

A Formal Verification of the Alternating Bit Protocol in μ CRL

Gertjan Kamsteeg

*Department of Computer Science
Leiden University*

Abstract

We present a formal verification proof for the Alternating Bit Protocol in the specification language and proof theory of μ CRL. A brief outline, dwelling on the more elaborate parts of the derivation, as well as a very detailed version of the proof are given. The proof follows the one presented in [1]. We also mention some shortcomings and possible solutions of the proof system.

μ CRL, developed to study processes with data [6,7] and to provide a formal means to mechanize proofs in process algebra, is based on algebraic specifications, classical first order logic with equality and without explicit quantification, and the process algebra ACP with some extensions.

Introduction

Verifying the ABP protocol, as a well-known communication protocol, has advantages as well as disadvantages. One of the drawbacks is that the presented proof may be viewed as 'just another verification' of the ABP protocol. On the other hand, because of its familiarity things are easy to grasp and we are able to emphasize on the proof system and the format of the proof rather than the proof itself. In this paper we present a formal verification proof for the Alternating Bit Protocol in the specification language and proof theory of μ CRL. The proof follows the one presented in [1]. We also mention some shortcomings and possible solutions of the proof system.

The given proof is as detailed as possible, in the sense that every single proof step is shown, cheating only on basic equality and substitution rules. That is, most of the proof is given as a series of rewrite steps rather than applications of a (derived) transitivity rule. However, if a still more detailed proof is needed, these rules can easily be added because they are implicitly - yet always correctly - applied. This way, the proof may be used to serve as the input of a mechanical proof checker. A brief outline of the proof, dwelling on the more elaborate parts of the derivation precedes the detailed version.

The specification language μ CRL (micro CRL), based on CRL (Common Representation Language) [10] was developed by J.F. Groote and A. Ponse both to study processes with data and to provide a formal means to mechanize proofs in process algebra. Syntax and semantics are defined in [6], while a proof theory is presented in [7]. It is based on algebraic specifications, classical first order logic with equality and without explicit quantification, and the process algebra ACP with some extensions. We refer to these reports for a full description of μ CRL.

Although μ CRL is provided with an induction mechanism and applying it would certainly help to generalize some lemmas, we will not use it. The reason for this is that it depends on generally undecidable external conditions, which have to be proved on meta-level. Therefore we find the induction rule - as it is - not very suitable for verification by mechanical proof checkers. Moreover, if at a later stage one wants to extend a specification, one has to check all proofs all over.

Because μ CRL is based on classical logic, we feel free to use the RAA (Reductio Ad Absurdum) rule whenever we like. In intuitionistic terms this means that every expressible predicate is considered decidable (especially equality). Note that this has nothing to do with the undecidability mentioned above, since the problem with the proviso on the induction rule is that it is not even directly expressible within the language. Of course we acknowledge that, if possible, a constructive proof (i.e. one without applications of RAA) is preferable to a nonconstructive one. If we have to choose however between many extra definitions in the specification and being nonconstructive but keeping things simple, we choose the latter. See also the remarks in the outline of the proof, where we give a suggestion for altering one of the communication rules to obtain a fully constructive proof for the

ABP protocol.

Format of the proofs

Syntax

Terms have the form x or $f(t_1, \dots, t_k)$ where x is a data (term) variable, f is a function symbol, $k \geq 0$ (in case $n = 0$ the parentheses are omitted) and t_1, \dots, t_k are terms. Atomic actions have the form $n(t_1, \dots, t_k)$ where n is an atomic action symbol (port name, label), $k \geq 0$ (in case $k = 0$ the parentheses are omitted) and t_1, \dots, t_k are terms. Processes have one of the following forms: a , x , $P(t_1, \dots, t_k)$, δ , τ , $p \triangleleft t \triangleright q$, $p + q$, $p \cdot q$, $p \parallel q$, $p \llbracket q$, $p \mid q$, $\delta(H, p)$, $\tau(I, p)$ or $\Sigma(d: D, p)$ where a is an atomic action, x is a process variable, P is a process identifier t, t_1, \dots, t_k are terms, p and q are processes, H and I are finite sequences of atomic action symbols represented by $\{n_1, \dots, n_k\}$ ($k \geq 0$), d is a data variable and D is a sort symbol. Finite sequences of atomic action symbols are considered syntactically (!) equal if they represent the same set. Predicates have the form $t = u$, $p = q$ or \perp where t and u are terms and p and q are processes. Formulas have the form π , $\neg\phi$, $\phi \rightarrow \psi$, $\phi \wedge \psi$, or $\phi \vee \psi$ where π is a predicate and ϕ and ψ are formulas.

Logic

The proof theory for μCRL is based on classical logic with equality and without explicit quantification. The (logical) inference rules are listed in table 1. In the rule REFL, the t on the left side of the $=$ must be syntactically equal to the t on the right side. That is, modulo the representation of finite sets of atomic action symbols.

$\frac{[\phi] \quad \vdots \quad \psi}{\phi \rightarrow \psi} \rightarrow\text{I}$	$\frac{\vdots \quad \phi \quad \vdots \quad \phi \rightarrow \psi}{\psi} \rightarrow\text{E}$
$\frac{[\neg\phi] \quad \vdots \quad \perp}{\phi} \text{RAA}$	
$\frac{}{t = t} \text{REFL}$	$\frac{\vdots \quad \phi[t/x] \quad \vdots \quad t = u}{\phi[u/x]} \text{REPL}$
$\frac{\vdots \quad \phi}{\phi[t/x]} \text{SUB}$	<p>if t free for x in ϕ and x not free in any assumption the premise depends on</p>

Table 1: Rules for logical deductions

Assumptions discharged with the application of a rule are stated between square brackets. φ and ψ range over formulas, t and u over data terms and x over variables. Vertical dots represent subderivations and $\varphi[t/x]$ denotes substitution of t for x in φ .

Preliminaries

Before explaining the format of proofs, we make the following remarks.

- The word *rule* is used to indicate rules and *axioms* (rules without premises), *lemmas* and *theorems* (derived rules) and also *rule schemes*. Schematic symbols (meta variables) are written in italics.
- The rule SUB will never be referred to explicitly. This is justified because free variables, not occurring free in an open assumption, are considered universally quantified. Therefore, any given - or earlier derived rule can be viewed as a *rule scheme* where variables act as schematic symbols.
- In the original formulation of μ CRL each formula is provided with a *context* E, V_d, V_p consisting of the names of the specification and variable sets for data and process variables respectively.

φ **from** E, V_d, V_p

In our proofs however, all variables will be declared beforehand. Consequently, there is no need for applications of the rule VAR:

$$\frac{\begin{array}{c} \vdots \\ \varphi \text{ from } E, V_d, V_p \end{array}}{\varphi \text{ from } E, V_d', V_p'} \text{VAR} ,$$

to alter a context. This practice is justified since there are no rules (except VAR itself) which modify contexts. Axioms, *introducing* a context, hold in every context in which their conclusions are *properties*. Therefore, as long as we make sure that all terms are well typed with respect to the given specification and declarations (easily checked), there is no need to change a context. At the end of a derivation, the context may be minimized by removing each variable from it not occurring in the derived formula.

Main axioms and rules

To provide for the references to the axioms and rules of ACP, standard concurrency, handshaking, and abstraction, these axioms and rules are listed in the following tables 2, 3, 4 and 5 respectively.

Sum operator

The sum operator (see table 6) is formally introduced in μ CRL as a part of the language. It is not just an abbreviation for the $+$ operator since it may abstract over infinite or even unspecified data types (sorts). Although in the latest versions of μ CRL α -congruent terms (i.e. terms only differing in the names of bound variables) are considered syntactically equal (to be precise, this is a consequence of the definition of substitution) we stick to the older version, in which terms must be explicitly α -converted by means of the axiom SUM2. The reason for this is that a mechanical proof checker easily verifies the conditions on SUB and SUM11, while introducing fresh variables with intelligible names is usually more complicated and can easily be done by hand.

The rule SUM11 will not be referred to explicitly. It is used mainly in combination with the rule REPL as follows:

$$\frac{\begin{array}{c} \vdots \\ p = \Sigma(d:D, q_1) \end{array} \quad \frac{\begin{array}{c} \vdots \\ q_1 = q_2 \end{array} \quad \text{SUM11}}{\Sigma(d:D, q_1) = \Sigma(d:D, q_2)} \quad \text{REPL}}{p = \Sigma(d:D, q_2)}$$

if d not free in the assumptions $q_1 = q_2$ depends on

Rewriting

The format of the proofs basically consists of a series of sequences of rewrite steps of the form

$$\begin{array}{l} [name] \\ p_1 = [r_1] \\ p_2 = [r_2] \\ \vdots \\ p_{n-1} = [r_{n-1}] \\ p_n; \end{array}$$

where $[name]$ will be the reference to the derivation of $p_1 = p_n$ in subsequent derivations, p_i ($1 \leq i \leq n$) is a process term or data term and $[r_i]$ ($1 \leq i \leq n$) is the reference to (the derivation of) the applied rule. Redexes are underlined and each rewrite sequence is terminated by a semicolon.

Thus, one rewrite step corresponds to one application of the rule REFL as follows:

The derivation

$$\frac{\begin{array}{c} \vdots \\ p_1 = p[t/x] \end{array} \quad \begin{array}{c} \vdots \\ t = u \end{array}}{p_1 = p[u/x]} \text{ REFL}$$

where x is supposed not to occur in p_1 , is represented by

$$\begin{array}{l} p_1 = [\cdot] \\ \vdots \\ p_i = [r] \\ p_{i+1} \\ \vdots \end{array}$$

where $p[t/x] \equiv p_i$, $p[u/x] \equiv p_{i+1}$, the first premise is satisfied by the preceding rewrite step (or by an application of REFL in the initial step) and $[r]$ is a reference to a derivation of the second premise $t = u$.

The *direction* of the equality $t = u$ does not follow from the reference to its derivation. However, this information could easily be added if required (as will be shown soon).

Linear, rather than tree-like, notation of proofs will improve readability of large derivations.

A1	$p_1 + p_2 = p_2 + p_1$	CF1	$n_1 n_2 = n_3$	if $\gamma(n_1, n_2) = n_3$
A2	$p_1 + (p_2 + p_3) = (p_1 + p_2) + p_3$	CF1'	$n_1(t_1, \dots, t_m) n_2(t_1, \dots, t_m) =$ $n_3(t_1, \dots, t_m)$	if $\gamma(n_1, n_2) = n_3$
A3	$p + p = p$	CF2	$a_1 a_2 = \delta$	if $\gamma(\text{label}(a_1), \text{label}(a_2)) = \delta$
A4	$(p_1 + p_2) \cdot p_3 = p_1 \cdot p_3 + p_2 \cdot p_3$	CF2'	$\neg(t_i = t'_i) \rightarrow$ $n_1(t_1, \dots, t_m) n_2(t'_1, \dots, t'_m) = \delta$	for $1 \leq i \leq m$
A5	$(p_1 \cdot p_2) \cdot p_3 = p_1 \cdot (p_2 \cdot p_3)$	CF2''	$n_1(t_1, \dots, t_m) n_2(t'_1, \dots, t'_m) = \delta$	if $m \neq m'$
A6	$p + \delta = p$	D1	$\partial(H, a) = a$	if $\text{label}(a) \notin H$
A7	$\delta \cdot p = \delta$	D2	$\partial(H, a) = \delta$	if $\text{label}(a) \in H$
CM1	$p_1 \parallel p_2 = p_1 \parallel p_2 + p_2 \parallel p_1 + p_1 p_2$	D3	$\partial(H, p_1 + p_2) = \partial(H, p_1) + \partial(H, p_2)$	
CM2	$a \parallel p = a \cdot p$	D4	$\partial(H, p_1 \cdot p_2) = \partial(H, x) \cdot \partial(H, p_2)$	
CM3	$a \cdot p_1 \parallel p_2 = a \cdot (p_1 \parallel p_2)$			
CM4	$(p_1 + p_2) \parallel p_3 = p_1 \parallel p_3 + p_2 \parallel p_3$			
CM5	$a_1 \cdot p a_2 = (a_1 a_2) \cdot p$			
CM6	$a_1 a_2 \cdot p = (a_1 a_2) \cdot p$			
CM7	$a \cdot p_1 b \cdot p_2 = (a b) \cdot (p_1 \parallel p_2)$			
CM8	$(p_1 + p_2) p_3 = p_1 p_3 + p_2 p_3$			
CM9	$p_1 (p_2 + p_3) = p_1 p_2 + p_1 p_3$			

Table 2: Axioms of ACP.

SC1	$(p_1 \parallel p_2) \parallel p_3 = p_1 \parallel (p_2 \parallel p_3)$	SC4	$(p_1 p_2) p_3 = p_1 p_2 p_3$
SC2	$p \parallel \delta = p \cdot \delta$	SC5	$p_1 (p_2 \parallel p_3) = (p_1 p_2) \parallel p_3$
SC3	$p_1 p_2 = p_2 p_1$		

Table 3: Axioms SC of standard concurrency.

HS	$p_1 p_2 p_3 = \delta$
----	----------------------------

Table 4: HS, Handshaking

TI1	$\tau(I, a) = a$	if $\text{label}(a) \notin I$	TI3	$\tau(p_1 + p_2) = \tau(p_1) + \tau(p_2)$
TI2	$\tau(I, a) = \tau$	if $\text{label}(a) \in I$	TI4	$\tau(p_1 \cdot p_2) = \tau(p_1) \cdot \tau(p_2)$

Table 5: Axioms TI for abstraction

Here, p, p_1, p_2, p_3 range over processes, a, a_1, a_2 over atomic actions, n_1, n_2, n_3 over labels of atomic actions, $t_1, t_i, t_m, t'_1, t'_i, t'_m$, over data terms and H and I over finite sets of labels of atomic actions.

SUM1	$\Sigma(d:D,p) = p$	if d not free in p
SUM2	$\Sigma(d_1:D,p) = \Sigma(d_2:D,p[d_2/d_1])$	if d_2 not free in $\Sigma(d_1:D,p)$ and free for d_1 in p
SUM3	$\Sigma(d:D,p) = \Sigma(d:D,p) + p$	
SUM4	$\Sigma(d:D,p_1 + p_2) = \Sigma(d:D,p_1) + \Sigma(d:D,p_2)$	
SUM5	$\Sigma(d:D,p_1 p_2) = \Sigma(d:D,p_1), p_2$	if d not free in p_2
SUM6	$\Sigma(d:D,p_1 \parallel p_2) = \Sigma(d:D,p_1) \parallel p_2$	if d not free in p_2
SUM7	$\Sigma(d:D,p_1 p_2) = \Sigma(d:D,p_1) p_2$	if d not free in p_2
SUM8	$\Sigma(d:D,\partial(H,p)) = \partial(H,\Sigma(d:D,p))$	
SUM9	$\Sigma(d:D,\tau(I,p)) = \tau(I,\Sigma(d:D,p))$	
	\vdots	
	$\frac{p_1 = p_2}{\Sigma(d:D,p_1) = \Sigma(d:D,p_2)}$	SUM11 if d not free in the assumptions the premise depends on

Table 6: Axioms SUM for the sum operator.

Here, p, p_1, p_2 range over processes, d, d_1, d_2 over data variables and H and I over finite sets of labels of atomic actions.

Parameterization of references to derivations

If the applied rule depends on certain premises, the references to the derivations of these premises are added as parameters.

For instance, consider the rule $\rightarrow E$ (see table 1). Let ψ have the form $t = u$. There are two premises. If $[r_1]$ and $[r_2]$ are the references to these premises (in the order in which they occur in the rule as stated in table 1), then we write

$$\begin{array}{l} \vdots \\ p_i = [\rightarrow E [r_1] [r_2]] \\ p_{i+1} \\ \vdots \end{array}$$

to rewrite p_i to p_{i+1} .

Another kind of parameterization is used if the name of the applied rule does not provide for all the necessary information. For instance, the rule CF2' (see table 2) is referred to by $[CF2'(i)]$. In fact, the rule REFL is similarly parameterized by the *position* of the subformula to be replaced. The role of this parameter is fulfilled by underlining the redexes.

If the rule REFL is parameterized by the term in its consequence, π denotes the position to the left of the $=$ sign and $[r]$ refers to a proof of $t = u$, then $[REPL(\pi) [REFL(t)] [r]]$ represents a proof of $u = t$.

If, in a proof, a formula is not obtained by rewriting, then the reference to the applied rule is placed to the right of the formula.

Assumptions

There are two rules containing an assumption in their premise: $\rightarrow I$ (\rightarrow introduction') and RAA (reductio ad absurdum').

These rules

$$\frac{\begin{array}{c} [\varphi] \\ \vdots \\ \psi \end{array}}{\varphi \rightarrow \psi} \rightarrow I \qquad \frac{\begin{array}{c} [\neg\varphi] \\ \vdots \\ \perp \end{array}}{\varphi} \text{RAA}$$

are represented by

$$\begin{array}{c} [name_1] \\ \llbracket [name_2] \\ \varphi \\ \triangleright \\ \vdots \\ \psi \\ \rrbracket [\rightarrow I] \\ \varphi \rightarrow \psi; \end{array} \qquad \begin{array}{c} [name_1] \\ \llbracket [name_2] \\ \neg\varphi \\ \triangleright \\ \vdots \\ \perp \\ \rrbracket [RAA] \\ \varphi; \end{array}$$

respectively, where the semicolon before the \rrbracket is omitted. The name of the last subderivation before the \rrbracket may also be omitted (because the last subderivation will never be referred to). Notation is taken from Dijkstra and Feijen [5]. The bracket pair \llbracket and \rrbracket delimits the scope of the assumption (here φ and $\neg\varphi$ respectively). The symbol \triangleright separates the hypotheses from the conclusions that may be drawn from it. Between \triangleright and \rrbracket the preceding assumptions are called 'open' (uncancelled) and are referred to by $[name_2]$. Above proof constructions may be nested. The nesting is done in reversed order to the discharge (cancelling) of assumptions.

Equalities obtained from the specification

To be able to use the equalities declared in the specification, we have the rule FACT for data equalities and REC for process equalities: If $t = u$ (is an instance of a declaration that) occurs in a **rew** section of the specification we have the axiom

$$\frac{}{t = u} \text{FACT} .$$

Here, by an *instance* we mean a substitution of terms for the free variables (that is, the variables occurring in the preceding **var** section) in the declaration. The free variables in the resulting equality must be asserted prior to the proof and with the proper sorts.

If $n(x_1 : S_k, \dots, x_k : S_k) = p$ (is an instance of a declaration that) occurs in a **proc** section of the specification we have the axiom

$$\frac{}{n(x_1, \dots, x_k) = p} \text{REC}$$

Here, by an instance we mean a substitution of terms for the formal parameters in the declaration. If [FACT] or [REC] occur in a rewrite sequence as references to the above rules, they should be parameterized by the declaration. However, since it will always be clear which declaration is used, the parameter is omitted. If FACT or REC are used as parameters themselves, for example in the rule $\rightarrow E$, the declarations must be included.

Example

As an example, consider the following (part of a) specification:

```

sort Bool
func T, F      : → Bool

sort Bit
func 0, 1      : → Bit
      zero       : Bit → Bool
rew zero(0)    = T
      zero(1)    = F

```

Furthermore, the following two axioms hold:

$\frac{}{\neg(T = F)} \text{ B1}$	
$\frac{}{\neg(b = T) \rightarrow b = F} \text{ B2}$	where b is an expression of sort Bool

Tabel 7: Axioms BOOL for Booleans

B1 is a very important axiom since it provides for the only means to prove two objects of the same sort unequal: For two syntactically distinguishable objects, say a and b , it is always possible to define a function f such that $f(a) = T$ and $f(b) = F$. Now suppose $a = b$. Then $f(a) = f(b)$. Hence $T = F$, leading to a contradiction with B1.

Suppose we want to prove $\neg(1 = 0)$. Recall that $\neg\varphi$ abbreviates $\varphi \rightarrow \perp$. The following formal derivation does the job (by the method outlined above):

```

[neq01]
[ [ass]
  1 = 0
▷
  [eqTF]
  I = [FACT]
  zero(0) = [ass]
  zero(1) = [FACT]
  F;

  ⊥ [→E [eqTF] [B1]]
] [→]
¬(1 = 0);

```

representing

$$A: \frac{\frac{}{\text{zero}(0) = \text{zero}(0)} \text{ REFL} \quad \frac{}{\text{zero}(0) = T} \text{ FACT}}{T = \text{zero}(0)} \text{ REPL}$$

$$B: \frac{\frac{}{1 = 1} \text{ REFL} \quad 1 = 0 \text{ }^{ass}}{0 = 1} \text{ REPL}$$

$$C: \frac{\frac{\frac{}{T = \text{zero}(0)} A \quad \frac{1 = 0 \text{ }^{ass}}{0 = 1} B}{T = \text{zero}(1)} \text{ REPL} \quad \frac{}{\text{zero}(1) = F} \text{ FACT}}{T = F} \text{ REPL}$$

$$\frac{\frac{1 = 0 \text{ }^{ass}}{T = F} C \quad \frac{}{\neg(T = F)} B1}{\frac{\perp}{\neg(1 = 0)}} \rightarrow E \quad \rightarrow I \text{ }^{ass} \quad \square$$

Note: the letters A , B and C only serve as abbreviations for the corresponding (sub)trees; They do not have any formal meaning.

RSP

The recursive specification principle (RSP, formulated in [3], see table 8) says that a guarded recursive specification has at most one solution. Thus, two solutions to a guarded system of process equations are considered equal. We take the definition of guardedness from [1] with the obvious extension to include summations (δ and τ do not count as guards). RSP, used as a reference in a proof, is parameterized as follows: $[\text{RSP}(G, \sigma_1, \sigma_2) R_1 R_2]$, where $G = \{G_1, \dots, G_m\}$ is the guarded system of process equations over E , V_d , V_p (the initial specification and variable declarations), $\sigma_1 =$

$[\lambda \bar{x}_j. p_j(\bar{x}_j)/n_j]_{j=1}^m$, $\sigma_2 = [\lambda \bar{x}_j. q_j(\bar{x}_j)/n_j]_{j=1}^m$ as described in [7] and R_1 and R_2 are sequences of references to the proofs of $G_1\sigma_1, \dots, G_m\sigma_1$ and $G_1\sigma_2, \dots, G_m\sigma_2$ respectively.

For $1 \leq k \leq m$

$$\frac{\begin{array}{cccc} \vdots & & \vdots & \\ G_1[\lambda \bar{x}_j. p_j/n_j]_{j=1}^m & \cdots & G_m[\lambda \bar{x}_j. p_j/n_j]_{j=1}^m & \\ \vdots & & \vdots & \\ G_1[\lambda \bar{x}_j. q_j/n_j]_{j=1}^m & \cdots & G_m[\lambda \bar{x}_j. q_j/n_j]_{j=1}^m & \end{array} \text{ RSP}}{p_k = q_k}$$

where

- G_1, \dots, G_m is a *guarded* system of process-equations over E , V_d , V_p ;
- For $1 \leq i \leq m$ the p_i and q_i are process terms (possibly containing variables from \bar{x}_j);
- The notation $[\dots]_{j=1}^m$ abbreviates the m given simultaneous substitutions

Table 8: RSP, Recursive specification principle.

