Q$.
   In other words, there exists a computation of M by which x is read completely and the stack is totally empty at the end of the computation. Even Z_0 is no longer on the stack. It doesn't matter in which state you end up.

   b. i. We give M_1 a new initial stack symbol Z_1 and a new initial state q_1, with transition

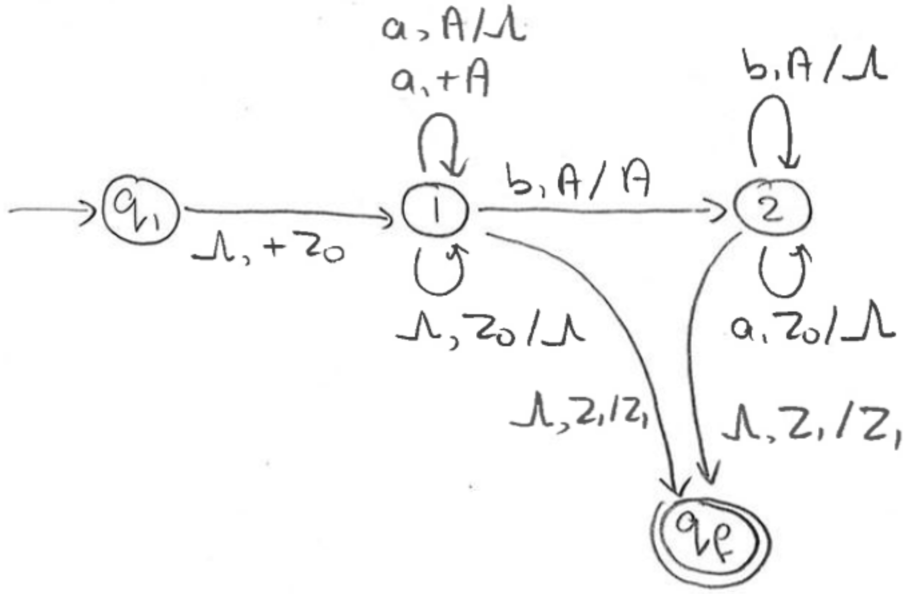


where q_0 is the initial state of M, and Z_0 the initial stack symbol of M.
Furthermore we provide every state $q \in Q$ with a transition



where q_f is a new state, the only accepting state of M_1.
For the rest, M_1 simulates M, from q_0 on.

ii. .



8. [13 pts]

   u_1: Not suitable, because it does not necessarily have length $\geq n$.

   u_2: Suitable.

   u_3: Not suitable, because the decomposition $v = a^n b^{2n+n-1}, w = b, x = \Lambda, y = a, z = a^{2n-1}$ satisfies the pumping lemma.

   u_4: Suitable.

End of exam.