Homework 4 Automata Theory 2023

Published on: Tuesday 5 December 2023.
Deadline for submission: Tuesday 19 December 2023, 23:59.
The assignment must be completed individually. A total of 100 points can be earned. Answers to be submitted via Brightspace. Submit a single file, e.g., a pdf or possibly a zip. Please include your name and student number in your submission. You may either type your answers or hand-write them. In the latter case, please hand in an easy-to-read scan / photos.

1. [50 pt] Let L_1 be the language consisting of all strings $x \in \{a,b\}^*$, such that

   - $n_b(x) \geq 1$, and
   - after the last occurrence of b, x contains at least $n_b(x)$ a's, and
   - $n_a(x) > n_b(x)$, i.e., in addition to the a's from the previous condition, x contains at least one more a (at some point in the string).

   Hence, the first five elements in the canonical (shortlex) order of L_1 are: $aba, baa, aaba, abaa, baaa, aaaba$. But also, e.g., $abbaa$ and $babaa$ are elements of L_1.

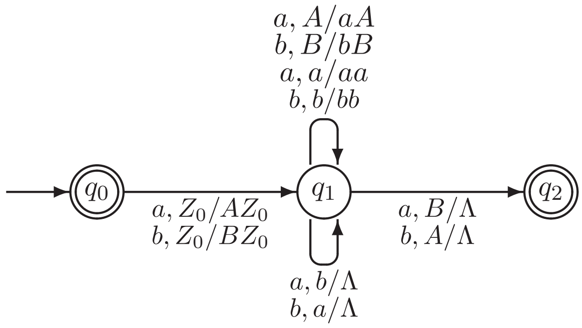
   (a) Give a pushdown automaton M_1, such that $L(M_1) = L_1$.
   M_1 should be based directly on properties of L_1. It must not be the result of applying a standard construction, e.g., to convert a context-free grammar into a pushdown automaton.
   Try to ensure that M_1 has no Λ-transitions. If you do not succeed in this, you lose 5 points.
   N.B.: It may be hard / impossible to construct a *deterministic* pushdown automaton for this language.

   (b) Explain how M_1 uses its states and/or stack to accept exactly L_1.

   (c) If your pushdown automaton M_1 is deterministic (and correct), then move on to part (d). Otherwise, mention one state, stack symbol and input σ (either Λ, or a or b), for which M_1 is nondeterministic.

   (d) Adjust M_1 in such a way, that the resulting pushdown automaton M_1' accepts L_1 *by empty stack*, i.e., not by final state.
   It is allowed to apply an ad hoc adjustment of M_1 for this. It is not allowed to introduce (extra) Λ-transitions in the automaton.

2. [20 pt] Consider the following pushdown automaton M_2:



(a) What is $L(M_2)$ for this automaton M_2? Express (in words or in formulas, but at least clearly and completely) what are the elements of M_2.

(b) Explain how M_2 uses its states and/or stack symbols to accept exactly the language you described at part (a).

3. [30 pt] Let G be the context-free grammar with start variable (and only variable) S, and the following productions:

$$S \to SaS \mid b \mid \Lambda$$

(a) Draw the nondeterministic bottom-up PDA $NB(G)$ for this grammar G.

(b) Give a derivation tree for $x = baa$ in G.

(c) Execute a successful computation in $NB(G)$ for the input $x = baa$, i.e., a computation that starts in the initial configuration for x and results in acceptance of x. The computation should correspond to the derivation tree of part (b).

Present this computation in a tabel of the following form:

state	stack (reversed)	remaining input	action
q_0	Z_0	baa	...
...	...	...	...

(see the lecture slides for an example).