CFAR

A finite conservative cluster is a finite set of processes, mutually accessible by performing some 'internal' steps. The Cluster Fair Abstraction rule (CFAR, first formulated in [11], see table 9) expresses that a finite conservative cluster will eventually be exited by performing a step to a process outside the cluster (provided at least one of the processes in the cluster can do such a step). It does so by abstracting from the internal actions. CFAR, used as a reference in a proof, is parameterized as follows: [CFAR(G, σ) R], where $G = \{G_1, \dots, G_m\}$ is the guarded system of process equations over E, V_d, V_p (the initial specification and variable declarations), σ is a substitution for the (fresh) names of process variables in G such that $G_1\sigma, \dots, G_m\sigma$ are provable and R is a sequence of references to the proofs of $G_1\sigma, \dots, G_m\sigma$. Note: the sum symbol in table 9 is *not* the sum operator. It just serves as an abbreviation for a (finitely) repeated +.

For $1 \leq k \leq m$

$$\frac{\begin{array}{c} \vdots \\ G_1[\lambda \bar{x}_j p_j / n_j]_{j=1}^m \quad \dots \quad G_m[\lambda \bar{x}_j p_j / n_j]_{j=1}^m \\ \vdots \end{array}}{\tau \cdot \tau_i(X_k[\lambda \bar{x}_k p_k / n_k]) = \tau \cdot \sum_{j=1}^m \tau_j(q_j)} \text{ CFAR}$$

where

- G_1, \dots, G_m ($m \geq 1$) is a guarded system of process equations over E, V_d, V_p of the form

$$G_i \equiv X_i = \sum_{j=1}^m a_{ij} X_j + q_i \quad (1 \leq i \leq m)$$

and for $1 \leq i, j \leq m$

- X_i has the form $n_i(x_{i1}, \dots, x_{ik_i})$ or n_i with n_i a process variable name and x_{i1}, \dots, x_{ik_i} data variables from V_d ;
- a_{ij} is a atomic action (possibly containing variables from \bar{x}_j). If $a_{ij} \equiv \delta$, then it may be omitted;
- p_i and q_i are process terms (possibly containing variables from \bar{x}_j). If $q_i \equiv \delta$, then it may be omitted

complying with the following conditions:

- No q_i (exit) contains any n_j ;
- All exits must be accessible from every X_i . That is, the graph consisting of m vertices v_1, \dots, v_m and directed edges from v_i to v_j iff X_j occurs in the right-hand side of G_i must be strongly connected;
- I is a set of action labels such that $\{label(a_{ij}) \mid 1 \leq i, j \leq m\} \subseteq I \cup \{\tau\}$.

Table 9: CFAR, Cluster Fair Abstraction Rule.

General lemmas and theorems

To keep derivations short and to avoid duplications as much as possible, we prove some general elementary lemmas and theorems. In all derivations w , x , y and z are process variables. The first two lemmas are very simple. However, they are applied a lot and with each application they save one derivation step. The lemmas are also included in the appendix as part of the ABP proof.

$$[\text{GEN1}] \quad \delta + x = x$$

proof:

$$[\text{GEN1}]$$

$$\underline{\delta + x} = [\text{A1}]$$

$$\underline{x + \delta} = [\text{A6}]$$

x ;

$$[\text{GEN2}] \quad \delta \parallel x = \delta$$

proof:

$$[\text{GEN2}]$$

$$\underline{\delta \parallel x} = [\text{CM2}]$$

$$\underline{\delta . x} = [\text{A7}]$$

δ ;

The next two lemmas, commutativity and associativity of the merge operator, depend on the axioms for standard concurrency, listed in table 3. Note that the merge as well as the communication merge associate to the right.

$$[\text{GEN3}] \quad x \mid y = y \mid x,$$

proof:

$$[\text{GEN3}]$$

$$\underline{x \mid y} = [\text{CM1}]$$

$$\underline{x \mid y + y \mid x + x \mid y} = [\text{A1}]$$

$$\underline{y \mid x + x \mid y + x \mid y} = [\text{SC3}]$$

$$\underline{y \mid x + x \mid y + y \mid x} = [\text{CM1}]$$

$y \mid x$;

$$[\text{GEN4}] \quad (x \mid y) \mid z = x \mid y \mid z$$

proof:

$$[\text{GEN4}]$$

$$\underline{(x \mid y) \mid z} = [\text{CM1}]$$

$$\underline{(x \mid y) \mid z + z \mid (x \mid y) + (x \mid y) \mid z} = [\text{CM1}]$$

$$\underline{(x \mid y + y \mid x + x \mid y) \mid z + z \mid (x \mid y) + (x \mid y) \mid z} = [\text{CM4}]$$

$$\underline{(x \mid y + y \mid x) \mid z + (x \mid y) \mid z + z \mid (x \mid y) + (x \mid y) \mid z} = [\text{CM4}]$$

$$\underline{(x \mid y) \mid z + (y \mid x) \mid z + (x \mid y) \mid z + z \mid (x \mid y) + (x \mid y) \mid z} = [\text{SC1}]$$

$$\underline{x \mid (y \mid z) + (y \mid x) \mid z + (x \mid y) \mid z + z \mid (x \mid y) + (x \mid y) \mid z} = [\text{SC1}]$$

$$\underline{x \mid (y \mid z) + y \mid (x \mid z) + (x \mid y) \mid z + z \mid (x \mid y) + (x \mid y) \mid z} = [\text{GEN3}]$$

$$\underline{x \mid (y \mid z) + y \mid (z \mid x) + (x \mid y) \mid z + z \mid (x \mid y) + (x \mid y) \mid z} = [\text{SC1}]$$

$$\underline{x \mid (y \mid z) + (y \mid z) \mid x + (x \mid y) \mid z + z \mid (x \mid y) + (x \mid y) \mid z} = [\text{SC5}]$$

$$\begin{aligned}
& x|(y|z) + y|(x|z) + z|(y|x) + (y|z)|x + x|(y|z) = [\text{SC1}] \\
& x|(y|z) + y|(x|z) + z|(y|x) + (y|z)|x + x|(y|z) = [\text{GEN3}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + x|(y|z) = [\text{CM1}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + x|(y|z) + z|(y + y|z) = [\text{CM9}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + (x|(y|z) + z|(y + y|z) + x|y|z) = [\text{CM9}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + (x|(y|z) + x|(z|y) + x|y|z) = [\text{SC5}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + ((x|y)|z + x|(z|y) + x|y|z) = [\text{SC5}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + ((x|y)|z + (x|z)|y + x|y|z) = [\text{HS}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + ((x|y)|z + (x|z)|y + \delta) = [\text{A6}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + ((x|y)|z + (x|z)|y) = [\text{A2}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + ((y|z)|x + ((x|y)|z + (x|z)|y)) = [\text{A1}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + ((x|y)|z + (x|z)|y + (y|z)|x) = [\text{A2}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + ((x|y)|z + (x|z)|y) + (y|z)|x = [\text{A2}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (x|y)|z + (x|z)|y + (y|z)|x;
\end{aligned}$$

Alternating Bit Protocol

The alternating bit protocol is a communication protocol concerning data transmission through an unreliable channel so that no information will get lost. It was first described in [2]. In the context of ACP the protocol was verified for the first time in [3]. Our specification and verification of the ABP are based on the ones given in [1].

The alternating bit protocol describes the behaviour of a sender S , a receiver R , a channel K from S to R and a channel L from R to S (see fig. 1).

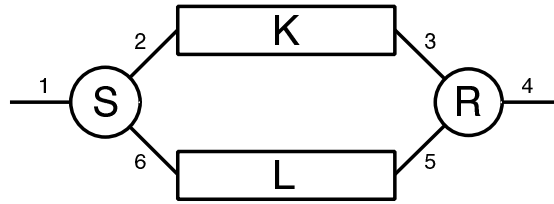


Figure 1: ABP configuration

The sender S waits for a data item d at port 1 and passes it on together with a bit n (initially 0) to port 2. Then it waits for either a bit or an error (supposed to be recognizable) at port 6. If it detects an error or a bit unequal to n at port 6, it again puts the combination dn at port 2 and waits for input at port 6, and so on, until it detects a bit equal to n . If so, S starts all over, waiting for data at port 1. Only now, instead of n , appending $(1 - n)$ to the data it gets from port 1.

Channel K waits for data and a bit at port 2, either relays this combination to port 3 or corrupts the information and puts an error at port 3. Then, in both cases, repeats the procedure, waiting for a bit at port 2.

The receiver R waits for either a data/bit combination dn or an error at port 3. If it detects an error or a data-bit combination of which the bit part is equal to m (initially 1) at port 3, it passes on the bit part to port 5 and again waits for input at port 3. This process is repeated until it detects a data/bit combination of which the bit part is equal to $1 - m$. If this is the case, it successively passes on the data part d to port 4 and the bit part $1 - m$ to port 5 and starts all over. Only now, instead of $1 - m$,

it expects m to be appended to the data it gets from port 3.

Finally, channel L waits for a bit at port 5, either relays this combination to port 6 or corrupts the information and puts an error at port 6. Then, in both cases, repeats the procedure, waiting for a bit at port 5.

Formal specification

The following specification of the alternating bit protocol is almost identical to the ones given in [1] and [7]. One difference between our approach and the one in [7] is that we do not have an explicit element `e' of sort `error'. The reading and writing of errors are represented by omitting parameters to the respective read and write actions. The advantage is that in (attempted) communication involving an error and a bit, we just apply CF2", while, if we had an explicit error element, no rule could be applied, since there is no rule that says what to do with parameters of different sorts. Moreover, we cannot prove two elements of different sort to be unequal (to be able to apply CF2"). Note that adding sort-inequality as an alternative to the proviso of CF2" would create ambiguities because in μ CRL overloading is allowed, making it possible to apply both CF1' and CF2", with different results (see also the outline of the proof). Of course there are other ways to solve this problem (for instance by adding a `third bit') but our solution is simple and introduces no extra functions and such.

Another difference is the presence of the function zero to connect the inequality of 0 and 1 to the given inequality of T and F (B1).

```

sort      Bool
func
  T, F      :  $\rightarrow$  Bool

sort      D

sort      Bit
func
  0, 1      :  $\rightarrow$  Bit
  invert    : Bit  $\rightarrow$  Bit
  zero      : Bit  $\rightarrow$  Bool

rew
  invert(0) = 1
  invert(1) = 0
  zero(0)   = T
  zero(1)   = F

act
  r1, s4     : D
  s2, r2, c2 : D  $\times$  Bit
  s3, r3, c3
  s3, r3, c3 : D  $\times$  Bit
  s5, r5, c5 : Bit
  s6, r6, c6
  s6, r6, c6 : Bit

comm
  r2|s2 = c2
  r3|s3 = c3
  r5|s5 = c5
  r6|s6 = c6

proc
  S          = S(0).S(1).S

```

$S(n:\text{bit})$	$= \sum(d:D, r1(d)).S(n,d)$
$S(d:D, n:\text{bit})$	$= s2(d,n).((r6(\text{invert}(n)) + r6).S(d,n) + r6(n))$
R	$= R(1).R(0).R$
$R(n:\text{bit})$	$= (\sum(d:D, r3(d,n)) + r3).s5(n).R(n) + \sum(d:D, r3(d, \text{invert}(n))).s4(d)).s5(\text{invert}(n))$
K	$= \sum(d:D, \sum(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3))).K$
L	$= \sum(n:\text{bit}, r5(n).(i.s6(n) + i.s6)).L$
ABP	$= \tau(\{c2, c3, c5, c6, i\}, \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S R K L))$

Formal specification of the ABP protocol

Correctness criterion

The following set of auxiliary equations is considered to be an *extension* of the above formal specification rather than a set of *macros* (as in [1]). This is done because there are no rules for handling macros. As a consequence, within the proof they can be treated as formal expressions. However, they can nevertheless be viewed as abbreviations and could be removed by substituting their right-hand side for their left-hand side in the proof and omitting the corresponding applications of [REC].

X	$= \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S R K L)$
$X1(d:D)$	$= \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S(d,0).S(1).S R K L)$
$X2(d:D)$	$= \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, ((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S s5(0).R(0).R K L)$
Y	$= \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S(1).S R(0).R K L)$
$Y1(d:D)$	$= \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S(d,1).S R(0).R K L)$
$Y2(d:D)$	$= \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, ((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).S s5(1).R K L)$

Extension of the formal specification of the ABP protocol

Together, the formal specification and the above-stated extension form the specification E . To complete the definition of the proof context, we give the set V_d of used data variables and the set V_p of used process variables.

$$V_d = \{ \langle m : \text{bit} \rangle, \langle m1 : \text{bit} \rangle, \langle m2 : \text{bit} \rangle, \langle m3 : \text{bit} \rangle, \langle n : \text{bit} \rangle, \langle n1 : \text{bit} \rangle, \langle n2 : \text{bit} \rangle, \langle p : \text{bit} \rangle, \langle q : \text{bit} \rangle, \langle d : D \rangle, \langle d1 : D \rangle, \langle e : D \rangle, \langle e1 : D \rangle \}$$

$$V_p = \{ x, x1, x2, x3, x4, x5, y, y1, y2, y3, z, z1, z2, z3, z4, z5, z6, w, w1, w2, w3 \}$$

Thus, E, V_d, V_p is the context in the correctness proof for the ABP protocol.

To show that the Alternating Bit Protocol is a *correct* communication protocol, we have to prove that it externally behaves as a one-element buffer, i.e. it satisfies the equation

ACP + SC + HS + TI + SUM + BOOL + RSP + CFAR ⊢ ABP = $\Sigma(d:D, r1(d).s4(d).ABP)$ from E, V_d, V_p .

This is the main objective of this report.

Outline of the proof

Our proof complies with the one given in [1]. To give an idea of the composition of the proof we have the following summary. By a 'normalizing lemma' we mean that all proof steps are performed outermost parallel in the standard direction (the one of the tables, except for some SUM axioms: The Σ symbols, like the + should be worked towards the top level), without expanding recursive process identifiers. Note that all normalizing proofs can easily be automated since no choices are made and the terms are reduced to normal forms modulo commutativity and associativity of the +.

[ABP1] and [ABP2] are general normalizing lemmas for frequently used terms. As an example we present the proof of [ABP1]:

[ABP1]

$((x1 + x2).x3 + x4).x5$ = [A4]

$((x1 + x2).x3).x5 + x4.x5$ = [A5]

$(x1 + x2).x3.x5 + x4.x5$ = [A4]

$x1.x3.x5 + x2.x3.x5 + x4.x5$;

[ABP3] proves that 1 is unequal to 0. First, assume $1 = 0$. Then we have

$T = \text{zero}(0) = \text{zero}(1) = F$

contradicting B1. Therefore, 1 must be unequal to 0. Note that the function zero is used to distinguish between 0 and 1.

[ABP3]

[[1]

1 = 0

▷ [2]

1 = [FACT]

zero(0) = [1]

zero(1) = [FACT]

F;

⊥ [→E [2] [B1]]

] [→]

¬(1 = 0);

[ABP4] ... [ABP8] are normalizing lemmas for some of the main processes. Their proofs consist of one [REC] step followed by some rewriting steps in the standard direction. For example:

[ABP6]

K = [REC]

$$\underline{\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n), (i.s3(d,n) + i.s3)))}.K = [\text{SUM5}]$$

$$\Sigma(d:D, \underline{\Sigma(n:\text{bit}, r2(d,n), (i.s3(d,n) + i.s3))}.K) = [\text{SUM5}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \underline{(r2(d,n), (i.s3(d,n) + i.s3))}.K) = [\text{A5}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n), (i.s3(d,n) + i.s3).K);$$

[ABP9] ... [ABP31] are normalizing lemmas for all occurring encapsulated left merges. They will be applied to the main expansions. Most of them reduce to δ :

[ABP17]

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, \underline{r2(d,m).x \parallel y}) = [\text{CM3}]$$

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, \underline{r2(d,m).(x \parallel y)}) = [\text{D4}]$$

$$\underline{\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, r2(d,m))}. \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, x \parallel y) = [\text{D2}]$$

$$\underline{\delta. \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, x \parallel y)} = [\text{A7}]$$

δ ;

[ABP32] ... [ABP79] are normalizing lemmas for all occurring 'basic' communication merges. Their scheme is about the same in all cases:

[ABP32]

$$\underline{r2(d,m).x \parallel s5(n).y} = [\text{CM7}]$$

$$\underline{(r2(d,m) \parallel s5(n))}.(x \parallel y) = [\text{CF2}]$$

$$\underline{\delta.(x \parallel y)} = [\text{A7}]$$

δ ;

If we had stated our specification as in [7], the following derivation would run into the difficulties mentioned earlier:

[ABP72]

$$\underline{r6(m).x \parallel s6.y} = [\text{CM7}]$$

$$\underline{(r6(m) \parallel s6)}.(x \parallel y) = [\text{CF2}']$$

$$\underline{\delta.(x \parallel y)} = [\text{A7}]$$

δ ;

Instead of $s6$ without parameter, we would have $s6(e)$, where e would be an object of sort *error*. In the second step we would have to reduce $(r6(m) \parallel s6(e)).(x \parallel y)$. And now we are stuck! There is no rule to apply: m and e are of different sorts, so we neither can prove them unequal in order to use $\text{CF2}'$ nor can we apply $\text{CF2}''$.

[ABP80] ... [ABP136] all involve main communication merges. Since these communication merges only occur in the expansions left-merged to another process, these left merges are included.

[ABP80] ... [ABP85], [ABP98] ... [ABP105] and [ABP113] ... [ABP132] are relatively simple. Some reduce to δ .

For instance:

[ABP81]

$$\underline{S(m).x} \parallel K \parallel z = [\text{ABP4}]$$

$$\underline{\Sigma(d:D, r1(d).S(d,m).x)} \parallel K \parallel z = [\text{ABP6}]$$

$$\underline{\Sigma(d:D, r1(d).S(d,m).x) \mid \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3).K))} \parallel z = [\text{SUM7}]$$

$$\underline{\Sigma(d:D, r1(d).S(d,m).x) \mid \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3).K))} \parallel z = [\text{SC3}]$$

$$\underline{\Sigma(d:D, \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3).K)) \mid r1(d).S(d,m).x)} \parallel z = [\text{SUM2}]$$

$$\underline{\Sigma(d:D, \Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1,n).(i.s3(d1,n) + i.s3).K)) \mid r1(d).S(d,m).x)} \parallel z = [\text{SUM7}]$$

$$\underline{\Sigma(d:D, \Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1,n).(i.s3(d1,n) + i.s3).K)) \mid r1(d).S(d,m).x)} \parallel z = [\text{SUM7}]$$

$$\underline{\Sigma(d:D, \Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1,n).(i.s3(d1,n) + i.s3).K) \mid r1(d).S(d,m).x))} \parallel z = [\text{ABP35}]$$

$$\underline{\Sigma(d:D, \Sigma(d1:D, \Sigma(n:\text{bit}, \delta))} \parallel z = [\text{SUM1}]$$

$$\underline{\Sigma(d:D, \Sigma(d1:D, \delta))} \parallel z = [\text{SUM1}]$$

$$\underline{\Sigma(d:D, \delta)} \parallel z = [\text{SUM1}]$$

$$\underline{\delta} \parallel z = [\text{GEN2}]$$

δ ;

[ABP86] ... [ABP92] involve two main communication merges *not* reducing to δ . The proofs contain some trickery. [ABP86] ... [ABP90] do the preparations for [ABP91] and [ABP92], being each others 'duals' (one can be obtained from the other by replacing 0's by 1's and vice versa). We could derive a somewhat more general lemma from [ABP91] and [ABP92] by using induction. However, as explained earlier, we want to avoid induction as much as possible. [ABP93] ... [ABP97] are similar: Again two main communication merges not reducing to δ are handled. [ABP106] ... [ABP112] (involving a double summation) and [ABP133] ... [ABP136] are also elaborate ones, however these are a little more general (no dual cases).

Although these lemmas are all based on the same trick, a general lemma, abstracting from port names and communication, cannot be expressed in the language of μCRL .

As a brief intermezzo we give a proof sketch with explanation of about the simplest case: Let $r, s, c : D$ be atomic actions with $r \mid s = c$, $d : D$ a data variable, p a process expression possibly containing d , $e : D$ a data expression with $e \neq d$ and q a process expression not containing d . We want to prove

$$\underline{\Sigma(d:D, r(d).p) \mid s(e).q} = c(e).(p[e/d] \parallel q).$$

First we prove

$$\underline{\Sigma(d:D, r(d).p) \mid s(e).q} = \underline{\Sigma(d:D, (r(d) \mid s(e)).(p \parallel q))} + c(e).(p[e/d] \parallel q): \quad [*]$$

[*]

$$\underline{\Sigma(d:D, r(d).p) \mid s(e).q} = [\text{SUM7}]$$

$$\underline{\Sigma(d:D, r(d).p \mid s(e).q)} = [\text{CM7}]$$

$$\begin{aligned}
& \Sigma(d:D, (r(d)|s(e)).(p|q)) = [\text{SUM3}] \\
& (\text{Since } d \text{ does not occur free in any open assumption}) \\
& \Sigma(d:D, (r(d)|s(e)).(p|q)) + (r(e)|s(e)).(p[e/d]|q) = [\text{CF1}'] \\
& \Sigma(d:D, (r(d)|s(e)).(p|q)) + c(e).(p[e/d]|q) = [\text{SUM1}] \\
& \Sigma(d:D, (r(d)|s(e)).(p|q)) + \Sigma(d:D, c(e)).(p[e/d]|q) = [\text{SUM4}] \\
& \Sigma(d:D, (r(d)|s(e)).(p|q)) + c(e).(p[e/d]|q);
\end{aligned}$$

Since we work with classical logic, for all d and e either $d = e$ or $\neg(d = e)$. In the first case we have

$$\begin{aligned}
& [\text{lemmaA}] \\
& \llbracket [1] \\
& \quad d = e \\
& \triangleright (r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = [1]^* \\
& \quad (r(e)|s(e)).(p[e/d]|q) + c(e).(p[e/d]|q) = [\text{CF1}'] \\
& \quad c(e).(p[e/d]|q) + c(e).(p[e/d]|q) = [\text{A3}] \\
& \quad c(e).(p[e/d]|q) \\
& \rrbracket [\rightarrow\text{I}] \\
& (d = e) \rightarrow (r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = c(e).(p[e/d]|q);
\end{aligned}$$

The $*$ in the first line indicates that [1] may be applied more than once (once extra for each occurrence of d in p). With $\rightarrow\text{I}$ the assumption [1] is discharged.

In the second case we have

$$\begin{aligned}
& [\text{lemmaB}] \\
& \llbracket [1] \\
& \quad \neg(d = e) \\
& \triangleright (r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = [\rightarrow\text{E} [1] [\text{CF2}']] \\
& \quad \delta.(p|q) + c(e).(p[e/d]|q) = [\text{A7}] \\
& \quad \delta + c(e).(p[e/d]|q) = [\text{GEN1}] \\
& \quad c(e).(p[e/d]|q) \\
& \rrbracket [\rightarrow\text{I}] \\
& \neg(d = e) \rightarrow (r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = c(e).(p[e/d]|q);
\end{aligned}$$

Now suppose

$$\neg((r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = c(e).(p[e/d]|q)). \quad [1]$$

If $d = e$, then, applying $\rightarrow\text{E}$, we have a contradiction because of *lemmaA*. So by $\rightarrow\text{I}$ we have $\neg(d = e)$ (discharging the assumption that $d = e$). Only then, applying $\rightarrow\text{E}$ again, we also have a contradiction, now because of *lemmaB*. Thus, by Reductio ad Absurdum (RAA), discharging the assumption [1], we get

$$(r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = c(e).(p[e/d]|q) \quad [\text{lemmaC}]$$

In a derivation format:

$$\begin{aligned}
& [\text{lemmaC}] \\
& \llbracket [1] \\
& \quad \neg((r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = c(e).(p[e/d]|q)) \\
& \triangleright [2]
\end{aligned}$$

$$\begin{array}{l}
\llbracket [3] \\
d = e \\
\triangleright [4] \\
(r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = [\rightarrow E [3] [lemmaA]] \\
c(e).(p[e/d]|q); \\
\perp [\rightarrow E [4] [1]] \\
\rrbracket [\rightarrow I] \\
\neg(d = e); \\
[5] \\
(r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = [\rightarrow E [2] [lemmaB]] \\
c(e).(p[e/d]|q); \\
\perp [\rightarrow E [5] [1]] \\
\rrbracket [RAA] \\
(r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = c(e).(p[e/d]|q);
\end{array}$$

Because there are no open assumptions left, and consequently d does not occur free in any open assumption, we may apply SUM11 to obtain

$$\begin{array}{l}
\Sigma(d:D, r(d).p) | s(e).q = [*] \\
\Sigma(d:D, (r(d)|s(e)).(p|q) + c(e).(p[e/d]|q)) = [lemmaC] \quad (\text{Here we apply SUM11}) \\
\Sigma(d:D, c(e).(p[e/d]|q)) = [SUM1] \\
c(e).(p[e/d]|q).
\end{array}$$

which concludes our proof sketch. Of course this is not a very constructive proof. However, if, instead of CF2', we had the axiom

$$(t_1 = t_1' \wedge \dots \wedge t_m = t_m') \vee n_1(t_1, \dots, t_m) | n_2(t_1', \dots, t_m') = \delta$$

which, in a classical world, is equivalent to CF2' but in an intuitionistic one slightly stronger, we could derive (*lemmaC*) without using RAA. In this particular case it would state

$$d = e \vee r(d)|s(e) = \delta.$$

It says that, no matter whether $=$ is decidable or not, either $d = e$ or $r(d)|s(e) = \delta$. This is not as strong as $d = e \vee \neg(d = e)$, saying (in intuitionistic logic) that $=$ is decidable (if r and s do not communicate at all, we always have $r(d)|s(e) = \delta$, no matter whether d and e are equal or not). Now, with the following provable constructions

$$\begin{array}{l}
\llbracket d = e \\
\triangleright (r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = c(e).(p[e/d]|q) \\
\rrbracket \\
\llbracket r(d)|s(e) = \delta \\
\triangleright (r(d)|s(e)).(p|q) + c(e).(p[e/d]|q) = c(e).(p[e/d]|q) \\
\rrbracket
\end{array}$$

we apply (intuitionistic) \vee -elimination

$$\frac{\varphi \vee \psi \quad \begin{array}{c} [\varphi] \\ \vdots \\ \chi \end{array} \quad \begin{array}{c} [\psi] \\ \vdots \\ \chi \end{array}}{\chi} \vee E$$

to obtain [lemmaC] constructively.

Another way to look at it is to regard our specification to be incomplete with respect to the sort D. If we would demand the specification to be 'effective' (see [3]), equality of (closed) objects of sort D would be decidable. Then $(d = e) \vee \neg(d = e)$ would be a plausible assumption (to be added as an axiom instead of RAA) and again [lemmaC] could be proved constructively by applying \vee -elimination.

We continue our main proof outline.

As an instance of the method just described, we present the 'easiest' one from the proof:

[ABP133]

[[1]

$n = m$

▷ $(r5(n) \mid s5(m)).((y1.s6(n) + y2).y3;x) + c5(m).((y1.s6(m) + y2).y3;x) = [1]$

$(r5(m) \mid s5(m)).((y1.s6(n) + y2).y3;x) + c5(m).((y1.s6(m) + y2).y3;x) = [CF1']$

$c5(m).((y1.s6(n) + y2).y3;x) + c5(m).((y1.s6(m) + y2).y3;x) = [1]$

$c5(m).((y1.s6(m) + y2).y3;x) + c5(m).((y1.s6(m) + y2).y3;x) = [A3]$

$c5(1).((y1.s6(m) + y2).y3;x)$

] [\rightarrow]

$n = m \rightarrow (r5(n) \mid s5(m)).((y1.s6(n) + y2).y3;x) + c5(m).((y1.s6(m) + y2).y3;x) = c5(m).((y1.s6(m) + y2).y3;x);$

[ABP134]

[[1]

$\neg(n = m)$

$(r5(n) \mid s5(m)).z1 + z2 = [\rightarrow E 1 CF2']$

$\delta.z1 + z2 = [A7]$

$\delta + z2 = [GEN1]$

$z2$

]

$\neg(n = m) \rightarrow (r5(n) \mid s5(m)).z1 + z2 = z2;$

[ABP135]

[[1]

$\neg((r5(n) \mid s5(m)).((y1.s6(n) + y2).y3;x) + c5(m).((y1.s6(m) + y2).y3;x) = c5(m).((y1.s6(m) + y2).y3;x))$

▷ [2]

[[3]

$n = m$

▷ [4]

$(r5(n) \mid s5(m)).((y1.s6(n) + y2).y3;x) + c5(m).((y1.s6(m) + y2).y3;x) = [\rightarrow E [3] [ABP133]]$

$$c5(m).((y1.s6(m) + y2).y3ix);$$

$$\perp [\rightarrow E [4] [1]]$$

$$\text{]} [\rightarrow I]$$

$$\neg(n = m);$$

$$[5]$$

$$\underline{(r5(n) \text{ } |s5(m)).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix)} = [\rightarrow E [2] [ABP134]]$$

$$c5(m).((y1.s6(m) + y2).y3ix);$$

$$\perp [\rightarrow E [5] [1]]$$

$$\text{]} [RAA]$$

$$(r5(n) \text{ } |s5(m)).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = c5(m).((y1.s6(m) + y2).y3ix);$$

$$[ABP136]$$

$$(s5(m).x \text{ } |L) \text{ } |z = [ABP7]$$

$$\underline{(s5(m).x \text{ } | \Sigma(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L))} \text{ } |z = [SC3]$$

$$\underline{(\Sigma(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L) \text{ } |s5(m).x)} \text{ } |z = [SUM7]$$

$$\Sigma(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L \text{ } |s5(m).x) \text{ } |z = [CM7]$$

$$\underline{\Sigma(n:\text{bit}, (r5(n) \text{ } |s5(m)).((i.s6(n) + i.s6).L|x))} \text{ } |z = [SUM3]$$

$$\underline{(\Sigma(n:\text{bit}, (r5(n) \text{ } |s5(m)).((i.s6(n) + i.s6).L|x)) + (r5(m) \text{ } |s5(m)).((i.s6(m) + i.s6).L|x))} \text{ } |z = [CF1']$$

$$\underline{(\Sigma(n:\text{bit}, (r5(n) \text{ } |s5(m)).((i.s6(n) + i.s6).L|x)) + c5(m).((i.s6(m) + i.s6).L|x))} \text{ } |z = [SUM1]$$

$$\underline{(\Sigma(n:\text{bit}, (r5(n) \text{ } |s5(m)).((i.s6(n) + i.s6).L|x)) + \Sigma(n:\text{bit}, c5(m).((i.s6(m) + i.s6).L|x)))} \text{ } |z = [SUM4]$$

$$\underline{\Sigma(n:\text{bit}, (r5(n) \text{ } |s5(m)).((i.s6(n) + i.s6).L|x) + c5(m).((i.s6(m) + i.s6).L|x))} \text{ } |z = [ABP135]$$

$$\underline{\Sigma(n:\text{bit}, c5(m).((i.s6(m) + i.s6).L|x))} \text{ } |z = [SUM1]$$

$$c5(m).((i.s6(m) + i.s6).L|x) \text{ } |z = [GEN3]$$

$$\underline{c5(m).(x|(i.s6(m) + i.s6).L)} \text{ } |z = [CM3]$$

$$c5(m).((x|(i.s6(m) + i.s6).L)|z) = [GEN4]$$

$$c5(m).(x|(i.s6(m) + i.s6).L|z);$$

Next, we come to the expansions of merged processes. This is about the level of the proof in [1]. [ABP137] does the bulk for [ABP138] and [ABP139]

$$X = \Sigma(d:D, r1(d).X1(d))$$

$$Y = \Sigma(d:D, r1(d).Y1(d))$$

Similarly, [ABP140] ... [ABP154] and [ABP157] do the preparations for [ABP155] and [ABP156]

$$X1(d) = c2(d,0).(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).(i.c6(1) + i.c6).X1(d))$$

$$Y1(d) = c2(d,1).(i.c3(d,1).s4(d).Y2(d) + i.c3.c5(0).(i.c6(0) + i.c6).Y1(d))$$

and for [ABP158] and [ABP159]

$$X2(d) = c5(0).(i.c6(0).Y + i.c6.c2(d,0).(i.c3(d,0) + i.c3).X2(d))$$

$$Y2(d) = c5(1).(i.c6(1).X + i.c6.c2(d,1).(i.c3(d,1) + i.c3).Y2(d))$$

Derivations are straightforward.

From these four lemmas, by applying CFAR, we obtain [ABP160] ... [ABP163]

$$\tau.\tau(\{c2,c3,c5,c6,i\},X1(d)) = \tau.s4(d).\tau(\{c2,c3,c5,c6,i\},X2(d))$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},Y1(d)) = \tau.s4(d).\tau(\{c2,c3,c5,c6,i\},Y2(d))$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},X2(d)) = \tau.\tau(\{c2,c3,c5,c6,i\},Y)$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},Y2(d)) = \tau.\tau(\{c2,c3,c5,c6,i\},X)$$

Substituting one for another in the above ten lemmas we get [ABP164] ... [ABP167]

$$\tau(\{c2,c3,c5,c6,i\},X) = \sum(d:D,r1(d).s4(d).\tau(\{c2,c3,c5,c6,i\},Y))$$

$$\tau(\{c2,c3,c5,c6,i\},Y) = \sum(d:D,r1(d).s4(d).\tau(\{c2,c3,c5,c6,i\},X))$$

$$\tau(\{c2,c3,c5,c6,i\},X) = \sum(d:D,r1(d).s4(d).\sum(d:D,r1(d).s4(d).\tau(\{c2,c3,c5,c6,i\},X)))$$

$$\tau(\{c2,c3,c5,c6,i\},Y) = \sum(d:D,r1(d).s4(d).\sum(d:D,r1(d).s4(d).\tau(\{c2,c3,c5,c6,i\},Y)))$$

Of which the latter two are proved equal by applying RSP on them: Both satisfy one process specification. Substituting the result in [ABP164] we finally have [ABP168]

$$ABP = \sum(d:D,r1(d).s4(d).ABP)$$

The full proof is given in the appendix.

Related work

In this section we give a brief explanation of the achievements of M.P.A. Sellink [9] and M. Bezem and J.F. Groote [4] as well a comparison of their work with ours.

[9] presents a representation of process algebra in terms of a variant of the *Calculus of Inductive Constructions* of T. Coquand and G. Huet [10], a powerful higher order typed lambda calculus, implemented in the interactive proof construction program COQ [11], which in turn is written in the typed functional programming language ML.

Via the so-called *Curry-Howard isomorphism* types are used to represent propositions. If and only if a type has an *inhabitant*, that is, if there exists an element of that type, the proposition corresponding to that type is considered true (hence, the empty type corresponds to false). An assumption is made by asserting a variable to represent an inhabitant of the type corresponding to that assumption. A sequence of open assumptions is called a *context*.

Translation of μ CRL axioms and rules to types can be done in various ways. The most direct one of course is to implement each language element (including variables), axiom and rule in the COQ system. This way, on the language level, no use is made of the special skills of the COQ system; on the other hand, we remain on familiar ground: theory and meta theory remain separated.

Yet, this is not the way things are done in [9]. Here, several notions of μ CRL are translated to similar,

though sometimes far more powerful notions in COQ. Language variables (data and process variables) as well as meta variables (from rule schemata) for instance, are both implemented by the variables of the COQ system; implication (\rightarrow I and \rightarrow E) and SUB are taken from the COQ system, where \rightarrow E and SUB are consequences of the same rule (\forall E); falsehood and equality are implemented as *inductive types*, the latter inducing the rule REPL (via $=$ E).

Sorts are viewed as inductive types, automatically providing for an induction mechanism. This is quite different from the original μ CRL where *constructors* and *functions* are not separated. It has the advantage however of being able to extend a specification without influencing the validity of deductions made so far.

Functions on sorts are defined by a general recursion mechanism that forces them to be total, while in μ CRL functions being total is preferable, but not necessary. Furthermore, no overloading of functions is possible, compelling to invent a fresh name for each newly introduced function.

Consequently, both the intermingling of language and meta level and the absence of constraints on the generation of the specification, the strength and scope of the obtained system are not quite clear. Perhaps it is possible to add some ML functions to the COQ system translating textual μ CRL specifications to COQ code.

There certainly exist propositions, unprovable by pure μ CRL, yet provable in the COQ system provided with context rendering the same specification. For instance, in COQ the inequality of the objects of any two-objects sort can simply be proved, while in pure μ CRL a function relating these two objects to \top and F of Bool has to be added to the specification (for \top and F can be proved unequal by the axiom B1).

Another difference between a pure μ CRL proof and a proof by the COQ system is due to the following. In μ CRL, actions are represented by their names (labels), possibly followed by some parameters. The label of an action only serves as a 'syntactical object' to distinguish actions from one another. In COQ, syntactical objects *do* exist as constructors of inductive types. They can neither be inspected however, nor compared. A solution to this problem as proposed in [9] is to represent each atomic $a(d)$ action by a sequence $(\text{ia } D a d)$, where label a is of (inductive type) act and parameter d is of type D . Actions without parameters are provided with a dummy parameter. This way, trivial notions (in μ CRL) such as syntactic equality and syntactic set membership of action labels, as used in rules concerning communication, encapsulation and hiding, can be *proved* in COQ. This produces lengthy deductions however, diverting one's attention from the main proof.

[4] consists of a formal verification of the alternating bit protocol according to the above framework. The outline of the proof in [4] is basically the same as ours. Yet, our specification section is much smaller ([4] uses several auxiliary functions), we have a slightly different fairness rule and our proof is more readable (although this may be a matter of habituation).

The large size of the specification section of [4] is probably because we make extensively use of the rule RAA, while [4] presents a constructive proof, requiring the introduction of equality functions to replace the assumption that action parameters are always either equal or unequal (that is, the assumption that equality on data is decidable). The remaining differences must be consequences of the deviation of the COQ implementation from pure μ CRL as well as personal preferences.

A report on a computer-checked μ CRL verification of Milner's scheduler by H. Korver and J. Springintveld will appear in [8].

Conclusions

Although μ CRL has its limitations, it showed to be a powerful tool with which non trivial protocols can be proven correct. For mechanical provers the length of the proof should not be a serious problem and the formal approach to data an indispensable advantage. The bulk of the work can easily be

automated since it only involves standard-direction rewriting. Even associativity and commutativity are no problem because rules only have to be *matched* - not *unified*. If we had omitted all such trivial rewrite steps the length of the proof would only be a fraction of what it is now: If the equalities in the communication and rewrite sections of the specification are viewed as rewrite rules, 70% of the lemmas in the appendix can be proved by rewriting both sides of the equations to a normal form modulo associativity of $.$, $+$ and $|$ and commutativity of $+$ and $|$ and comparing the results. This percentage may even become greater if derived lemmas can be used as rewrite rules as well.

Perhaps if we could abstract a little more (for instance from atomic actions and from communication) we could state some more general theorems and develop some powerful 'proof tools'. Especially the expansion theorem and the communication lemma mentioned in the proof outline are worthwhile to be presented in a more general setting.

The flaw on communication may be put right by imposing some restrictions on overloading on atomic actions (for instance by forbidding it altogether).

Finally, induction can be made more effective if we distinguish between *constructors* and *functions* with some restrictions on both of them. This approach is well known from some type theories with inductive types and some typed functional languages (such as ML, Haskell). Some adaptations with respect to the semantics may be needed, but this will not raise any serious problems.

For example, instead of

```

sort      Bit
func
  0, 1      : → Bit
  invert    : Bit → Bit
  zero      : Bit → Bool
rew
  invert(0) = 1
  invert(1) = 0
  zero(0)   = T
  zero(1)   = F

```

we could have

```

sort      Bit
cons
  0, 1      : → Bit
func
  invert    : Bit → Bit
  zero      : Bit → Bool
rew
  invert(0) = 1
  invert(1) = 0
  zero(0)   = T
  zero(1)   = F

```

where the parameters of the functions on the left-hand side must be constructor terms (i.e. either variables, constructors without parameters or constructors parameterized by constructor terms). Further conditions must hold to guarantee that function terms rewrite (in a finite number of steps) to constructor terms. The condition on sorts and constructors could be that each sort name and constructor name must be unique and sorts must be well founded in the following sense. Let (*) be the property of a sort that there is at least one constructor of which all parameters are of sorts with property (*). This means that a sort with a constructor without parameters automatically has property (*). Now, *all* sorts of a set of sorts depending only on each other are well founded if they *all* have property (*). (Note that this is different from saying that a sort is well founded if it has property (*).)

Example: In the following specification the sorts *forest* and *tree* are defined by mutual induction. Yet, they are both well founded, because 0, 1 and *nil* have no parameters and therefore *bit* and *forest* have property (*) and consequently, so has *tree*. The specification defines trees with vertices labelled by 0 or 1 and with ordered edges.

```
bit ::= 0 | 1  
forest ::= nil | cons(tree,forest)  
tree ::= vertex(bit,forest)
```

Induction can now be explained in terms of the constructors, independent of the defined functions or of external verifications, such that extending a specification by introducing extra functions does not affect preceding proofs.

Appendix

This appendix contains the full proof of the correctness of the ABP protocol. The keywords **datavar**, **procv** are added to initialize the sets V_q and V_p and the keyword **proof** marks the beginning of the derivation.

sort Bool

func

T, F : \rightarrow Bool

sort D

sort Bit

func

0, 1 : \rightarrow Bit

invert : Bit \rightarrow Bit

zero : Bit \rightarrow Bool

rew

invert(0) = 1

invert(1) = 0

zero(0) = T

zero(1) = F

act

r1, s4 : D

s2, r2, c2 : D \times Bit

s3, r3, c3

s3, r3, c3 : D \times Bit

s5, r5, c5 : Bit

s6, r6, c6

s6, r6, c6 : Bit

comm

r2 | s2 = c2

r3 | s3 = c3

r5 | s5 = c5

r6 | s6 = c6

proc

S = S(0).S(1).S

S(n:bit) = $\sum(d:D, r1(d)).S(n,d)$

S(d:D, n:bit) = s2(d, n).((r6(invert(n)) + r6).S(d, n) + r6(n))

R = R(1).R(0).R

R(n:bit) = ($\sum(d:D, r3(d, n)) + r3$).s5(n).R(n) + $\sum(d:D, r3(d, invert(n))).s4(d)$).s5(invert(n))

K = $\sum(d:D, \sum(n:bit, r2(d, n).(i.s3(d, n) + i.s3))$).K

L = $\sum(n:bit, r5(n).(i.s6(n) + i.s6))$.L

ABP = $\tau(\{c2, c3, c5, c6, i\}, \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S; R; K; L))$

X = $\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S; R; K; L)$

X1(d:D) = $\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S(d, 0).S(1), S; R; K; L)$

$$X2(d:D) = \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, ((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).Sis5(0).R(0).RiKiL)$$

$$Y = \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, S(1).SiR(0).RiKiL)$$

$$Y1(d:D) = \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, S(d,1).SiR(0).RiKiL)$$

$$Y2(d:D) = \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, ((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).Sis5(1).RiKiL)$$

datavar

m, m1, m2, m3, n, n1, n2, p, q : bit;

d, d1, e, e1 : D

procvar

x, x1, x2, x3, x4, x5, y, y1, y2, y3, z, z1, z2, z3, z4, z5, z6, w, w1, w2, w3

proof

[GEN1]

$$\underline{\delta} + x = [A1]$$

$$x + \underline{\delta} = [A6]$$

x;

[GEN2]

$$\underline{\delta} \underline{x} = [CM2]$$

$$\underline{\delta} \underline{x} = [A7]$$

δ ;

[GEN3]

$$x \underline{y} = [CM1]$$

$$\underline{x} \underline{y} + \underline{y} \underline{x} + x \underline{y} = [A1]$$

$$y \underline{x} + x \underline{y} + x \underline{y} = [SC3]$$

$$\underline{y} \underline{x} + x \underline{y} + y \underline{x} = [CM1]$$

y;x;

[GEN4]

$$\underline{(x \underline{y})} \underline{z} = [CM1]$$

$$\underline{(x \underline{y})} \underline{z} + \underline{z} \underline{(x \underline{y})} + (x \underline{y}) \underline{z} = [CM1]$$

$$\underline{(x \underline{y} + \underline{y} \underline{x} + x \underline{y})} \underline{z} + \underline{z} \underline{(x \underline{y})} + (x \underline{y}) \underline{z} = [CM4]$$

$$\underline{(x \underline{y} + \underline{y} \underline{x})} \underline{z} + (x \underline{y}) \underline{z} + \underline{z} \underline{(x \underline{y})} + (x \underline{y}) \underline{z} = [CM4]$$

$$\underline{(x \underline{y})} \underline{z} + \underline{(y \underline{x})} \underline{z} + (x \underline{y}) \underline{z} + \underline{z} \underline{(x \underline{y})} + (x \underline{y}) \underline{z} = [SC1]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{x})} \underline{z} + (x \underline{y}) \underline{z} + \underline{z} \underline{(x \underline{y})} + (x \underline{y}) \underline{z} = [SC1]$$

$$x \underline{(y \underline{z})} + \underline{y} \underline{(x \underline{z})} + (x \underline{y}) \underline{z} + \underline{z} \underline{(x \underline{y})} + (x \underline{y}) \underline{z} = [GEN3]$$

$$x \underline{(y \underline{z})} + \underline{y} \underline{(z \underline{x})} + (x \underline{y}) \underline{z} + \underline{z} \underline{(x \underline{y})} + (x \underline{y}) \underline{z} = [SC1]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + \underline{(x \underline{y})} \underline{z} + \underline{z} \underline{(x \underline{y})} + (x \underline{y}) \underline{z} = [SC5]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{z} \underline{(x \underline{y})} + (x \underline{y}) \underline{z} = [GEN3]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{z} \underline{(y \underline{x})} + (x \underline{y}) \underline{z} = [SC1]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{(z \underline{y})} \underline{x} + \underline{(x \underline{y})} \underline{z} = [CM1]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{(z \underline{y})} \underline{x} + \underline{(x \underline{y} + \underline{y} \underline{x} + x \underline{y})} \underline{z} = [CM8]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{(z \underline{y})} \underline{x} + \underline{(x \underline{y} + \underline{y} \underline{x})} \underline{z} + (x \underline{y}) \underline{z} = [CM8]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{(z \underline{y})} \underline{x} + \underline{(x \underline{y})} \underline{z} + \underline{(y \underline{x})} \underline{z} + (x \underline{y}) \underline{z} = [SC3]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{(z \underline{y})} \underline{x} + \underline{(z \underline{(x \underline{y})})} \underline{z} + \underline{(y \underline{x})} \underline{z} + (x \underline{y}) \underline{z} = [SC5]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{(z \underline{y})} \underline{x} + \underline{(z \underline{x})} \underline{y} + \underline{(y \underline{x})} \underline{z} + (x \underline{y}) \underline{z} = [SC3]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{(z \underline{y})} \underline{x} + \underline{(x \underline{z})} \underline{y} + \underline{(y \underline{x})} \underline{z} + (x \underline{y}) \underline{z} = [SC5]$$

$$x \underline{(y \underline{z})} + \underline{(y \underline{z})} \underline{x} + x \underline{(y \underline{z})} + \underline{(z \underline{y})} \underline{x} + \underline{(x \underline{z})} \underline{y} + \underline{(y \underline{x})} \underline{z} + (x \underline{y}) \underline{z} = [SC3]$$

$$\begin{aligned}
& x|(y|z) + (y|z)|x + x|(y|z) + (z|y)|x + (x|(z|y) + z|(y|x) + (x|y)|z) = [\text{SC5}] \\
& x|(y|z) + (y|z)|x + x|(y|z) + (z|y)|x + (x|(z|y) + (z|y)|x + (x|y)|z) = [\text{SC3}] \\
& x|(y|z) + (y|z)|x + x|(y|z) + (z|y)|x + (x|(z|y) + (y|z)|x + (x|y)|z) = [\text{SC4}] \\
& x|(y|z) + (y|z)|x + x|(y|z) + (z|y)|x + (x|(z|y) + (y|z)|x + x|y|z) = [\text{A1}] \\
& x|(y|z) + (y|z)|x + x|(y|z) + (z|y)|x + ((y|z)|x + x|(z|y) + x|y|z) = [\text{A2}] \\
& x|(y|z) + (y|z)|x + x|(y|z) + (z|y)|x + ((y|z)|x + x|(z|y)) + x|y|z = [\text{A2}] \\
& x|(y|z) + (y|z)|x + x|(y|z) + (z|y)|x + (y|z)|x + x|(z|y) + x|y|z = [\text{A2}] \\
& x|(y|z) + (y|z)|x + x|(y|z) + ((z|y)|x + (y|z)|x) + x|(z|y) + x|y|z = [\text{A2}] \\
& x|(y|z) + (y|z)|x + (x|(y|z) + ((z|y)|x + (y|z)|x)) + x|(z|y) + x|y|z = [\text{A1}] \\
& x|(y|z) + (y|z)|x + ((z|y)|x + (y|z)|x + x|(y|z)) + x|(z|y) + x|y|z = [\text{A2}] \\
& x|(y|z) + (y|z)|x + ((z|y)|x + (y|z)|x) + x|(y|z) + x|(z|y) + x|y|z = [\text{A2}] \\
& x|(y|z) + ((y|z)|x + ((z|y)|x + (y|z)|x)) + x|(y|z) + x|(z|y) + x|y|z = [\text{A2}] \\
& x|(y|z) + ((y|z)|x + (z|y)|x + (y|z)|x) + x|(y|z) + x|(z|y) + x|y|z = [\text{CM4}] \\
& x|(y|z) + ((y|z) + z|y + y|z)|x + x|(y|z) + x|(z|y) + x|y|z = [\text{CM4}] \\
& x|(y|z) + (y|z + z|y + y|z)|x + x|(y|z) + x|(z|y) + x|y|z = [\text{CM1}] \\
& x|(y|z) + (y|z)|x + x|(y|z) + x|(z|y) + x|y|z = [\text{A2}] \\
& x|(y|z) + (y|z)|x + (x|(y|z) + x|(z|y)) + x|y|z = [\text{A2}] \\
& x|(y|z) + (y|z)|x + (x|(y|z) + x|(z|y) + x|y|z) = [\text{CM9}] \\
& x|(y|z) + (y|z)|x + (x|(y|z + z|y) + x|y|z) = [\text{CM9}] \\
& x|(y|z) + (y|z)|x + x|(y|z + z|y + y|z) = [\text{CM1}] \\
& x|(y|z) + (y|z)|x + x|(y|z) = [\text{CM1}] \\
& x|y|z;
\end{aligned}$$

[EXP3]

$$\begin{aligned}
& x|y|z = [\text{CM1}] \\
& x|(y|z) + (y|z)|x + x|(y|z) = [\text{CM1}] \\
& x|(y|z) + (y|z + z|y + y|z)|x + x|(y|z) = [\text{CM4}] \\
& x|(y|z) + ((y|z + z|y)|x + (y|z)|x) + x|(y|z) = [\text{CM4}] \\
& x|(y|z) + ((y|z)|x + (z|y)|x + (y|z)|x) + x|(y|z) = [\text{A2}] \\
& x|(y|z) + ((y|z)|x + (z|y)|x) + (y|z)|x + x|(y|z) = [\text{A2}] \\
& x|(y|z) + (y|z)|x + (z|y)|x + (y|z)|x + x|(y|z) = [\text{SC1}] \\
& x|(y|z) + y|(z|x) + (z|y)|x + (y|z)|x + x|(y|z) = [\text{GEN3}] \\
& x|(y|z) + y|(x|z) + (z|y)|x + (y|z)|x + x|(y|z) = [\text{SC1}] \\
& x|(y|z) + y|(x|z) + z|(y|x) + (y|z)|x + x|(y|z) = [\text{GEN3}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + x|(y|z) = [\text{CM1}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + x|(y|z + z|y + y|z) = [\text{CM9}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + (x|(y|z + z|y) + x|y|z) = [\text{CM9}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + (x|(y|z) + x|(z|y) + x|y|z) = [\text{SC5}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + ((x|y)|z + x|(z|y) + x|y|z) = [\text{SC5}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + ((x|y)|z + (x|z)|y + x|y|z) = [\text{HS}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + ((x|y)|z + (x|z)|y + \delta) = [\text{A6}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (y|z)|x + ((x|y)|z + (x|z)|y) = [\text{A2}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + ((y|z)|x + ((x|y)|z + (x|z)|y)) = [\text{A1}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + ((x|y)|z + (x|z)|y + (y|z)|x) = [\text{A2}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + ((x|y)|z + (x|z)|y) + (y|z)|x = [\text{A2}] \\
& x|(y|z) + y|(x|z) + z|(x|y) + (x|y)|z + (x|z)|y + (y|z)|x;
\end{aligned}$$

[EXP4]

$$w|x|y|z = [\text{CM1}]$$

$$w|(x|y|z) + (x|y|z)|w + w|(x|y|z) = [\text{EXP3}]$$

$$\begin{aligned}
& w|(x|y|z) + \\
& (x|(y|z) + y|(x|z) + z|(x|y) + (x|y)|z + (x|z)|y + (y|z)|x)|w + \\
& w|(x|y|z) = [\text{CM4}]
\end{aligned}$$

$$(x|y)\ll(z|w) + ((x|z)\ll y)\ll w + ((y|z)\ll x)\ll w + w|(x|y|z) = [\text{GEN3}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + \underline{((x|z)\ll y)\ll w} + ((y|z)\ll x)\ll w + w|(x|y|z) = [\text{SC1}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(y|w) + ((y|z)\ll x)\ll w + w|(x|y|z) = [\text{GEN3}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + \underline{((y|z)\ll x)\ll w} + w|(x|y|z) = [\text{SC1}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(x|w) + w|(x|y|z) = [\text{GEN3}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + w|\underline{(x|y|z)} = [\text{EXP3}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{w|(x|(y|z) + y|(x|z) + z|(x|y) + (x|y)\ll z + (x|z)\ll y + (y|z)\ll x)} = [\text{CM9}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{(w|(x|(y|z) + y|(x|z) + z|(x|y) + (x|y)\ll z + (x|z)\ll y)} + w|((y|z)\ll x) = [\text{CM9}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{(w|(x|(y|z) + y|(x|z) + z|(x|y) + (x|y)\ll z)} + w|((x|z)\ll y) + w|((y|z)\ll x) = [\text{CM9}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{(w|(x|(y|z) + y|(x|z) + z|(x|y))} + w|((x|y)\ll z) + w|((x|z)\ll y) + w|((y|z)\ll x) = [\text{CM9}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{(w|(x|(y|z) + y|(x|z) + z|(x|y))} + w|((x|y)\ll z) + w|((x|z)\ll y) + w|((y|z)\ll x) = [\text{CM9}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{(w|(x|(y|z))} + w|(y|(x|z)) + w|(z|(x|y)) + w|((x|y)\ll z) + w|((x|z)\ll y) + w|((y|z)\ll x) = [\text{SC5}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{((w|x)\ll(y|z))} + \underline{w|(y|(x|z))} + w|(z|(x|y)) + w|((x|y)\ll z) + w|((x|z)\ll y) + w|((y|z)\ll x) = [\text{SC5}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{((w|x)\ll(y|z))} + (w|y)\ll(x|z) + \underline{w|(z|(x|y))} + w|((x|y)\ll z) + w|((x|z)\ll y) + w|((y|z)\ll x) = [\text{SC5}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{((w|x)\ll(y|z))} + (w|y)\ll(x|z) + (w|z)\ll(x|y) + \underline{w|((x|y)\ll z)} + w|((x|z)\ll y) + w|((y|z)\ll x) = [\text{SC5}]$$

$$w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + \\ (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x) + \\ \underline{((w|x)\ll(y|z))} + (w|y)\ll(x|z) + (w|z)\ll(x|y) + \underline{(w|x|y)\ll z} + w|((x|z)\ll y) + w|((y|z)\ll x) = [\text{HS}]$$

$$\frac{((w|x)\ll(y|z) + (w|y)\ll(x|z) + (w|z)\ll(x|y)) + ((x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x))}{((w|x)\ll(y|z) + (w|y)\ll(x|z) + (w|z)\ll(x|y)) + ((x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x))} = [A2]$$

$$\frac{w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + ((w|x)\ll(y|z) + (w|y)\ll(x|z) + (w|z)\ll(x|y)) + ((x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x))}{((w|x)\ll(y|z) + (w|y)\ll(x|z) + (w|z)\ll(x|y)) + ((x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x))} = [A2]$$

$$\frac{w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + ((w|x)\ll(y|z) + (w|y)\ll(x|z)) + (w|z)\ll(x|y) + ((x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x))}{((x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x))} = [A2]$$

$$\frac{w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + (w|x)\ll(y|z) + (w|y)\ll(x|z) + (w|z)\ll(x|y) + ((x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x))}{((x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x))} = [A2]$$

$$\frac{w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + (w|x)\ll(y|z) + (w|y)\ll(x|z) + (w|z)\ll(x|y) + ((x|y)\ll(w|z) + (x|z)\ll(w|y)) + (y|z)\ll(w|x)}{((x|y)\ll(w|z) + (x|z)\ll(w|y)) + (y|z)\ll(w|x)} = [A2]$$

$$\frac{w\ll(x|y|z) + x\ll(w|y|z) + y\ll(w|x|z) + z\ll(w|x|y) + (w|x)\ll(y|z) + (w|y)\ll(x|z) + (w|z)\ll(x|y) + (x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x)}{(x|y)\ll(w|z) + (x|z)\ll(w|y) + (y|z)\ll(w|x)}$$

[ABP1]

$$\frac{(x1 + x2).x3 + x4}.x5 = [A4]$$

$$\frac{(x1 + x2).x3}.x5 + x4.x5 = [A5]$$

$$(x1 + x2).x3.x5 + x4.x5 = [A4]$$

$$x1.x3.x5 + x2.x3.x5 + x4.x5;$$

[ABP2]

$$\frac{x1.x2 + x3.x4}.x5 = [A4]$$

$$(x1.x2).x5 + (x3.x4).x5 = [A5]$$

$$\frac{x1.x2}.x5 + x3.x4.x5 = [A5]$$

$$x1.x2.x5 + x3.x4.x5;$$

[ABP3]

$$\begin{array}{l} [\quad [1] \\ \quad 1 = 0 \end{array}$$

$$\begin{array}{l} \triangleright [2] \\ \quad \underline{1} = [\text{FACT}] \end{array}$$

$$\text{zero}(0) = [1]$$

$$\underline{\text{zero}(1)} = [\text{FACT}]$$

F;

$$\begin{array}{l} \perp [\rightarrow E [2] [B1]] \\] [\rightarrow I] \\ \neg(1 = 0); \end{array}$$

[ABP4]

 $\underline{S(m).x} = [\text{REC}]$ $\underline{\sum(d:D, r1(d).S(d,m)).x} = [\text{SUM5}]$ $\sum(d:D, (r1(d).S(d,m)).x) = [\text{A5}]$ $\sum(d:D, r1(d).S(d,m).x);$

[ABP5]

 $\underline{R(m).x} = [\text{REC}]$ $\underline{((\sum(d:D, r3(d,m)) + r3).s5(m).R(m) + \sum(d:D, r3(d, invert(m)).s4(d)).s5(invert(m))))}.x = [\text{A4}]$ $\underline{((\sum(d:D, r3(d,m)) + r3).s5(m).R(m)).x + (\sum(d:D, r3(d, invert(m)).s4(d)).s5(invert(m))))}.x = [\text{A5}]$ $\underline{((\sum(d:D, r3(d,n)) + r3).s5(m).R(m)).x + \sum(d:D, r3(d, invert(m)).s4(d)).s5(invert(m)).x} = [\text{SUM5}]$ $\underline{((\sum(d:D, r3(d,n)) + r3).s5(m).R(m)).x + \sum(d:D, (r3(d, invert(m)).s4(d)).s5(invert(m)).x} = [\text{A5}]$ $\underline{((\sum(d:D, r3(d,n)) + r3).s5(m).R(m)).x + \sum(d:D, r3(d, invert(m)).s4(d).s5(invert(m)).x} = [\text{A5}]$ $\underline{(\sum(d:D, r3(d,n)) + r3).s5(m).R(m).x + \sum(d:D, r3(d, invert(m)).s4(d).s5(invert(m)).x} = [\text{A5}]$ $\underline{(\sum(d:D, r3(d,n)) + r3).s5(m).R(m).x + \sum(d:D, r3(d, invert(m)).s4(d).s5(invert(m)).x} = [\text{A4}]$ $\underline{\sum(d:D, r3(d,n).s5(m).R(m).x + r3.s5(m).R(m).x + \sum(d:D, r3(d, invert(m)).s4(d).s5(invert(m)).x} = [\text{SUM5}]$ $\sum(d:D, r3(d,n).s5(m).R(m).x + r3.s5(m).R(m).x + \sum(d:D, r3(d, invert(m)).s4(d).s5(invert(m)).x);$

[ABP6]

 $\underline{K} = [\text{REC}]$ $\underline{\sum(d:D, \sum(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3)).K} = [\text{SUM5}]$ $\sum(d:D, \sum(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3)).K) = [\text{SUM5}]$ $\sum(d:D, \sum(n:\text{bit}, (r2(d,n).(i.s3(d,n) + i.s3)).K) = [\text{A5}]$ $\sum(d:D, \sum(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3).K);$

[ABP7]

 $\underline{L} = [\text{REC}]$ $\underline{\sum(n:\text{bit}, r5(n).(i.s6(n) + i.s6)).L} = [\text{SUM5}]$ $\sum(n:\text{bit}, (r5(n).(i.s6(n) + i.s6)).L) = [\text{A5}]$ $\sum(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L);$

[ABP8]

 $\underline{S(d,m).x} = [\text{REC}]$ $\underline{(s2(d,m).((r6(invert(m)) + r6).S(d,m) + r6(m))).x} = [\text{A5}]$ $s2(d,m).((r6(invert(m)) + r6).S(d,m) + r6(m)).x;$

[ABP9]

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_2(d, m)} \cdot \underline{x} \parallel \underline{y}) = [\text{CM3}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_2(d, m)} \cdot (x \parallel y)) = [\text{D4}]$$

$$\underline{\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_2(d, m)})} \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y) = [\text{D2}]$$

$$\underline{\delta \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y)} = [\text{A7}]$$

δ ;

[ABP10]

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_3} \cdot \underline{x} \parallel \underline{y}) = [\text{CM3}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_3} \cdot (x \parallel y)) = [\text{D4}]$$

$$\underline{\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_3})} \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y) = [\text{D2}]$$

$$\underline{\delta \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y)} = [\text{A7}]$$

δ ;

[ABP11]

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_3(d, m)} \cdot \underline{x} \parallel \underline{y}) = [\text{CM3}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_3(d, m)} \cdot (x \parallel y)) = [\text{D4}]$$

$$\underline{\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_3(d, m)})} \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y) = [\text{D2}]$$

$$\underline{\delta \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y)} = [\text{A7}]$$

δ ;

[ABP12]

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_4(d)} \cdot \underline{x} \parallel \underline{y}) = [\text{CM3}]$$

$$\underline{\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_4(d)})} \cdot (x \parallel y) = [\text{D4}]$$

$$\underline{\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_4(d)})} \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y) = [\text{D1}]$$

$$s_4(d) \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y);$$

[ABP13]

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_5(m)} \cdot \underline{x} \parallel \underline{y}) = [\text{CM3}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_5(m)} \cdot (x \parallel y)) = [\text{D4}]$$

$$\underline{\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_5(m)})} \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y) = [\text{D2}]$$

$$\underline{\delta \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x \parallel y)} = [\text{A7}]$$

δ ;

[ABP14]

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \underline{s_6} \cdot \underline{x} \parallel \underline{y}) = [\text{CM3}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},s6.(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},s6).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D2]$$

$$\delta.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [A7]$$

δ ;

[ABP15]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},s6(m).x|y) = [CM3]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},s6(m).(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},s6(m)).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D2]$$

$$\delta.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [A7]$$

δ ;

[ABP16]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r1(d).x|y) = [CM3]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r1(d).(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r1(d)).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D1]$$

$$r1(d).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y);$$

[ABP17]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r2(d,m).x|y) = [CM3]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r2(d,m).(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r2(d,m)).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D2]$$

$$\delta.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [A7]$$

δ ;

[ABP18]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r3.x|y) = [CM3]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r3.(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r3).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D2]$$

$$\delta.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [A7]$$

δ ;

[ABP19]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r3(d,m).x|y) = [CM3]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r3(d,m).(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r3(d,m)).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D2]$$

$$\delta.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [A7]$$

$\delta;$

[ABP20]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r5(m).x|y) = [CM3]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r5(m).(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r5(m)).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D2]$$

$$\delta.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [A7]$$

$\delta;$

[ABP21]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r6.x|y) = [CM3]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r6.(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r6).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D2]$$

$$\delta.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [A7]$$

$\delta;$

[ABP22]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r6(m).x|y) = [CM3]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r6(m).(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r6(m)).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D2]$$

$$\delta.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [A7]$$

$\delta;$

[ABP23]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},i.x|y) = [CM3]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},i.(x|y)) = [D4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},i).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y) = [D1]$$

$$i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x|y);$$

[ABP24]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S(m).x|y) = [ABP4]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},\Sigma(d:D,r1(d).S(d,m).x|y)) = [SUM6]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},\Sigma(d:D,r1(d).S(d,m).x|y)) = [SUM8]$$

$$\Sigma(d:D,\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},r1(d).S(d,m).x|y)) = [ABP16]$$

$$\Sigma(d:D,r1(d)).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S(d,m).x|y);$$

[ABP25]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{R(m).x} \parallel y) = [\text{ABP5}]$$

 $\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \sum(d:D,r3(d,\text{invert}(m)).s4(d).s5(\text{invert}(m)).x)) \parallel y)} = [\text{CM4}]$$

 $\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \parallel y + \sum(d:D,r3(d,\text{invert}(m)).s4(d).s5(\text{invert}(m)).x)) \parallel y)} = [\text{SUM6}]$$

 $\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \parallel y + \sum(d:D,r3(d,\text{invert}(m)).s4(d).s5(\text{invert}(m)).x)) \parallel y)} = [\text{D3}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \parallel y)} + \underline{(\sum(d:D,r3(d,\text{invert}(m)).s4(d).s5(\text{invert}(m)).x)) \parallel y)}) = [\text{SUM8}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \parallel y)} + \underline{\sum(d:D, \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, r3(d,\text{invert}(m)).s4(d).s5(\text{invert}(m)).x)) \parallel y)}) = [\text{ABP19}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \parallel y)} + \underline{\sum(d:D, \delta)}) = [\text{SUM1}]$$

$$\underline{\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \parallel y)} + \delta)} = [\text{A6}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \parallel y}) = [\text{CM4}]$$

$$\underline{\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\sum(d:D, r3(d,m).s5(m).R(m).x) \parallel y} + r3.s5(m).R(m).x) \parallel y)} = [\text{D3}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\sum(d:D, r3(d,m).s5(m).R(m).x) \parallel y)} + \underline{\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, r3.s5(m).R(m).x) \parallel y)}) = [\text{ABP18}]$$

$$\underline{\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\sum(d:D, r3(d,m).s5(m).R(m).x) \parallel y} + \delta)} = [\text{A6}]$$

$$\underline{\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\sum(d:D, r3(d,m).s5(m).R(m).x) \parallel y})} = [\text{SUM8}]$$

$$\underline{\sum(d:D, \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, r3(d,m).s5(m).R(m).x) \parallel y))} = [\text{ABP19}]$$

$$\underline{\sum(d:D, \delta)} = [\text{SUM1}]$$

 $\delta;$

[ABP26]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{K} \parallel y) = [\text{ABP6}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\sum(d:D, \sum(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K)) \parallel y}) = [\text{SUM6}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\sum(d:D, \sum(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K)) \parallel y}) = [\text{SUM6}]$$

$$\underline{\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\sum(d:D, \sum(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K)) \parallel y}))} = [\text{SUM8}]$$

$$\underline{\sum(d:D, \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\sum(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K)) \parallel y}))} = [\text{SUM8}]$$

$$\underline{\sum(d:D, \sum(n:\text{bit}, \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K)) \parallel y))} = [\text{ABP17}]$$

$$\underline{\sum(d:D, \sum(n:\text{bit}, \delta))} = [\text{SUM1}]$$

$$\Sigma(d:D,\delta) = [\text{SUM1}]$$

$\delta;$

[ABP27]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\underline{L}} \parallel y) = [\text{ABP7}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\Sigma(n:\text{bit},r5(n).(i.s6(n) + i.s6).L)} \parallel y) = [\text{SUM6}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{\Sigma(n:\text{bit},r5(n).(i.s6(n) + i.s6).L)} \parallel y) = [\text{SUM8}]$$

$$\Sigma(n:\text{bit}, \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, r5(n).(i.s6(n) + i.s6).L) \parallel y) = [\text{ABP20}]$$

$$\Sigma(n:\text{bit}, \delta) = [\text{SUM1}]$$

$\delta;$

[ABP28]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{S(d,m).x} \parallel y) = [\text{ABP8}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{s2(d,m).(r6(\text{invert}(m)) + r6).S(d,m) + r6(m)}.x) \parallel y) = [\text{ABP9}]$$

$\delta;$

[ABP29]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{(r6(m1) + r6).x1 + r6(m2)}.x2) \parallel y) = [\text{ABP1}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2} \parallel y) = [\text{CM4}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m1).x1.x2 + r6.x1.x2} \parallel y + r6(m2).x2) \parallel y) = [\text{D3}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m1).x1.x2 + r6.x1.x2} \parallel y) +$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m2).x2} \parallel y) = [\text{ABP22}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m1).x1.x2 + r6.x1.x2} \parallel y) + \delta = [\text{A6}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m1).x1.x2 + r6.x1.x2} \parallel y) = [\text{CM4}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m1).x1.x2} \parallel y + r6.x1.x2) \parallel y) = [\text{D3}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m1).x1.x2} \parallel y) + \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6.x1.x2} \parallel y) = [\text{ABP21}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m1).x1.x2} \parallel y) + \delta = [\text{A6}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{r6(m1).x1.x2} \parallel y) = [\text{ABP22}]$$

$\delta;$

[ABP30]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{(i.x1 + i.x2).x3} \parallel y) = [\text{ABP2}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{(i.x1.x3 + i.x2.x3)} \parallel y) = [\text{CM4}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{i.x1.x3} \parallel y + i.x2.x3) \parallel y) = [\text{D3}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{i.x1.x3} \parallel y) + \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, \underline{i.x2.x3} \parallel y) = [\text{ABP23}]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},i,x1.x3\|y) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x2.x3\|y) = [\text{ABP23}]$$

$$i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x1.x3\|y) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},x2.x3\|y);$$

[ABP31]

$$\underline{r2(d,m).x \text{ } |s4(e).y} = [\text{CM7}]$$

$$(\underline{r2(d,m) \text{ } |s4(e)}).(x\|y) = [\text{CF2}]$$

$$\underline{\delta.(x\|y)} = [\text{A7}]$$

 $\delta;$

[ABP32]

$$\underline{r2(d,m).x \text{ } |s5(n).y} = [\text{CM7}]$$

$$(\underline{r2(d,m) \text{ } |s5(n)}).(x\|y) = [\text{CF2}]$$

$$\underline{\delta.(x\|y)} = [\text{A7}]$$

 $\delta;$

[ABP33]

$$\underline{r2(d,m).x \text{ } |s6.y} = [\text{CM7}]$$

$$(\underline{r2(d,m) \text{ } |s6}).(x\|y) = [\text{CF2}]$$

$$\underline{\delta.(x\|y)} = [\text{A7}]$$

 $\delta;$

[ABP34]

$$\underline{r2(d,m).x \text{ } |s6(n).y} = [\text{CM7}]$$

$$(\underline{r2(d,m) \text{ } |s6(n)}).(x\|y) = [\text{CF2}]$$

$$\underline{\delta.(x\|y)} = [\text{A7}]$$

 $\delta;$

[ABP35]

$$\underline{r2(d,m).x \text{ } |r1(e).y} = [\text{CM7}]$$

$$(\underline{r2(d,m) \text{ } |r1(e)}).(x\|y) = [\text{CF2}]$$

$$\underline{\delta.(x\|y)} = [\text{A7}]$$

 $\delta;$

[ABP36]

$$\underline{r2(d,m).x \text{ } |r6.y} = [\text{CM7}]$$

$$(\underline{r2(d,m) \text{ } |r6}).(x\|y) = [\text{CF2}]$$

$$\underline{\delta.(x\|y)} = [\text{A7}]$$

δ ;

[ABP37]

$r2(d,m).x \mid r6(n).y = [CM7]$

$(r2(d,m) \mid r6(n)).(x \mid y) = [CF2]$

$\delta.(x \mid y) = [A7]$

δ ;

[ABP38]

$r2(d,m).x \mid i.y = [CM7]$

$(r2(d,m) \mid i).(x \mid y) = [CF2]$

$\delta.(x \mid y) = [A7]$

δ ;

[ABP39]

$r3.x \mid s2(e,n).y = [CM7]$

$(r3 \mid s2(e,n)).(x \mid y) = [CF2]$

$\delta.(x \mid y) = [A7]$

δ ;

[ABP40]

$r3.x \mid s3(e,n).y = [CM7]$

$(r3 \mid s3(e,n)).(x \mid y) = [CF2]$

$\delta.(x \mid y) = [A7]$

δ ;

[ABP41]

$r3.x \mid s6.y = [CM7]$

$(r3 \mid s6).(x \mid y) = [CF2]$

$\delta.(x \mid y) = [A7]$

δ ;

[ABP42]

$r3.x \mid s6(n).y = [CM7]$

$(r3 \mid s6(n)).(x \mid y) = [CF2]$

$\delta.(x \mid y) = [A7]$

δ ;

42

[ABP43]

$r3.x \text{ | } r1(e).y = [CM7]$

$(r3 \text{ | } r1(e)).(x|y) = [CF2]$

$\delta.(x|y) = [A7]$

$\delta;$

[ABP44]

$r3.x \text{ | } r2(e,n).y = [CM7]$

$(r3 \text{ | } r2(e,n)).(x|y) = [CF2]$

$\delta.(x|y) = [A7]$

$\delta;$

[ABP45]

$r3.x \text{ | } r5(n).y = [CM7]$

$(r3 \text{ | } r5(n)).(x|y) = [CF2]$

$\delta.(x|y) = [A7]$

$\delta;$

[ABP46]

$r3.x \text{ | } r6.y = [CM7]$

$(r3 \text{ | } r6).(x|y) = [CF2]$

$\delta.(x|y) = [A7]$

$\delta;$

[ABP47]

$r3.x \text{ | } r6(n).y = [CM7]$

$(r3 \text{ | } r6(n)).(x|y) = [CF2]$

$\delta.(x|y) = [A7]$

$\delta;$

[ABP48]

$r3.x \text{ | } i.y = [CM7]$

$(r3 \text{ | } i).(x|y) = [CF2]$

$\delta.(x|y) = [A7]$

$\delta;$

[ABP49]

$r3(d,m).x \text{ | } s2(e,n).y = [CM7]$

$$\langle r3(d,m) \mid s2(e,n) \rangle.(x,y) = [CF2]$$

$$\delta.(x,y) = [A7]$$

$\delta;$

[ABP50]

$$\langle r3.x(d,m) \mid s3.y \rangle = [CM7]$$

$$\langle r3(d,m) \mid s3 \rangle.(x,y) = [CF2"]$$

$$\delta.(x,y) = [A7]$$

$\delta;$

[ABP51]

$$\langle r3(d,m).x \mid s6.y \rangle = [CM7]$$

$$\langle r3(d,m) \mid s6 \rangle.(x,y) = [CF2]$$

$$\delta.(x,y) = [A7]$$

$\delta;$

[ABP52]

$$\langle r3(d,m).x \mid s6(n).y \rangle = [CM7]$$

$$\langle r3(d,m) \mid s6(n) \rangle.(x,y) = [CF2]$$

$$\delta.(x,y) = [A7]$$

$\delta;$

[ABP53]

$$\langle r3(d,m).x \mid r1(e).y \rangle = [CM7]$$

$$\langle r3(d,m) \mid r1(e) \rangle.(x,y) = [CF2]$$

$$\delta.(x,y) = [A7]$$

$\delta;$

[ABP54]

$$\langle r3(d,m).x \mid r2(e,n).y \rangle = [CM7]$$

$$\langle r3(d,m) \mid r2(e,n) \rangle.(x,y) = [CF2]$$

$$\delta.(x,y) = [A7]$$

$\delta;$

[ABP55]

$$\langle r3(d,m).x \mid r5(n).y \rangle = [CM7]$$

$$\langle r3(d,m) \mid r5(n) \rangle.(x,y) = [CF2]$$

44

$$\delta_{\cdot}(x|y) = [A7]$$

δ ;

[ABP56]

$$\underline{r3(d,m).x \mid r6.y} = [CM7]$$

$$(r3(d,m) \mid r6).(x|y) = [CF2]$$

$$\delta_{\cdot}(x|y) = [A7]$$

δ ;

[ABP57]

$$\underline{r3(d,m).x \mid r6(n).y} = [CM7]$$

$$(r3(d,m) \mid r6(n)).(x|y) = [CF2]$$

$$\delta_{\cdot}(x|y) = [A7]$$

δ ;

[ABP58]

$$\underline{r3(d,m).x \mid i.y} = [CM7]$$

$$(r3(d,m) \mid i).(x|y) = [CF2]$$

$$\delta_{\cdot}(x|y) = [A7]$$

δ ;

[ABP59]

$$\underline{r5(m).x \mid s2(e,n).y} = [CM7]$$

$$(r5(m) \mid s2(e,n)).(x|y) = [CF2]$$

$$\delta_{\cdot}(x|y) = [A7]$$

δ ;

[ABP60]

$$\underline{r5(m).x \mid s3.y} = [CM7]$$

$$(r5(m) \mid s3).(x|y) = [CF2]$$

$$\delta_{\cdot}(x|y) = [A7]$$

δ ;

[ABP61]

$$\underline{r5(m).x \mid s3(e,n).y} = [CM7]$$

$$(r5(m) \mid s3(e,n)).(x|y) = [CF2]$$

$$\delta_{\cdot}(x|y) = [A7]$$

$\delta;$

[ABP62]

$$r5(m).x \mid s4(e).y = [CM7]$$

$$(r5(m) \mid s4(e)).(x|y) = [CF2]$$

$$\underline{\delta.(x|y)} = [A7]$$

$\delta;$

[ABP63]

$$r5(m).x \mid r1(e).y = [CM7]$$

$$(r5(m) \mid r1(e)).(x|y) = [CF2]$$

$$\underline{\delta.(x|y)} = [A7]$$

$\delta;$

[ABP64]

$$r5(m).x \mid r2(e,n).y = [CM7]$$

$$(r5(m) \mid r2(e,n)).(x|y) = [CF2]$$

$$\underline{\delta.(x|y)} = [A7]$$

$\delta;$

[ABP65]

$$r5(m).x \mid r6.y = [CM7]$$

$$(r5(m) \mid r6).(x|y) = [CF2]$$

$$\underline{\delta.(x|y)} = [A7]$$

$\delta;$

[ABP66]

$$r5(m).x \mid r6(n).y = [CM7]$$

$$(r5(m) \mid r6(n)).(x|y) = [CF2]$$

$$\underline{\delta.(x|y)} = [A7]$$

$\delta;$

[ABP67]

$$r5(m).x \mid i.y = [CM7]$$

$$(r5(m) \mid i).(x|y) = [CF2]$$

$$\underline{\delta.(x|y)} = [A7]$$

$\delta;$

46

[ABP68]

$\underline{r6.x \text{ ls3.y}} = [\text{CM7}]$

$(\underline{r6 \text{ ls3}}).(\text{x|y}) = [\text{CF2}]$

$\underline{\delta.(\text{x|y})} = [\text{A7}]$

$\delta;$

[ABP69]

$\underline{r6.x \text{ ls3(e,n).y}} = [\text{CM7}]$

$(\underline{r6.x \text{ ls3(e,n)}}).(\text{x|y}) = [\text{CF2}]$

$\underline{\delta.(\text{x|y})} = [\text{A7}]$

$\delta;$

[ABP70]

$\underline{r6.x \text{ ls4(e).y}} = [\text{CM7}]$

$(\underline{r6 \text{ ls4(e)}}).(\text{x|y}) = [\text{CF2}]$

$\underline{\delta.(\text{x|y})} = [\text{A7}]$

$\delta;$

[ABP71]

$\underline{r6.x \text{ ls5(n).y}} = [\text{CM7}]$

$(\underline{r6 \text{ ls5(n)}}).(\text{x|y}) = [\text{CF2}]$

$\underline{\delta.(\text{x|y})} = [\text{A7}]$

$\delta;$

[ABP72]

$\underline{r6(m).x \text{ ls6.y}} = [\text{CM7}]$

$(\underline{r6(m) \text{ ls6}}).(\text{x|y}) = [\text{CF2}']$

$\underline{\delta.(\text{x|y})} = [\text{A7}]$

$\delta;$

[ABP73]

$\underline{r6.x \text{ li.y}} = [\text{CM7}]$

$(\underline{r6 \text{ li}}).(\text{x|y}) = [\text{CF2}]$

$\underline{\delta.(\text{x|y})} = [\text{A7}]$

$\delta;$

[ABP74]

$\underline{r6(m).x \text{ ls3.y}} = [\text{CM7}]$

$$(r6(m) \text{ } \underline{\text{ls3}}). (x \text{ } y) = [\text{CF2}]$$

$$\underline{\delta}. (x \text{ } y) = [\text{A7}]$$

$\delta;$

[ABP75]

$$r6(m).x \text{ } \underline{\text{ls3}}(e.n).y = [\text{CM7}]$$

$$(r6(m) \text{ } \underline{\text{ls3}}(e.n)). (x \text{ } y) = [\text{CF2}]$$

$$\underline{\delta}. (x \text{ } y) = [\text{A7}]$$

$\delta;$

[ABP76]

$$r6(m).x \text{ } \underline{\text{ls4}}(e).y = [\text{CM7}]$$

$$(r6(m) \text{ } \underline{\text{ls4}}(e)). (x \text{ } y) = [\text{CF2}]$$

$$\underline{\delta}. (x \text{ } y) = [\text{A7}]$$

$\delta;$

[ABP77]

$$r6(m).x \text{ } \underline{\text{ls5}}(n).y = [\text{CM7}]$$

$$(r6(m) \text{ } \underline{\text{ls5}}(n)). (x \text{ } y) = [\text{CF2}]$$

$$\underline{\delta}. (x \text{ } y) = [\text{A7}]$$

$\delta;$

[ABP78]

$$r6.x \text{ } \underline{\text{ls6}}(n).y = [\text{CM7}]$$

$$(r6 \text{ } \underline{\text{ls6}}(n)). (x \text{ } y) = [\text{CF2}"]$$

$$\underline{\delta}. (x \text{ } y) = [\text{A7}]$$

$\delta;$

[ABP79]

$$r6(m).x \text{ } \underline{\text{li}}.y = [\text{CM7}]$$

$$(r6(m) \text{ } \underline{\text{li}}). (x \text{ } y) = [\text{CF2}]$$

$$\underline{\delta}. (x \text{ } y) = [\text{A7}]$$

$\delta;$

[ABP80]

$$(\underline{\text{S}}(m).x \text{ } \underline{\text{R}}(n).y) \underline{\text{z}} = [\text{ABP4}]$$

$$(\underline{\Sigma}(d:D, r1(d). \underline{\text{S}}(d,m).x) \text{ } \underline{\text{R}}(n).y) \underline{\text{z}} = [\text{ABP5}]$$

$$\left(\frac{\sum(d:D, r1(d).S(d,m).x)}{\sum(d:D, r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y + \sum(d:D, r3(d, invert(n)).s4(d).s5(invert(n)).y)} \right) \parallel z = [\text{SUM7}]$$

$$\frac{\sum(d:D, r1(d).S(d,m).x)}{\sum(d:D, r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y + \sum(d:D, r3(d, invert(n)).s4(d).s5(invert(n)).y)} \parallel z = [\text{SC3}]$$

$$\frac{\sum(d:D, (\sum(d:D, r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y + \sum(d:D, r3(d, invert(n)).s4(d).s5(invert(n)).y))}{r1(d).S(d,m).x} \parallel z = [\text{CM8}]$$

$$\frac{\sum(d:D, (\sum(d:D, r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y) \parallel r1(d).S(d,m).x + \sum(d:D, r3(d, invert(n)).s4(d).s5(invert(n)).y) \parallel r1(d).S(d,m).x)}{\parallel z = [\text{SUM2}]$$

$$\frac{\sum(d:D, (\sum(d:D, r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y) \parallel r1(d).S(d,m).x + \sum(d1:D, r3(d1, invert(n)).s4(d1).s5(invert(n)).y) \parallel r1(d).S(d,m).x)}{\parallel z = [\text{SUM7}]$$

$$\frac{\sum(d:D, (\sum(d:D, r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y) \parallel r1(d).S(d,m).x + \sum(d1:D, r3(d1, invert(n)).s4(d1).s5(invert(n)).y) \parallel r1(d).S(d,m).x)}{\parallel z = [\text{ABP53}]$$

$$\frac{\sum(d:D, (\sum(d:D, r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y) \parallel r1(d).S(d,m).x + \sum(d1:D, \delta))}{\parallel z = [\text{SUM1}]$$

$$\frac{\sum(d:D, (\sum(d:D, r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y) \parallel r1(d).S(d,m).x + \delta)}{\parallel z = [\text{A6}]$$

$$\frac{\sum(d:D, (\sum(d:D, r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y) \parallel r1(d).S(d,m).x)}{\parallel z = [\text{CM8}]$$

$$\frac{\sum(d:D, \sum(d:D, r3(d,n).s5(n).R(n).y) \parallel r1(d).S(d,m).x + r3.s5(n).R(n).y \parallel r1(d).S(d,m).x)}{\parallel z = [\text{ABP43}]$$

$$\frac{\sum(d:D, \sum(d:D, r3(d,n).s5(n).R(n).y) \parallel r1(d).S(d,m).x + \delta)}{\parallel z = [\text{A6}]$$

$$\frac{\sum(d:D, \sum(d:D, r3(d,n).s5(n).R(n).y) \parallel r1(d).S(d,m).x)}{\parallel z = [\text{SUM2}]$$

$$\frac{\sum(d:D, \sum(d1:D, r3(d1, n).s5(n).R(n).y) \parallel r1(d).S(d,m).x)}{\parallel z = [\text{SUM7}]$$

$$\frac{\sum(d:D, \sum(d1:D, r3(d1, n).s5(n).R(n).y) \parallel r1(d).S(d,m).x)}{\parallel z = [\text{ABP53}]$$

$$\frac{\sum(d:D, \sum(d1:D, \delta))}{\parallel z = [\text{SUM1}]$$

$$\frac{\sum(d:D, \delta)}{\parallel z = [\text{SUM1}]$$

$$\frac{\delta}{\parallel z = [\text{GEN2}]$$

$\delta;$

[ABP81]

$$\frac{(\underline{S(m).x} \parallel K)}{\parallel z = [\text{ABP4}]$$

$$\frac{(\sum(d:D, r1(d).S(d,m).x) \parallel K)}{\parallel z = [\text{ABP6}]$$

$$\frac{(\sum(d:D, r1(d).S(d,m).x) \parallel \sum(d:D, \sum(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3.K)))}{\parallel z = [\text{SUM7}]$$

$$\frac{\sum(d:D, r1(d).S(d,m).x \parallel \sum(d:D, \sum(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3.K)))}{\parallel z = [\text{SC3}]$$

$$\frac{\sum(d:D, \sum(d:D, \sum(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3.K)) \parallel r1(d).S(d,m).x)}{\parallel z = [\text{SUM2}]$$

$$\Sigma(d:D, \Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1, n). (i.s3(d1, n) + i.s3). K)) | r1(d). S(d, m). x) \parallel z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1, n). (i.s3(d1, n) + i.s3). K) | r1(d). S(d, m). x) \parallel z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1, n). (i.s3(d1, n) + i.s3). K | r1(d). S(d, m). x)) \parallel z = [\text{ABP35}]$$

$$\Sigma(d:D, \Sigma(d1:D, \Sigma(n:\text{bit}, \delta))) \parallel z = [\text{SUM1}]$$

$$\Sigma(d:D, \Sigma(d1:D, \delta)) \parallel z = [\text{SUM1}]$$

$$\Sigma(d:D, \delta) \parallel z = [\text{SUM1}]$$

$$\delta \parallel z = [\text{GEN2}]$$

$\delta;$

[ABP82]

$$(S(m).x | L) \parallel z = [\text{ABP4}]$$

$$(\Sigma(d:D, r1(d). S(d, m). x) | L) \parallel z = [\text{ABP7}]$$

$$(\Sigma(d:D, r1(d). S(d, m). x) | \Sigma(n:\text{bit}, r5(n). (i.s6(n) + i.s6). L)) \parallel z = [\text{SUM7}]$$

$$\Sigma(d:D, r1(d). S(d, m). x | \Sigma(n:\text{bit}, r5(n). (i.s6(n) + i.s6). L)) \parallel z = [\text{SC3}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r5(n). (i.s6(n) + i.s6). L) | r1(d). S(d, m). x) \parallel z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r5(n). (i.s6(n) + i.s6). L | r1(d). S(d, m). x)) \parallel z = [\text{ABP63}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \delta)) \parallel z = [\text{SUM1}]$$

$$\Sigma(d:D, \delta) \parallel z = [\text{SUM1}]$$

$$\delta \parallel z = [\text{GEN2}]$$

$\delta;$

[ABP83]

$$(R(m).x | K) \parallel z = [\text{ABP5}]$$

$$((\Sigma(d:D, r3(d, m). s5(m). R(m). x) + r3.s5(m). R(m). x + \Sigma(d:D, r3(d, \text{invert}(m)). s4(d). s5(\text{invert}(m)). x)) | K) \parallel z = [\text{ABP6}]$$

$$\left(\frac{\Sigma(d:D, r3(d, m). s5(m). R(m). x) + r3.s5(m). R(m). x + \Sigma(d:D, r3(d, \text{invert}(m)). s4(d). s5(\text{invert}(m)). x)}{\Sigma(d:D, \Sigma(n:\text{bit}, r2(d, n). (i.s3(d, n) + i.s3). K)} \right) \parallel z = [\text{SC3}]$$

$$\left(\frac{\Sigma(d:D, \Sigma(n:\text{bit}, r2(d, n). (i.s3(d, n) + i.s3). K)}{\Sigma(d:D, r3(d, m). s5(m). R(m). x) + r3.s5(m). R(m). x + \Sigma(d:D, r3(d, \text{invert}(m)). s4(d). s5(\text{invert}(m)). x)} \right) \parallel z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d, n). (i.s3(d, n) + i.s3). K) | \frac{\Sigma(d:D, r3(d, m). s5(m). R(m). x) + r3.s5(m). R(m). x + \Sigma(d:D, r3(d, \text{invert}(m)). s4(d). s5(\text{invert}(m)). x)}{\Sigma(d:D, \Sigma(n:\text{bit}, r2(d, n). (i.s3(d, n) + i.s3). K)} \parallel z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d, n). (i.s3(d, n) + i.s3). K) | \left(\frac{\Sigma(d:D, r3(d, m). s5(m). R(m). x) + r3.s5(m). R(m). x + \Sigma(d:D, r3(d, \text{invert}(m)). s4(d). s5(\text{invert}(m)). x)}{\Sigma(d:D, \Sigma(n:\text{bit}, r2(d, n). (i.s3(d, n) + i.s3). K)} \right) \parallel z = [\text{SC3}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x)}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{CM8}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x}{r2(d,n).(i.s3(d,n) + i.s3).K} + \frac{\Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x)}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{SUM2}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x}{r2(d,n).(i.s3(d,n) + i.s3).K} + \frac{\Sigma(d1:D, r3(d1, \text{invert}(m)).s4(d1).s5(\text{invert}(m)).x)}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x}{r2(d,n).(i.s3(d,n) + i.s3).K} + \frac{\Sigma(d1:D, r3(d1, \text{invert}(m)).s4(d1).s5(\text{invert}(m)).x)}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{ABP54}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x}{r2(d,n).(i.s3(d,n) + i.s3).K} + \frac{\Sigma(d1:D, \delta)}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{SUM1}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x}{r2(d,n).(i.s3(d,n) + i.s3).K + \delta} \right))) \parallel z = [\text{A6}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{CM8}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x)}{r2(d,n).(i.s3(d,n) + i.s3).K} + \frac{r3.s5(m).R(m).x}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{ABP44}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x)}{r2(d,n).(i.s3(d,n) + i.s3).K + \delta} \right))) \parallel z = [\text{A6}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x)}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{SUM2}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d1:D, r3(d1, m).s5(m).R(m).x)}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d1:D, r3(d1, m).s5(m).R(m).x)}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{ABP54}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \left(\frac{\Sigma(d1:D, \delta)}{r2(d,n).(i.s3(d,n) + i.s3).K} \right))) \parallel z = [\text{SUM1}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \delta)) \parallel z = [\text{SUM1}]$$

$$\Sigma(d:D, \delta) \parallel z = [\text{SUM1}]$$

$$\delta \parallel z = [\text{GEN2}]$$

$\delta;$

[ABP84]

$$\left(\frac{R(m).x}{L} \right) \parallel z = [\text{ABP5}]$$

$$\left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x)}{L} \right) \parallel z = [\text{ABP7}]$$

$$\left(\frac{\left(\frac{\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x)}{\Sigma(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L)} \right) \right) \parallel z = [\text{SC3}]$$

$$\left(\frac{\sum(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L)}{\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \sum(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x)} \right) \Big|_z = [\text{SUM7}]$$

$$\frac{\sum(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L)}{\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \sum(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x)} \Big|_z = [\text{SC3}]$$

$$\frac{\sum(n:\text{bit}, (\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \sum(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x))}{r5(n).(i.s6(n) + i.s6).L} \Big|_z = [\text{CM8}]$$

$$\frac{\sum(n:\text{bit}, (\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid r5(n).(i.s6(n) + i.s6).L + \sum(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x) \mid r5(n).(i.s6(n) + i.s6).L)}{\Big|_z = [\text{SUM7}]$$

$$\frac{\sum(n:\text{bit}, (\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid r5(n).(i.s6(n) + i.s6).L + \sum(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x) \mid r5(n).(i.s6(n) + i.s6).L)}{\Big|_z = [\text{ABP55}]$$

$$\frac{\sum(n:\text{bit}, (\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid r5(n).(i.s6(n) + i.s6).L + \sum(d:D, \delta)}{\Big|_z = [\text{SUM1}]$$

$$\frac{\sum(n:\text{bit}, (\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid r5(n).(i.s6(n) + i.s6).L + \delta)}{\Big|_z = [\text{A6}]$$

$$\frac{\sum(n:\text{bit}, (\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid r5(n).(i.s6(n) + i.s6).L)}{\Big|_z = [\text{CM8}]$$

$$\frac{\sum(n:\text{bit}, \sum(d:D, r3(d,m).s5(m).R(m).x) \mid r5(n).(i.s6(n) + i.s6).L + r3.s5(m).R(m).x \mid r5(n).(i.s6(n) + i.s6).L)}{\Big|_z = [\text{ABP45}]$$

$$\frac{\sum(n:\text{bit}, \sum(d:D, r3(d,m).s5(m).R(m).x) \mid r5(n).(i.s6(n) + i.s6).L + \delta)}{\Big|_z = [\text{A6}]$$

$$\frac{\sum(n:\text{bit}, \sum(d:D, r3(d,m).s5(m).R(m).x) \mid r5(n).(i.s6(n) + i.s6).L)}{\Big|_z = [\text{SUM7}]$$

$$\frac{\sum(n:\text{bit}, \sum(d:D, r3(d,m).s5(m).R(m).x) \mid r5(n).(i.s6(n) + i.s6).L)}{\Big|_z = [\text{ABP55}]$$

$$\frac{\sum(n:\text{bit}, \sum(d:D, \delta)}{\Big|_z = [\text{SUM1}]$$

$$\frac{\sum(n:\text{bit}, \delta)}{\Big|_z = [\text{SUM1}]$$

$$\delta \Big|_z = [\text{GEN2}]$$

$\delta;$

[ABP85]

$$\frac{(R(m).x \mid (i.y1 + i.y2).y3)}{\Big|_z = [\text{ABP5}]$$

$$\left(\frac{(\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \sum(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x))}{(i.y1 + i.y2).y3} \right) \Big|_z = [\text{ABP2}]$$

$$\left(\frac{(\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \sum(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x))}{(i.y1.y3 + i.y2.y3)} \right) \Big|_z = [\text{CM8}]$$

$$\left(\frac{(\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \sum(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x) \mid (i.y1.y3 + i.y2.y3))}{\Big|_z = [\text{SUM7}]$$

$$\left(\frac{(\sum(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) +$$

$$\begin{aligned}
& \Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x \mid (i.y1.y3 + i.y2.y3)) \mid z = [\text{CM9}] \\
(& (\Sigma(d:D, r3(d, m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \\
& \Sigma(d:D, \\
& \quad r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x \mid i.y1.y3 + \\
& \quad \underline{r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x \mid i.y2.y3} \mid z = [\text{ABP58}] \\
(& (\Sigma(d:D, r3(d, m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \\
& \Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x \mid i.y1.y3 + \delta) \mid z = [\text{A6}] \\
(& (\Sigma(d:D, r3(d, m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \\
& \Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x \mid i.y1.y3) \mid z = [\text{ABP58}] \\
& ((\Sigma(d:D, r3(d, m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \underline{\Sigma(d:D, \delta)}) \mid z = [\text{SUM1}] \\
& ((\Sigma(d:D, r3(d, m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \delta) \mid z = [\text{A6}] \\
& ((\Sigma(d:D, r3(d, m).s5(m).R(m).x) + r3.s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3)) \mid z = [\text{CM8}] \\
& (\Sigma(d:D, r3(d, m).s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \underline{r3.s5(m).R(m).x \mid (i.y1.y3 + i.y2.y3)}) \mid z = [\text{CM9}] \\
(& \Sigma(d:D, r3(d, m).s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \\
& (r3.s5(m).R(m).x \mid i.y1.y3 + \underline{r3.s5(m).R(m).x \mid i.y2.y3} \mid z = [\text{ABP48}] \\
& (\Sigma(d:D, r3(d, m).s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \underline{(r3.s5(m).R(m).x \mid i.y1.y3 + \delta)}) \mid z = [\text{A6}] \\
& (\Sigma(d:D, r3(d, m).s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \underline{r3.s5(m).R(m).x \mid i.y1.y3}) \mid z = [\text{ABP48}] \\
& (\underline{\Sigma(d:D, r3(d, m).s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3) + \delta}) \mid z = [\text{A6}] \\
& (\underline{\Sigma(d:D, r3(d, m).s5(m).R(m).x) \mid (i.y1.y3 + i.y2.y3)}) \mid z = [\text{SUM7}] \\
& \Sigma(d:D, r3(d, m).s5(m).R(m).x \mid (i.y1.y3 + i.y2.y3)) \mid z = [\text{CM9}] \\
& \Sigma(d:D, r3(d, m).s5(m).R(m).x \mid i.y1.y3 + \underline{r3(d, m).s5(m).R(m).x \mid i.y2.y3}) \mid z = [\text{ABP58}] \\
& \Sigma(d:D, r3(d, m).s5(m).R(m).x \mid i.y1.y3 + \delta) \mid z = [\text{A6}] \\
& \Sigma(d:D, r3(d, m).s5(m).R(m).x \mid i.y1.y3) \mid z = [\text{ABP58}] \\
& \underline{\Sigma(d:D, \delta)} \mid z = [\text{SUM1}] \\
& \underline{\delta} \mid z = [\text{GEN2}] \\
& \delta; \\
& [\text{ABP86}] \\
[& [1] \\
& d1 = d \\
> & (r3(d1, m) \mid s3(d, m)).(s4(d1).x \mid y) + c3(d, m).(s4(d).x \mid y) = [1] \\
& \underline{(r3(d, m) \mid s3(d, m)).(s4(d1).x \mid y) + c3(d, m).(s4(d).x \mid y)} = [\text{CF1}] \\
& c3(d, m).(s4(d1).x \mid y) + c3(d, m).(s4(d).x \mid y) = [1]
\end{aligned}$$

$$\underline{c3(d,m).(s4(d).x|y) + c3(d,m).(s4(d).x|y)} = [A3]$$

$$c3(d,m).(s4(d).x|y)$$

] [→I]

$$d1 = d \rightarrow (r3(d1,m) |s3(d,m)).(s4(d1).x|y) + c3(d,m).(s4(d).x|y) = c3(d,m).(s4(d).x|y);$$

[ABP87]

[[1]

$$\neg(d1 = d)$$

$$\triangleright \underline{(r3(d1,m) |s3(d,m)).z1 + z2} = [\rightarrow E [1] [CF2'(1)]]$$

$$\underline{\delta.z1 + z2} = [A7]$$

$$\underline{\delta + z2} = [GEN1]$$

$$z2$$

] [→I]

$$\neg(d1 = d) \rightarrow (r3(d1,m) |s3(d,m)).z1 + z2 = z2;$$

[ABP88]

[[1]

$$\neg((r3(d1,m) |s3(d,m)).(s4(d1).x|y) + c3(d,m).(s4(d).x|y) = c3(d,m).(s4(d).x|y))$$

\triangleright [2]

[[3]

$$d1 = d$$

\triangleright [4]

$$(r3(d1,m) |s3(d,m)).(s4(d1).x|y) + c3(d,m).(s4(d).x|y) = [\rightarrow E [3] [ABP86]]$$

$$c3(d,m).(s4(d).x|y);$$

$$\perp [\rightarrow E [4] [1]]$$

] [→I]

$$\neg(d1 = d);$$

[5]

$$(r3(d1,m) |s3(d,m)).(s4(d1).x|y) + c3(d,m).(s4(d).x|y) = [\rightarrow E [2] [ABP87]]$$

$$c3(d,m).(s4(d).x|y);$$

$$\perp [\rightarrow E [5] [1]]$$

] [RAA]

$$(r3(d1,m) |s3(d,m)).(s4(d1).x|y) + c3(d,m).(s4(d).x|y) = c3(d,m).(s4(d).x|y);$$

[ABP89]

$$\underline{\Sigma(d1:D,(r3(d1,m) |s3(d,m)).(s4(d1).x|y))} = [SUM3]$$

$$\Sigma(d1:D,(r3(d1,m) |s3(d,m)).(s4(d1).x|y)) + \underline{(r3(d,m) |s3(d,m)).(s4(d).x|y)} = [CF1']$$

$$\Sigma(d1:D,(r3(d1,m) |s3(d,m)).(s4(d1).x|y)) + \underline{c3(d,m).(s4(d).x|y)} = [SUM1]$$

$$\underline{\Sigma(d1:D,(r3(d1,m) |s3(d,m)).(s4(d1).x|y))} + \underline{\Sigma(d1:D,c3(d,m).(s4(d).x|y))} = [SUM4]$$

$$\Sigma(d1:D,\underline{(r3(d1,m) |s3(d,m)).(s4(d1).x|y) + c3(d,m).(s4(d).x|y)}) = [ABP88]$$

$$\underline{\Sigma(d1:D, c3(d,m).s4(d).x|y))} = [\text{SUM1}]$$

$$c3(d,m).s4(d).x|y;$$

[ABP90]

$$\underline{R(m).x |s3(d,n).y} = [\text{ABP5}]$$

$$\underline{(\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \Sigma(d:D, r3(d, invert(m)).s4(d).s5(invert(m)).x)) |s3(d,n).y} = [\text{CM8}]$$

$$\underline{(\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) |s3(d,n).y + \Sigma(d:D, r3(d, invert(m)).s4(d).s5(invert(m)).x) |s3(d,n).y} = [\text{SUM2}]$$

$$\underline{(\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) |s3(d,n).y + \Sigma(d1:D, r3(d1, invert(m)).s4(d1).s5(invert(m)).x) |s3(d,n).y} = [\text{SUM7}]$$

$$\underline{(\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) |s3(d,n).y + \Sigma(d1:D, r3(d1, invert(m)).s4(d1).s5(invert(m)).x) |s3(d,n).y} = [\text{CM7}]$$

$$\underline{(\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) |s3(d,n).y + \Sigma(d1:D, (r3(d1, invert(m)) |s3(d,n)).s4(d1).s5(invert(m)).x|y))} = [\text{CM8}]$$

$$\underline{\Sigma(d:D, r3(d,m).s5(m).R(m).x) |s3(d,n).y + r3.s5(m).R(m).x |s3(d,n).y + \Sigma(d1:D, (r3(d1, invert(m)) |s3(d,n)).s4(d1).s5(invert(m)).x|y))} = [\text{ABP40}]$$

$$\underline{\Sigma(d:D, r3(d,m).s5(m).R(m).x) |s3(d,n).y + \delta + \Sigma(d1:D, (r3(d1, invert(m)) |s3(d,n)).s4(d1).s5(invert(m)).x|y))} = [\text{A6}]$$

$$\underline{\Sigma(d:D, r3(d,m).s5(m).R(m).x) |s3(d,n).y + \Sigma(d1:D, (r3(d1, invert(m)) |s3(d,n)).s4(d1).s5(invert(m)).x|y))} = [\text{SUM2}]$$

$$\underline{\Sigma(d1:D, r3(d1,m).s5(m).R(m).x) |s3(d,n).y + \Sigma(d1:D, (r3(d1, invert(m)) |s3(d,n)).s4(d1).s5(invert(m)).x|y))} = [\text{SUM7}]$$

$$\underline{\Sigma(d1:D, r3(d1,m).s5(m).R(m).x) |s3(d,n).y + \Sigma(d1:D, (r3(d1, invert(m)) |s3(d,n)).s4(d1).s5(invert(m)).x|y))} = [\text{CM7}]$$

$$\underline{\Sigma(d1:D, (r3(d1,m) |s3(d,n)).s5(m).R(m).x|y) + \Sigma(d1:D, (r3(d1, invert(m)) |s3(d,n)).s4(d1).s5(invert(m)).x|y))};$$

[ABP91]

$$\underline{(R(1).x |s3(d,0).y)} \parallel_Z = [\text{ABP90}]$$

$$\underline{(\Sigma(d1:D, (r3(d1, 1) |s3(d,0)).s5(1).R(1).x|y) + \Sigma(d1:D, (r3(d1, invert(1)) |s3(d,0)).s4(d1).s5(invert(1)).x|y))} \parallel_Z = [\rightarrow E [\text{ABP3} [\text{CF2'(2)}]]]$$

$$\underline{(\Sigma(d1:D, \delta.s5(1).R(1).x|y) + \Sigma(d1:D, (r3(d1, invert(1)) |s3(d,0)).s4(d1).s5(invert(1)).x|y))} \parallel_Z = [\text{A7}]$$

$$\underline{(\Sigma(d1:D, \delta) + \Sigma(d1:D, (r3(d1, invert(1)) |s3(d,0)).s4(d1).s5(invert(1)).x|y))} \parallel_Z = [\text{SUM1}]$$

$$\underline{(\delta + \Sigma(d1:D, (r3(d1, invert(1)) |s3(d,0)).s4(d1).s5(invert(1)).x|y))} \parallel_Z = [\text{GEN1}]$$

$$\underline{\Sigma(d1:D, (r3(d1, invert(1)) |s3(d,0)).s4(d1).s5(invert(1)).x|y)} \parallel_Z = [\text{FACT}]$$

$$\underline{\Sigma(d1:D, (r3(d1, 0) |s3(d,0)).s4(d1).s5(invert(1)).x|y)} \parallel_Z = [\text{FACT}]$$

$$\Sigma(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s4(d1).s5(0).x|y)) \parallel z = [\text{ABP89}]$$

$$c3(d,0).(s4(d).s5(0).x|y) \parallel z = [\text{CM3}]$$

$$c3(d,0).((s4(d).s5(0).x|y)|z) = [\text{GEN4}]$$

$$c3(d,0).(s4(d).s5(0).x|y|z);$$

[ABP92]

$$(R(0).x \text{ } |s3(d,1).y) \parallel z = [\text{ABP90}]$$

$$\left(\begin{array}{l} \Sigma(d1:D, (r3(d1,0) \text{ } |s3(d,1)).(s5(0).R(0).x|y)) + \\ \Sigma(d1:D, (r3(d1, \text{invert}(0)) \text{ } |s3(d,1)).(s4(d1).s5(\text{invert}(0)).x|y)) \end{array} \right) \parallel z = [\text{SC3}]$$

$$\left(\begin{array}{l} \Sigma(d1:D, (s3(d,1) \text{ } |r3(d1,0)).(s5(0).R(0).x|y)) + \\ \Sigma(d1:D, (r3(d1, \text{invert}(0)) \text{ } |s3(d,1)).(s4(d1).s5(\text{invert}(0)).x|y)) \end{array} \right) \parallel z = [\rightarrow E [\text{ABP3} [\text{CF2}'(2)]]]$$

$$\left(\begin{array}{l} \Sigma(d1:D, \delta.(s5(0).R(0).x|y)) + \\ \Sigma(d1:D, (r3(d1, \text{invert}(0)) \text{ } |s3(d,1)).(s4(d1).s5(\text{invert}(0)).x|y)) \end{array} \right) \parallel z = [\text{A7}]$$

$$(\Sigma(d1:D, \delta) + \Sigma(d1:D, (r3(d1, \text{invert}(0)) \text{ } |s3(d,1)).(s4(d1).s5(\text{invert}(0)).x|y))) \parallel z = [\text{SUM1}]$$

$$(\delta + \Sigma(d1:D, (r3(d1, \text{invert}(0)) \text{ } |s3(d,1)).(s4(d1).s5(\text{invert}(0)).x|y))) \parallel z = [\text{GEN1}]$$

$$\Sigma(d1:D, (r3(d1, \text{invert}(0)) \text{ } |s3(d,1)).(s4(d1).s5(\text{invert}(0)).x|y)) \parallel z = [\text{FACT}]$$

$$\Sigma(d1:D, (r3(d1, 1) \text{ } |s3(d,1)).(s4(d1).s5(\text{invert}(0)).x|y)) \parallel z = [\text{FACT}]$$

$$\Sigma(d1:D, (r3(d1, 1) \text{ } |s3(d,1)).(s4(d1).s5(1).x|y)) \parallel z = [\text{ABP89}]$$

$$c3(d,1).(s4(d).s5(1).x|y) \parallel z = [\text{CM3}]$$

$$c3(d,1).((s4(d).s5(1).x|y)|z) = [\text{GEN4}]$$

$$c3(d,1).(s4(d).s5(1).x|y|z);$$

[ABP93]

$$\begin{array}{l} [\quad [1] \\ \quad d1 = d \end{array}$$

$$\triangleright (r3(d1,m) \text{ } |s3(d,m)).z + c3(d,m).z = [1]$$

$$(r3(d,m) \text{ } |s3(d,m)).z + c3(d,m).z = [\text{CF1}]$$

$$c3(d,m).z + c3(d,m).z = [\text{A3}]$$

$$c3(d,m).z$$

] [\rightarrow]

$$d1 = d \rightarrow (r3(d1,m) \text{ } |s3(d,m)).z + c3(d,m).z = c3(d,m).z;$$

[ABP94]

$$\begin{array}{l} [\quad [1] \\ \quad -(d1 = d) \end{array}$$

$$\triangleright (r3(d1,m) \text{ } |s3(d,m)).z1 + z2 = [\rightarrow E [1] [\text{CF2}'(1)]]$$

$$\delta.z1 + z2 = [A7]$$

$$\delta + z2 = [GEN1]$$

$$z2$$

] [\rightarrow]

$$\neg(d1 = d) \rightarrow (r3(d1,m) \text{ } |s3(d,m)).z1 + z2 = z2;$$

[ABP95]

[[1]

$$\neg((r3(d1,m) \text{ } |s3(d,m)).z + c3(d,m).z = c3(d,m).z)$$

\triangleright [2]

[[3]

$$d1 = d$$

\triangleright [4]

$$(r3(d1,m) \text{ } |s3(d,m)).z + c3(d,m).z = [\rightarrow E [3] [ABP93]]$$

$$c3(d,m).z;$$

$$\perp [\rightarrow E [4] [1]]$$

] [\rightarrow !]

$$\neg(d1 = d);$$

[5]

$$(r3(d1,m) \text{ } |s3(d,m)).z + c3(d,m).z = [\rightarrow E [2] [ABP94]]$$

$$c3(d,m).z;$$

$$\perp [\rightarrow E [5] [1]]$$

] [RAA]

$$(r3(d1,m) \text{ } |s3(d,m)).z + c3(d,m).z = c3(d,m).z;$$

[ABP96]

$$\underline{(R(0).x \text{ } |s3(d,0).y)} \parallel z = [ABP90]$$

$$\left(\sum(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y)) + \sum(d1:D, (r3(d1, \text{invert}(0)) \text{ } |s3(d,0)).(s4(d1).s5(\text{invert}(0)).x|y)) \right) \parallel z = [\text{FACT}]$$

$$\left(\sum(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y)) + \sum(d1:D, (r3(d1,1) \text{ } |s3(d,0)).(s4(d1).s5(\text{invert}(0)).x|y)) \right) \parallel z = [\rightarrow E [ABP3] [CF2'(2)]]$$

$$\left(\sum(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y)) + \sum(d1:D, \delta.(s4(d1).s5(\text{invert}(0)).x|y)) \right) \parallel z = [A7]$$

$$\left(\sum(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y)) + \sum(d1:D, \delta) \right) \parallel z = [\text{SUM1}]$$

$$\left(\underline{\sum(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y))} + \delta \right) \parallel z = [A6]$$

$$\sum(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y)) \parallel z = [\text{SUM3}]$$

$$\left(\sum(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y)) + \underline{(r3(d,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y)} \right) \parallel z = [\text{CF1}']$$

$$\left(\sum(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y)) + \underline{c3(d,0).(s5(0).R(0).x|y)} \right) \parallel z = [\text{SUM1}]$$

$$\left(\underline{\sum(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y))} + \sum(d1:D, c3(d,0).(s5(0).R(0).x|y)) \right) \parallel z = [\text{SUM4}]$$

$$\Sigma(d1:D, (r3(d1,0) \text{ } |s3(d,0)).(s5(0).R(0).x|y) + c3(d,0).(s5(0).R(0).x|y)) \parallel_z = [\text{ABP95}]$$

$$\Sigma(d1:D, c3(d,0).(s5(0).R(0).x|y)) \parallel_z = [\text{SUM1}]$$

$$c3(d,0).(s5(0).R(0).x|y) \parallel_z = [\text{CM3}]$$

$$c3(d,0).(s5(0).R(0).x|y) \parallel_z = [\text{GEN4}]$$

$$c3(d,0).(s5(0).R(0).x|y|z);$$

[ABP97]

$$(R(1).x \text{ } |s3(d,1).y) \parallel_z = [\text{ABP90}]$$

$$(\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y)) + \Sigma(d1:D, (r3(d1, \text{invert}(1)) \text{ } |s3(d,1)).(s4(d1).s5(\text{invert}(1)).x|y))) \parallel_z = [\text{FACT}]$$

$$(\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(0).R(1).x|y)) + \Sigma(d1:D, (r3(d1,0) \text{ } |s3(d,1)).(s4(d1).s5(\text{invert}(1)).x|y))) \parallel_z = [\text{→E [ABP3] [CF2'(2)]}]$$

$$(\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y)) + \Sigma(d1:D, \delta.(s4(d1).s5(\text{invert}(1)).x|y))) \parallel_z = [\text{A7}]$$

$$(\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y)) + \Sigma(d1:D, \delta)) \parallel_z = [\text{SUM1}]$$

$$(\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y)) + \delta) \parallel_z = [\text{A6}]$$

$$\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y)) \parallel_z = [\text{SUM3}]$$

$$(\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y)) + (r3(d,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y)) \parallel_z = [\text{CF1'}]$$

$$(\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y)) + c3(d,1).(s5(1).R(1).x|y)) \parallel_z = [\text{SUM1}]$$

$$(\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y)) + \Sigma(d1:D, c3(d,1).(s5(1).R(1).x|y))) \parallel_z = [\text{SUM4}]$$

$$\Sigma(d1:D, (r3(d1,1) \text{ } |s3(d,1)).(s5(1).R(1).x|y) + c3(d,1).(s5(1).R(1).x|y)) \parallel_z = [\text{ABP95}]$$

$$\Sigma(d1:D, c3(d,1).(s5(1).R(1).x|y)) \parallel_z = [\text{SUM1}]$$

$$c3(d,1).(s5(1).R(1).x|y) \parallel_z = [\text{CM3}]$$

$$c3(d,1).(s5(1).R(1).x|y) \parallel_z = [\text{GEN4}]$$

$$c3(d,1).(s5(1).R(1).x|y|z);$$

[ABP98]

$$(R(m).x \text{ } |s3.y) \parallel_z = [\text{ABP5}]$$

$$((\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x)) \text{ } |s3.y)) \parallel_z = [\text{CM8}]$$

$$((\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s3.y + \Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x) \text{ } |s3.y) \parallel_z = [\text{SUM7}]$$

$$((\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s3.y + \Sigma(d:D, r3(d, \text{invert}(m)).s4(d).s5(\text{invert}(m)).x) \text{ } |s3.y) \parallel_z = [\text{ABP50}]$$

$$((\Sigma(d:D, r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s3.y + \Sigma(d:D, \delta)) \parallel_z = [\text{SUM1}]$$

$$(\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s3.y + \delta)} \Big|_z = [A6]$$

$$(\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s3.y)} \Big|_z = [CM8]$$

$$(\underline{\sum(d:D,r3(d,m).s5(m).R(m).x) \text{ } |s3.y + r3.s5(m).R(m).x \text{ } |s3.y)} \Big|_z = [SUM7]$$

$$(\underline{\sum(d:D,r3(d,m).s5(m).R(m).x \text{ } |s3.y) + r3.s5(m).R(m).x \text{ } |s3.y)} \Big|_z = [ABP50]$$

$$(\underline{\sum(d:D,\delta) + r3.s5(m).R(m).x \text{ } |s3.y)} \Big|_z = [SUM1]$$

$$(r3.s5(m).R(m).x \text{ } |s3.y + \delta) \Big|_z = [A6]$$

$$(r3.s5(m).R(m).x \text{ } |s3.y) \Big|_z = [CM7]$$

$$(r3 \text{ } |s3).(s5(m).R(m).x|y) \Big|_z = [CF1]$$

$$c3.(s5(m).R(m).x|y) \Big|_z = [CM3]$$

$$c3.((s5(m).R(m).x|y)|z) = [GEN4]$$

$$c3.(s5(m).R(m).x|y|z);$$

[ABP99]

$$(\underline{R(m).x \text{ } |s6(n).y)} \Big|_z = [ABP5]$$

$$\left(\frac{\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x + \sum(d:D,r3(d,\text{invert}(m)).s4(d).s5(\text{invert}(m)).x)) \text{ } |s6(n).y)} \Big|_z = [CM8]$$

$$\left(\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s6(n).y + \sum(d:D,r3(d,\text{invert}(m)).s4(d).s5(\text{invert}(m)).x) \text{ } |s6(n).y)} \Big|_z = [SUM7]$$

$$\left(\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s6(n).y + \sum(d:D,r3(d,\text{invert}(m)).s4(d).s5(\text{invert}(m)).x) \text{ } |s6(n).y)} \Big|_z = [ABP52]$$

$$(\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s6(n).y + \sum(d:D,\delta)} \Big|_z = [SUM1]$$

$$(\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s6(n).y + \delta)} \Big|_z = [A6]$$

$$(\underline{(\sum(d:D,r3(d,m).s5(m).R(m).x) + r3.s5(m).R(m).x) \text{ } |s6(n).y)} \Big|_z = [CM8]$$

$$(\underline{\sum(d:D,r3(d,m).s5(m).R(m).x) \text{ } |s6(n).y + r3.s5(m).R(m).x \text{ } |s6(n).y)} \Big|_z = [ABP42]$$

$$(\underline{\sum(d:D,r3(d,m).s5(m).R(m).x) \text{ } |s6(n).y + \delta)} \Big|_z = [A6]$$

$$(\underline{\sum(d:D,r3(d,m).s5(m).R(m).x) \text{ } |s6(n).y)} \Big|_z = [SUM7]$$

$$\underline{\sum(d:D,r3(d,m).s5(m).R(m).x \text{ } |s6(n).y)} \Big|_z = [ABP52]$$

$$\underline{\sum(d:D,\delta)} \Big|_z = [SUM1]$$

$$\underline{\delta} \Big|_z = [GEN2]$$

$\delta;$

[ABP100]

$$(\underline{R(m).x \text{ } |s6(n).y)} \Big|_z = [ABP5]$$

$$\langle K \langle i.y1 + i.y2 \rangle .y3 \rangle z = [\text{ABP6}]$$

$$\langle \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K)) \langle i.y1 + i.y2 \rangle .y3 \rangle z = [\text{ABP2}]$$

$$\langle \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K)) \langle i.y1.y3 + i.y2.y2 \rangle \rangle z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K) \langle i.y1.y3 + i.y2.y3 \rangle \rangle z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K \langle i.y1.y3 + i.y2.y3 \rangle \rangle \rangle z = [\text{CM9}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K \langle i.y1.y3 + r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K \langle i.y2.y3 \rangle \rangle \rangle \rangle z = [\text{ABP38}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K \langle i.y1.y3 + \delta \rangle \rangle \rangle z = [\text{A6}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K \langle i.y1.y3 \rangle \rangle \rangle z = [\text{ABP38}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \delta) \rangle \rangle z = [\text{SUM1}]$$

$$\langle \Sigma(d:D, \delta) \rangle z = [\text{SUM1}]$$

$$\langle \delta \rangle z = [\text{GEN2}]$$

$\delta;$

[ABP103]

$$\langle K \langle s6(n).y \rangle \rangle z = [\text{ABP6}]$$

$$\langle \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K)) \langle s6(n).y \rangle \rangle z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K) \langle s6(n).y \rangle \rangle z = [\text{SUM2}]$$

$$\Sigma(d:D, \Sigma(n1:\text{bit}, r2(d,n1).(\text{i.s3}(d,n1) + \text{i.s3}).K) \langle s6(n).y \rangle \rangle z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n1:\text{bit}, r2(d,n1).(\text{i.s3}(d,n1) + \text{i.s3}).K \langle s6(n).y \rangle \rangle \rangle z = [\text{ABP34}]$$

$$\Sigma(d:D, \Sigma(n1:\text{bit}, \delta) \rangle \rangle z = [\text{SUM1}]$$

$$\langle \Sigma(d:D, \delta) \rangle z = [\text{SUM1}]$$

$$\langle \delta \rangle z = [\text{GEN2}]$$

$\delta;$

[ABP104]

$$\langle K \langle s6.y \rangle \rangle z = [\text{ABP6}]$$

$$\langle \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K)) \langle s6.y \rangle \rangle z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K) \langle s6.y \rangle \rangle z = [\text{SUM7}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(\text{i.s3}(d,n) + \text{i.s3}).K \langle s6.y \rangle \rangle \rangle z = [\text{ABP33}]$$

$$\Sigma(d:D, \Sigma(n:\text{bit}, \delta) \rangle \rangle z = [\text{SUM1}]$$

$$\langle \Sigma(d:D, \delta) \rangle z = [\text{SUM1}]$$

$$\delta \parallel z = [\text{GEN2}]$$

δ ;

[ABP105]

$$(\underline{S(d,m).x} \parallel R(n).y) \parallel z = [\text{ABP8}]$$

$$(s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x \parallel R(n).y) \parallel z = [\text{ABP5}]$$

$$\left(\frac{s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x}{(\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y + \Sigma(d:D,r3(d,\text{invert}(n)).s4(d).s5(\text{invert}(n)).y)})} \right) \parallel z = [\text{SC3}]$$

$$\left(\frac{(\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y + \Sigma(d:D,r3(d,\text{invert}(n)).s4(d).s5(\text{invert}(n)).y)})}{s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x} \right) \parallel z = [\text{CM8}]$$

$$\left(\frac{(\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x + \underline{\Sigma(d:D,r3(d,\text{invert}(n)).s4(d).s5(\text{invert}(n)).y)})}{s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x} \right) \parallel z = [\text{SUM2}]$$

$$\left(\frac{(\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x + \underline{\Sigma(d1:D,r3(d1,\text{invert}(n)).s4(d1).s5(\text{invert}(n)).y)})}{s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x} \right) \parallel z = [\text{SUM7}]$$

$$\left(\frac{(\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x + \underline{\Sigma(d1:D,r3(d1,\text{invert}(n)).s4(d1).s5(\text{invert}(n)).y)})}{s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x} \right) \parallel z = [\text{ABP49}]$$

$$\left(\frac{(\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x + \underline{\Sigma(d1:D,\delta)})}{s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x} \right) \parallel z = [\text{SUM1}]$$

$$((\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x + \delta)) \parallel z = [\text{A6}]$$

$$((\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x)) \parallel z = [\text{CM8}]$$

$$\left(\frac{\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x + r3.s5(n).R(n).y}{s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x} \right) \parallel z = [\text{ABP39}]$$

$$(\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x + \delta)) \parallel z = [\text{A6}]$$

$$(\underline{\Sigma(d:D,r3(d,n).s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x)) \parallel z = [\text{SUM2}]$$

$$(\underline{\Sigma(d1:D,r3(d1,n).s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x)) \parallel z = [\text{SUM7}]$$

$$\underline{\Sigma(d1:D,r3(d1,n).s5(n).R(n).y} \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x) \parallel z = [\text{ABP49}]$$

$$\underline{\Sigma(d1:D,\delta)} \parallel z = [\text{SUM1}]$$

$$\delta \parallel z = [\text{GEN2}]$$

δ ;

[ABP106]

$$\begin{bmatrix} [1] \\ n = m \end{bmatrix}$$

$$\triangleright (r2(d,n) \parallel s2(d,m)).((y1.s3(d,n) + y2).y3;x) + c2(d,m).((y1.s3(d,m) + y2).y3;x) = [1]$$

$$(\underline{r2(d,m) \parallel s2(d,m)}).((y1.s3(d,n) + y2).y3;x) + c2(d,m).((y1.s3(d,m) + y2).y3;x) = [\text{CF1}']$$

$$c2(d,m).((y1.s3(d,n) + y2).y3x) + c2(d,m).((y1.s3(d,m) + y2).y3x) = [1]$$

$$\underline{c2(d,m).((y1.s3(d,m) + y2).y3x) + c2(d,m).((y1.s3(d,m) + y2).y3x)} = [A3]$$

$$c2(d,m).((y1.s3(d,m) + y2).y3x)$$

] [→]

n = m →

$$(r2(d,n) \text{ls2}(d,m)).((y1.s3(d,n) + y2).y3x) + c2(d,m).((y1.s3(d,m) + y2).y3x) = c2(d,m).((y1.s3(d,m) + y2).y3x)$$

[ABP107]

[[1]

$$\neg(n = m)$$

▷ $(r2(d,n) \text{ls2}(d,m)).z1 + z2 = [→E [1] [CF2'(2)]]$

$$\underline{\delta.z1 + z2} = [A7]$$

$$\underline{\delta + z2} = [GEN1]$$

z2

] [→]

$$\neg(n = m) \rightarrow (r2(d,n) \text{ls2}(d,m)).z1 + z2 = z2;$$

[ABP108]

[[1]

$$\neg((r2(d,n) \text{ls2}(d,m)).((y1.s3(d,n) + y2).y3x) + c2(d,m).((y1.s3(d,m) + y2).y3x) = c2(d,m).((y1.s3(d,m) + y2).y3x))$$

▷ [2]

[[3]

$$n = m$$

▷ [4]

$$\underline{(r2(d,n) \text{ls2}(d,m)).((y1.s3(d,n) + y2).y3x) + c2(d,m).((y1.s3(d,m) + y2).y3x)} = [→E [3] [ABP106]]$$

$$c2(d,m).((y1.s3(d,m) + y2).y3x);$$

$$\perp [→E [4] [1]]$$

] [→]

$$\neg(n = m);$$

[5]

$$\underline{(r2(d,n) \text{ls2}(d,m)).((y1.s3(d,n) + y2).y3x) + c2(d,m).((y1.s3(d,m) + y2).y3x)} = [→E [2] [ABP107]]$$

$$c2(d,m).((y1.s3(d,m) + y2).y3x);$$

$$\perp [→E [5] [1]]$$

] [RAA]

$$\underline{(r2(d,n) \text{ls2}(d,m)).((y1.s3(d,n) + y2).y3x) + c2(d,m).((y1.s3(d,m) + y2).y3x)} = c2(d,m).((y1.s3(d,m) + y2).y3x);$$

[ABP109]

[[1]

$$d1 = d$$

$$\triangleright (r2(d1,n) \mid s2(d,m)).((y1.s3(d1,n) + y2).y3ix) + c2(d,m).((y1.s3(d,m) + y2).y3ix) = [1]$$

$$(r2(d,n) \mid s2(d,m)).((y1.s3(d1,n) + y2).y3ix) + c2(d,m).((y1.s3(d,m) + y2).y3ix) = [1]$$

$$\underline{(r2(d,n) \mid s2(d,m)).((y1.s3(d,n) + y2).y3ix) + c2(d,m).((y1.s3(d,m) + y2).y3ix)} = [ABP108]$$

$$c2(d,m).((y1.s3(d,m) + y2).y3ix)$$

] [→]

$$d1 = d \rightarrow$$

$$(r2(d1,n) \mid s2(d,m)).((y1.s3(d1,n) + y2).y3ix) + c2(d,m).((y1.s3(d,m) + y2).y3ix) = c2(d,m).((y1.s3(d,m) + y2).y3ix);$$

[ABP110]

[[1]

$$\neg(d1 = d)$$

$$\triangleright \underline{(r2(d1,n) \mid s2(d,m)).z1 + z2} = [\rightarrow E [1] [CF2'(1)]]$$

$$\underline{\delta.z1 + z2} = [A7]$$

$$\underline{\delta + z2} = [GEN1]$$

$$z2$$

] [→]

$$\neg(d1 = d) \rightarrow$$

$$(r2(d1,n) \mid s2(d,m)).z1 + z2 = z2;$$

[ABP111]

[[1]

$$\neg((r2(d1,n) \mid s2(d,m)).((y1.s3(d1,n) + y2).y3ix) + c2(d,m).((y1.s3(d,m) + y2).y3ix) = c2(d,m).((y1.s3(d,m) + y2).y3ix))$$

\triangleright [2]

[[3]

$$d1 = d$$

\triangleright [4]

$$\underline{(r2(d1,n) \mid s2(d,m)).((y1.s3(d1,n) + y2).y3ix) + c2(d,m).((y1.s3(d,m) + y2).y3ix)} = [\rightarrow E [3] [ABP109]]$$

$$c2(d,m).((y1.s3(d,m) + y2).y3ix);$$

$$\perp [\rightarrow E [4] [1]]$$

] [→]

$$\neg(d1 = d);$$

[5]

$$\underline{(r2(d1,n) \mid s2(d,m)).((y1.s3(d1,n) + y2).y3ix) + c2(d,m).((y1.s3(d,m) + y2).y3ix)} = [\rightarrow E [2] [ABP110]]$$

$$c2(d,m).((y1.s3(d,m) + y2).y3ix);$$

$$\perp [\rightarrow E [5] [1]]$$

] [RAA]

$$\underline{(r2(d1,n) \mid s2(d,m)).((y1.s3(d1,n) + y2).y3ix) + c2(d,m).((y1.s3(d,m) + y2).y3ix)} = c2(d,m).((y1.s3(d,m) + y2).y3ix);$$

[ABP112]

$$\underline{c2(d,m).(((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x(i.s3(d,m) + i.s3).K))} \parallel_z = [\text{CM3}]$$

$$c2(d,m).(((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x(i.s3(d,m) + i.s3).K)iz);$$

[ABP113]

$$\underline{(S(d,m).x \parallel L)} \parallel_z = [\text{ABP8}]$$

$$(s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x \parallel L) \parallel_z = [\text{ABP7}]$$

$$\underline{(s2(d,m).((r6(\text{invert}(m)) + r6).S(d,n) + r6(m)).x \parallel \sum(n:\text{bit},r5(n).(i.s6(n) + i.s6).L))} \parallel_z = [\text{SC3}]$$

$$\underline{(\sum(n:\text{bit},r5(n).(i.s6(n) + i.s6).L) \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x)} \parallel_z = [\text{SUM7}]$$

$$\underline{\sum(n:\text{bit},r5(n).(i.s6(n) + i.s6).L \parallel s2(d,m).((r6(\text{invert}(m)) + r6).S(d,m) + r6(m)).x)} \parallel_z = [\text{ABP59}]$$

$$\underline{\sum(n:\text{bit},\delta)} \parallel_z = [\text{SUM1}]$$

$$\underline{\delta} \parallel_z = [\text{GEN2}]$$

$\delta;$

[ABP114]

$$\underline{(i.x1 + i.x2).x3 \parallel L)} \parallel_z = [\text{ABP2}]$$

$$\underline{(i.x1.x3 + i.x2.x3) \parallel L)} \parallel_z = [\text{ABP7}]$$

$$\underline{(i.x1.x3 + i.x2.x3) \parallel \sum(n:\text{bit},r5(n).(i.s6(n) + i.s6).L)} \parallel_z = [\text{SC3}]$$

$$\underline{(\sum(n:\text{bit},r5(n).(i.s6(n) + i.s6).L) \parallel (i.x1.x3 + i.x2.x3))} \parallel_z = [\text{SUM7}]$$

$$\underline{\sum(n:\text{bit},r5(n).(i.s6(n) + i.s6).L \parallel (i.x1.x3 + i.x2.x3))} \parallel_z = [\text{CM9}]$$

$$\underline{\sum(n:\text{bit},r5(n).(i.s6(n) + i.s6).L \parallel i.x1.x3 + r5(n).(i.s6(n) + i.s6).L \parallel i.x2.x3)} \parallel_z = [\text{ABP67}]$$

$$\underline{\sum(n:\text{bit},r5(n).(i.s6(n) + i.s6).L \parallel i.x1.x3 + \delta)} \parallel_z = [\text{A6}]$$

$$\underline{\sum(n:\text{bit},r5(n).(i.s6(n) + i.s6).L \parallel i.x1.x3)} \parallel_z = [\text{ABP67}]$$

$$\underline{\sum(n:\text{bit},\delta)} \parallel_z = [\text{SUM1}]$$

$$\underline{\delta} \parallel_z = [\text{GEN2}]$$

$\delta;$

[ABP115]

$$\underline{(((r6(m1) + r6).x1 + r6(m2)).x2 \parallel R(n).y)} \parallel_z = [\text{ABP1}]$$

$$\underline{((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \parallel R(n).y)} \parallel_z = [\text{ABP5}]$$

$$\left(\underline{(r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \parallel (\sum(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y + \sum(d:D,r3(d,\text{invert}(n)).s4(d).s5(\text{invert}(n)).y))} \right) \parallel_z = [\text{SC3}]$$

$$\left(\underline{(\sum(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y + \sum(d:D,r3(d,\text{invert}(n)).s4(d).s5(\text{invert}(n)).y)) \parallel (r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2)} \right) \parallel_z = [\text{CM8}]$$

$$\left(\underline{(\sum(d:D,r3(d,n).s5(n).R(n).y) + r3.s5(n).R(n).y) \parallel (r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) +$$

$$\Sigma(d:D, r3(d,n).s5(n).R(n).y \mid (r6(m1).x1.x2 + r6.x1.x2) + r3(d,n).s5(n).R(n).y \mid r6(m2).x2) \parallel z = [ABP57]$$

$$\Sigma(d:D, r3(d,n).s5(n).R(n).y \mid (r6(m1).x1.x2 + r6.x1.x2) + \delta) \parallel z = [A6]$$

$$\Sigma(d:D, r3(d,n).s5(n).R(n).y \mid (r6(m1).x1.x2 + r6.x1.x2)) \parallel z = [CM9]$$

$$\Sigma(d:D, r3(d,n).s5(n).R(n).y \mid r6(m1).x1.x2 + r3(d,n).s5(n).R(n).y \mid r6.x1.x2) \parallel z = [ABP56]$$

$$\Sigma(d:D, r3(d,n).s5(n).R(n).y \mid r6(m1).x1.x2 + \delta) \parallel z = [A6]$$

$$\Sigma(d:D, r3(d,n).s5(n).R(n).y \mid r6(m1).x1.x2) \parallel z = [ABP57]$$

$$\Sigma(d:D, \delta) \parallel z = [SUM1]$$

$$\delta \parallel z = [GEN2]$$

$\delta;$

[ABP116]

$$(((r6(m1) + r6).x1 + r6(m2)).x2 \mid K) \parallel z = [ABP1]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \mid K) \parallel z = [ABP6]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \mid \Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K))) \parallel z = [SC3]$$

$$(\Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K) \mid (r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2)) \parallel z = [SUM7]$$

$$\Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K) \mid (r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2)) \parallel z = [SUM7]$$

$$\Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K \mid (r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2)) \parallel z = [CM9]$$

$$\Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K \mid (r6(m1).x1.x2 + r6.x1.x2) + r2(d,n).(i.s3(d,n) + i.s3).K \mid r6(m2).x2) \parallel z = [ABP37]$$

$$\Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K \mid (r6(m1).x1.x2 + r6.x1.x2) + \delta)) \parallel z = [A6]$$

$$\Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K \mid (r6(m1).x1.x2 + r6.x1.x2)) \parallel z = [CM9]$$

$$\Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K \mid r6(m1).x1.x2 + r2(d,n).(i.s3(d,n) + i.s3).K \mid r6.x1.x2) \parallel z = [ABP36]$$

$$\Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K \mid r6(m1).x1.x2 + \delta)) \parallel z = [A6]$$

$$\Sigma(d:D, \Sigma(n:bit, r2(d,n).(i.s3(d,n) + i.s3).K \mid r6(m1).x1.x2)) \parallel z = [ABP37]$$

$$\Sigma(d:D, \Sigma(n:bit, \delta)) \parallel z = [SUM1]$$

$$\Sigma(d:D, \delta) \parallel z = [SUM1]$$

$$\delta \parallel z = [GEN2]$$

$\delta;$

[ABP117]

$$(((r6(m1) + r6).x1 + r6(m2)).x2 \mid L) \parallel z = [ABP1]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \underline{L}) \underline{L} \underline{L} z = [\text{ABP7}]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \underline{\Sigma}(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L)) \underline{L} z = [\text{SC3}]$$

$$(\underline{\Sigma}(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L) \underline{L}(r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2)) \underline{L} z = [\text{SUM7}]$$

$$\underline{\Sigma}(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L \underline{L}(r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2)) \underline{L} z = [\text{CM9}]$$

$$\underline{\Sigma}(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L \underline{L}(r6(m1).x1.x2 + r6.x1.x2) + r5(n).(i.s6(n) + i.s6).L \underline{L} r6(m2).x2) \underline{L} z = [\text{ABP66}]$$

$$\underline{\Sigma}(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L \underline{L}(r6(m1).x1.x2 + r6.x1.x2) + \delta) \underline{L} z = [\text{A6}]$$

$$\underline{\Sigma}(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L \underline{L}(r6(m1).x1.x2 + r6.x1.x2)) \underline{L} z = [\text{CM9}]$$

$$\underline{\Sigma}(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L \underline{L} r6(m1).x1.x2 + r5(n).(i.s6(n) + i.s6).L \underline{L} r6.x1.x2) \underline{L} z = [\text{ABP65}]$$

$$\underline{\Sigma}(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L \underline{L} r6(m1).x1.x2 + \delta) \underline{L} z = [\text{A6}]$$

$$\underline{\Sigma}(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L \underline{L} r6(m1).x1.x2) \underline{L} z = [\text{ABP66}]$$

$$\underline{\Sigma}(n:\text{bit}, \delta) \underline{L} z = [\text{SUM1}]$$

$$\delta \underline{L} z = [\text{SUM1}]$$

δ ;

[ABP118]

$$(((r6(m1) + r6).x1 + r6(m2).x2) \underline{L}(i.y1 + i.y2).y3) \underline{L} z = [\text{ABP1}]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \underline{L}(i.y1 + i.y2).y3) \underline{L} z = [\text{ABP2}]$$

$$(((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \underline{L}(i.y1.y2 + i.y2.y3)) \underline{L} z = [\text{CM8}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \underline{L}(i.y1.y3 + i.y2.y3) + r6(m2).x2 \underline{L}(i.y1.y3 + i.y2.y3)) \underline{L} z = [\text{CM9}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \underline{L}(i.y1.y3 + i.y2.y3) + (r6(m2).x2 \underline{L} i.y1.y3 + r6(m2).x2 \underline{L} i.y2.y3)) \underline{L} z = [\text{ABP79}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \underline{L}(i.y1.y3 + i.y2.y3) + (r6(m2).x2 \underline{L} i.y1.y3 + \delta)) \underline{L} z = [\text{A6}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \underline{L}(i.y1.y3 + i.y2.y3) + r6(m2).x2 \underline{L} i.y1.y3) \underline{L} z = [\text{ABP79}]$$

$$(((r6(m1).x1.x2 + r6.x1.x2) \underline{L}(i.y1.y3 + i.y2.y3) + \delta) \underline{L} z = [\text{A6}]$$

$$(((r6(m1).x1.x2 + r6.x1.x2) \underline{L}(i.y1.y3 + i.y2.y3)) \underline{L} z = [\text{CM8}]$$

$$(r6(m1).x1.x2 \underline{L}(i.y1.y3 + i.y2.y3) + r6.x1.x2 \underline{L}(i.y1.y3 + i.y2.y3)) \underline{L} z = [\text{CM9}]$$

$$(r6(m1).x1.x2 \underline{L}(i.y1.y3 + i.y2.y3) + (r6.x1.x2 \underline{L} i.y1.y3 + r6.x1.x2 \underline{L} i.y2.y3)) \underline{L} z = [\text{ABP73}]$$

$$(r6(m1).x1.x2 \underline{L}(i.y1.y3 + i.y2.y3) + (r6.x1.x2 \underline{L} i.y1.y3 + \delta)) \underline{L} z = [\text{A6}]$$

$$(r6(m1).x1.x2 \underline{L}(i.y1.y3 + i.y2.y3) + r6.x1.x2 \underline{L} i.y1.y3) \underline{L} z = [\text{ABP73}]$$

$$(r6(m1).x1.x2 \underline{L}(i.y1.y3 + i.y2.y3) + \delta) \underline{L} z = [\text{A6}]$$

$$(r6(m1).x1.x2 \underline{L}(i.y1.y3 + i.y2.y3)) \underline{L} z = [\text{CM9}]$$

$$(r6(m1).x1.x2 \text{ li.y1.y3} + r6(m1).x1.x2 \text{ li.y2.y3}) \parallel z = [\text{ABP79}]$$

$$(r6(m1).x1.x2 \text{ li.y1.y3} + \delta) \parallel z = [\text{A6}]$$

$$(r6(m1).x1.x2 \text{ li.y1.y3}) \parallel z = [\text{ABP79}]$$

$$\delta \parallel z = [\text{GEN2}]$$

δ ;

[ABP119]

$$(((r6(m1) + r6).x1 + r6(m2)).x2 \text{ ls3(d,m).y}) \parallel z = [\text{ABP1}]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \text{ ls3(d,m).y}) \parallel z = [\text{CM8}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ ls3(d,m).y} + r6(m2).x2 \text{ ls3(d,0).y}) \parallel z = [\text{ABP75}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ ls3(d,m).y} + \delta) \parallel z = [\text{A6}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ ls3(d,m).y}) \parallel z = [\text{CM8}]$$

$$(r6(m1).x1.x2 \text{ ls3(d,m).y} + r6.x1.x2 \text{ ls3(d,m).y}) \parallel z = [\text{ABP69}]$$

$$(r6(m1).x1.x2 \text{ ls3(d,m).y} + \delta) \parallel z = [\text{A6}]$$

$$(r6(m1).x1.x2 \text{ ls3(d,m).y}) \parallel z = [\text{ABP75}]$$

$$\delta \parallel z = [\text{GEN2}]$$

δ ;

[ABP120]

$$(((r6(m1) + r6).x1 + r6(m2)).x2 \text{ ls3.y}) \parallel z = [\text{ABP1}]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \text{ ls3.y}) \parallel z = [\text{CM8}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ ls3.y} + r6(m2).x2 \text{ ls3.y}) \parallel z = [\text{ABP74}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ ls3.y} + \delta) \parallel z = [\text{A6}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ ls3.y}) \parallel z = [\text{CM8}]$$

$$(r6(m1).x1.x2 \text{ ls3.y} + r6.x1.x2 \text{ ls3.y}) \parallel z = [\text{ABP68}]$$

$$(r6(m1).x1.x2 \text{ ls3.y} + \delta) \parallel z = [\text{A6}]$$

$$(r6(m1).x1.x2 \text{ ls3.y}) \parallel z = [\text{ABP74}]$$

$$\delta \parallel z = [\text{GEN2}]$$

δ ;

[ABP121]

$$(((r6(m1) + r6).x1 + r6(m2)).x2 \text{ ls4(d).y}) \parallel z = [\text{ABP1}]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \text{ ls4(d).y}) \parallel z = [\text{CM8}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ls4}(d).y + \underline{r6(m2).x2 \text{ls4}(d).y}) \parallel z = [\text{ABP76}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ls4}(d).y + \delta) \parallel z = [\text{A6}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ls4}(d).y) \parallel z = [\text{CM8}]$$

$$(r6(m1).x1.x2 \text{ls4}(d).y + \underline{r6.x1.x2 \text{ls4}(d).y}) \parallel z = [\text{ABP70}]$$

$$(r6(m1).x1.x2 \text{ls4}(d).y + \delta) \parallel z = [\text{A6}]$$

$$(r6(m1).x1.x2 \text{ls4}(d).y) \parallel z = [\text{ABP76}]$$

$$\delta \parallel z = [\text{GEN2}]$$

δ ;

[ABP122]

$$(((r6(m1) + r6).x1 + r6(m2)).x2 \text{ls5}(n).y) \parallel z = [\text{ABP1}]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) \text{ls5}(n).y) \parallel z = [\text{CM8}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ls5}(n).y + \underline{r6(m2).x2 \text{ls5}(n).y}) \parallel z = [\text{ABP77}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ls5}(n).y + \delta) \parallel z = [\text{A6}]$$

$$((r6(m1).x1.x2 + r6.x1.x2) \text{ls5}(n).y) \parallel z = [\text{CM8}]$$

$$(r6(m1).x1.x2 \text{ls5}(n).y + \underline{r6.x1.x2 \text{ls5}(n).y}) \parallel z = [\text{ABP71}]$$

$$(r6(m1).x1.x2 \text{ls5}(n).y + \delta) \parallel z = [\text{A6}]$$

$$(r6(m1).x1.x2 \text{ls5}(n).y) \parallel z = [\text{ABP77}]$$

$$\delta \parallel z = [\text{GEN2}]$$

δ ;

[ABP123]

$$(((r6(\text{invert}(0)) + r6).x1 + r6(0)).x2 \text{ls6}(1).y) \parallel z = [\text{ABP1}]$$

$$((r6(\text{invert}(0)).x1.x2 + r6.x1.x2 + r6(0).x2) \text{ls6}(1).y) \parallel z = [\text{CM8}]$$

$$((r6(\text{invert}(0)).x1.x2 + r6.x1.x2) \text{ls6}(1).y + \underline{r6(0).x2 \text{ls6}(1).y}) \parallel z = [\text{CM7}]$$

$$((r6(\text{invert}(0)).x1.x2 + r6.x1.x2) \text{ls6}(1).y + \underline{r6(0) \text{ls6}(1)}.(x2.y)) \parallel z = [\text{SC3}]$$

$$((r6(\text{invert}(0)).x1.x2 + r6.x1.x2) \text{ls6}(1).y + \underline{s6(1) \text{r6}(0)}.(x2.y)) \parallel z = [\rightarrow E [\text{ABP3} [\text{CF2}'(1)]]]$$

$$((r6(\text{invert}(0)).x1.x2 + r6.x1.x2) \text{ls6}(1).y + \underline{\delta.(x2.y)}) \parallel z = [\text{A7}]$$

$$((r6(\text{invert}(0)).x1.x2 + r6.x1.x2) \text{ls6}(1).y + \delta) \parallel z = [\text{A6}]$$

$$((r6(\text{invert}(0)).x1.x2 + r6.x1.x2) \text{ls6}(1).y) \parallel z = [\text{CM8}]$$

$$(r6(\text{invert}(0)).x1.x2 \text{ls6}(1).y + \underline{r6.x1.x2 \text{ls6}(1).y}) \parallel z = [\text{ABP72}]$$

$$\langle \langle r6(\text{invert}(0)).x1.x2 \text{ } \underline{\text{ls6}(1).y + \delta} \rangle \rangle_z = [\text{A6}]$$

$$\langle \langle r6(\text{invert}(0)).x1.x2 \text{ } \underline{\text{ls6}(1).y} \rangle \rangle_z = [\text{CM7}]$$

$$\langle \langle (r6(\text{invert}(0)) \text{ } \underline{\text{ls6}(1)}).(x1.x2.y) \rangle \rangle_z = [\text{FACT}]$$

$$\langle \langle (r6(1) \text{ } \underline{\text{ls6}(1)}).(x1.x2.y) \rangle \rangle_z = [\text{CF1}']$$

$$\langle \langle c6(1).(x1.x2.y) \rangle \rangle_z = [\text{CM3}]$$

$$c6(1).(\langle \langle x1.x2.y \rangle \rangle_z) = [\text{GEN4}]$$

$$c6(1).(x1.x2.y|z);$$

[ABP124]

$$\langle \langle ((r6(\text{invert}(1)) + r6).x1 + r6(1)).x2 \text{ } \underline{\text{ls6}(0).y} \rangle \rangle_z = [\text{ABP1}]$$

$$\langle \langle (r6(\text{invert}(1)).x1.x2 + r6.x1.x2 + r6(1).x2) \text{ } \underline{\text{ls6}(0).y} \rangle \rangle_z = [\text{CM8}]$$

$$\langle \langle (r6(\text{invert}(1)).x1.x2 + r6.x1.x2) \text{ } \underline{\text{ls6}(0).y} + r6(1).x2 \text{ } \underline{\text{ls6}(0).y} \rangle \rangle_z = [\text{CM7}]$$

$$\langle \langle (r6(\text{invert}(1)).x1.x2 + r6.x1.x2) \text{ } \underline{\text{ls6}(0).y} + (r6(1) \text{ } \underline{\text{ls6}(0)}).(x2.y) \rangle \rangle_z = [\rightarrow \text{E} [\text{ABP3} [\text{CF2}'(1)]]]$$

$$\langle \langle (r6(\text{invert}(1)).x1.x2 + r6.x1.x2) \text{ } \underline{\text{ls6}(0).y} + \underline{\delta.(x2.y)} \rangle \rangle_z = [\text{A7}]$$

$$\langle \langle (r6(\text{invert}(1)).x1.x2 + r6.x1.x2) \text{ } \underline{\text{ls6}(0).y + \delta} \rangle \rangle_z = [\text{A6}]$$

$$\langle \langle (r6(\text{invert}(1)).x1.x2 + r6.x1.x2) \text{ } \underline{\text{ls6}(0).y} \rangle \rangle_z = [\text{CM8}]$$

$$\langle \langle r6(\text{invert}(1)).x1.x2 \text{ } \underline{\text{ls6}(0).y} + r6.x1.x2 \text{ } \underline{\text{ls6}(0).y} \rangle \rangle_z = [\text{ABP72}]$$

$$\langle \langle r6(\text{invert}(1)).x1.x2 \text{ } \underline{\text{ls6}(0).y + \delta} \rangle \rangle_z = [\text{A6}]$$

$$\langle \langle r6(\text{invert}(1)).x1.x2 \text{ } \underline{\text{ls6}(0).y} \rangle \rangle_z = [\text{CM7}]$$

$$\langle \langle r6(\text{invert}(1)) \text{ } \underline{\text{ls6}(0)}).(x1.x2.y) \rangle \rangle_z = [\text{FACT}]$$

$$\langle \langle r6(0) \text{ } \underline{\text{ls6}(0)}).(x1.x2.y) \rangle \rangle_z = [\text{CF1}']$$

$$\langle \langle c6(0).(x1.x2.y) \rangle \rangle_z = [\text{CM3}]$$

$$c6(0).(\langle \langle x1.x2.y \rangle \rangle_z) = [\text{GEN4}]$$

$$c6(0).(x1.x2.y|z);$$

[ABP125]

$$\langle \langle ((r6(\text{invert}(0)) + r6).x1 + r6(0)).x2 \text{ } \underline{\text{ls6}(0).y} \rangle \rangle_z = [\text{ABP1}]$$

$$\langle \langle (r6(\text{invert}(0)).x1.x2 + r6.x1.x2 + r6(0).x2) \text{ } \underline{\text{ls6}(0).y} \rangle \rangle_z = [\text{CM8}]$$

$$\langle \langle (r6(\text{invert}(0)).x1.x2 + r6.x1.x2) \text{ } \underline{\text{ls6}(0).y} + r6(0).x2 \text{ } \underline{\text{ls6}(0).y} \rangle \rangle_z = [\text{CM7}]$$

$$\langle \langle (r6(\text{invert}(0)).x1.x2 + r6.x1.x2) \text{ } \underline{\text{ls6}(0).y} + (r6(0) \text{ } \underline{\text{ls6}(0)}).(x2.y) \rangle \rangle_z = [\text{CF1}']$$

$$\langle \langle (r6(\text{invert}(0)).x1.x2 + r6.x1.x2) \text{ } \underline{\text{ls6}(0).y} + c6(0).(x2.y) \rangle \rangle_z = [\text{CM8}]$$

$$(r6(\text{invert}(0)).x1.x2 \text{ls6}(0).y + r6.x1.x2 \text{ls6}(0).y + c6(0).(x1y))\|z = [\text{ABP72}]$$

$$(r6(\text{invert}(0)).x1.x2 \text{ls6}(0).y + \delta + c6(0).(x2y))\|z = [\text{A6}]$$

$$(r6(\text{invert}(0)).x1.x2 \text{ls6}(0).y + c6(0).(x2y))\|z = [\text{CM7}]$$

$$((r6(\text{invert}(0)) \text{ls6}(0)).(x1.x2y) + c6(0).(x2y))\|z = [\text{FACT}]$$

$$((r6(1) \text{ls6}(0)).(x1.x2y) + c6(0).(x2y))\|z = [\rightarrow E [\text{ABP3} [\text{CF2}'(1)]]]$$

$$(\delta.(x1.x2y) + c6(0).(x2y))\|z = [\text{A7}]$$

$$(\delta + c6(0).(x2y))\|z = [\text{GEN1}]$$

$$c6(0).(x2y)\|z = [\text{CM3}]$$

$$c6(0).((x2y)z) = [\text{GEN4}]$$

$$c6(0).(x2y)z;$$

[ABP126]

$$(((r6(\text{invert}(1)) + r6).x1 + r6(1)).x2 \text{ls6}(1).y)\|z = [\text{ABP1}]$$

$$((r6(\text{invert}(1)).x1.x2 + r6.x1.x2 + r6(1).x2) \text{ls6}(1).y)\|z = [\text{CM8}]$$

$$((r6(\text{invert}(1)).x1.x2 + r6.x1.x2) \text{ls6}(1).y + r6(1).x2 \text{ls6}(1).y)\|z = [\text{CM7}]$$

$$((r6(\text{invert}(1)).x1.x2 + r6.x1.x2) \text{ls6}(1).y + (r6(1) \text{ls6}(1)).(x2y))\|z = [\text{CF1}']$$

$$((r6(\text{invert}(1)).x1.x2 + r6.x1.x2) \text{ls6}(1).y + c6(1).(x2y))\|z = [\text{CM8}]$$

$$(r6(\text{invert}(1)).x1.x2 \text{ls6}(1).y + r6.x1.x2 \text{ls6}(1).y + c6(1).(x1y))\|z = [\text{ABP72}]$$

$$(r6(\text{invert}(1)).x1.x2 \text{ls6}(1).y + \delta + c6(1).(x2y))\|z = [\text{A6}]$$

$$(r6(\text{invert}(1)).x1.x2 \text{ls6}(1).y + c6(1).(x2y))\|z = [\text{CM7}]$$

$$((r6(\text{invert}(1)) \text{ls6}(1)).(x1.x2y) + c6(1).(x2y))\|z = [\text{FACT}]$$

$$((r6(0) \text{ls6}(1)).(x1.x2y) + c6(1).(x2y))\|z = [\text{SC3}]$$

$$((\text{ls6}(1) | r6(0)).(x1.x2y) + c6(1).(x2y))\|z = [\rightarrow E [\text{ABP3} [\text{CF2}'(1)]]]$$

$$(\delta.(x1.x2y) + c6(1).(x2y))\|z = [\text{A7}]$$

$$(\delta + c6(1).(x2y))\|z = [\text{GEN1}]$$

$$c6(1).(x2y)\|z = [\text{CM3}]$$

$$c6(1).((x2y)z) = [\text{GEN4}]$$

$$c6(1).(x2y)z;$$

[ABP127]

$$(((r6(m1) + r6).x1 + r6(m2)).x2 \text{ls6}.y)\|z = [\text{ABP1}]$$

$$((r6(m1).x1.x2 + r6.x1.x2 + r6(m2).x2) | s6.y) \parallel z = [CM8]$$

$$((r6(m1).x1.x2 + r6.x1.x2) | s6.y + r6(m2).x2 | s6.y) \parallel z = [ABP78]$$

$$((r6(m1).x1.x2 + r6.x1.x2) | s6.y + \delta) \parallel z = [A6]$$

$$((r6(m1).x1.x2 + r6.x1.x2) | s6.y) \parallel z = [CM8]$$

$$(r6(m1).x1.x2 | s6.y + r6.x1.x2 | s6.y) \parallel z = [CM7]$$

$$(r6(m1).x1.x2 | s6.y + (r6 | s6).(x1.x2.y)) \parallel z = [CF1]$$

$$(r6(m1).x1.x2 | s6.y + c6.(x1.x2.y)) \parallel z = [ABP78]$$

$$(\delta + c6.(x1.x2.y)) \parallel z = [GEN1]$$

$$c6.(x1.x2.y) \parallel z = [CM3]$$

$$c6.(x1.x2.y) \parallel z = [GEN4]$$

$$c6.(x1.x2.y) \parallel z;$$

[ABP128]

$$(s3(d,m).x | L) \parallel z = [ABP7]$$

$$(s3(d,m).x | \Sigma(n:bit,r5(n).(i.s6(n) + i.s6).L)) \parallel z = [SC3]$$

$$(\Sigma(n:bit,r5(n).(i.s6(n) + i.s6).L) | s3(d,m).x) \parallel z = [SUM7]$$

$$\Sigma(n:bit,r5(n).(i.s6(n) + i.s6).L | s3(d,m).x) \parallel z = [ABP61]$$

$$\Sigma(n:bit,\delta) \parallel z = [SUM1]$$

$$\delta \parallel z = [GEN2]$$

$\delta;$

[ABP129]

$$(s3.x | L) \parallel z = [ABP7]$$

$$(s3.x | \Sigma(n:bit,r5(n).(i.s6(n) + i.s6).L)) \parallel z = [SC3]$$

$$(\Sigma(n:bit,r5(n).(i.s6(n) + i.s6).L) | s3.x) \parallel z = [SUM7]$$

$$\Sigma(n:bit,r5(n).(i.s6(n) + i.s6).L | s3.x) \parallel z = [ABP60]$$

$$\Sigma(n:bit,\delta) \parallel z = [SUM1]$$

$$\delta \parallel z = [GEN2]$$

$\delta;$

[ABP130]

$$(s4(d).x | K) \parallel z = [ABP6]$$

$$(s4(d).x | \Sigma(d:D,\Sigma(n:bit,r2(d,n).(i.s3(d,n) + i.s3).K)) \parallel z = [SC3]$$

74

$$\langle \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3).K)) \mid s4(d,x) \rangle_z = [\text{SUM2}]$$

$$\langle \Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1,n).(i.s3(d1,n) + i.s3).K)) \mid s4(d,x) \rangle_z = [\text{SUM7}]$$

$$\Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1,n).(i.s3(d1,n) + i.s3).K) \mid s4(d,x) \rangle_z = [\text{SUM7}]$$

$$\Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1,n).(i.s3(d1,n) + i.s3).K \mid s4(d,x) \rangle_z = [\text{ABP31}]$$

$$\Sigma(d1:D, \Sigma(n:\text{bit}, \delta)) \rangle_z = [\text{SUM1}]$$

$$\langle \Sigma(d1:D, \delta) \rangle_z = [\text{SUM1}]$$

$$\delta \rangle_z = [\text{GEN2}]$$

$\delta;$

[ABP131]

$$\langle s4(d,x) \mid L \rangle_z = [\text{ABP7}]$$

$$\langle s4(d,x) \mid \Sigma(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L) \rangle_z = [\text{SC3}]$$

$$\langle \Sigma(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L) \mid s4(d,x) \rangle_z = [\text{SUM7}]$$

$$\Sigma(n:\text{bit}, r5(n).(i.s6(n) + i.s6).L \mid s4(d,x) \rangle_z = [\text{ABP62}]$$

$$\langle \Sigma(n:\text{bit}, \delta) \rangle_z = [\text{SUM1}]$$

$$\delta \rangle_z = [\text{GEN2}]$$

$\delta;$

[ABP132]

$$\langle s5(m,x) \mid K \rangle_z = [\text{ABP6}]$$

$$\langle s5(m,x) \mid \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3).K)) \rangle_z = [\text{SC3}]$$

$$\langle \Sigma(d:D, \Sigma(n:\text{bit}, r2(d,n).(i.s3(d,n) + i.s3).K)) \mid s5(m,x) \rangle_z = [\text{SUM2}]$$

$$\langle \Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1,n).(i.s3(d1,n) + i.s3).K)) \mid s5(m,x) \rangle_z = [\text{SUM7}]$$

$$\Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1,n).(i.s3(d1,n) + i.s3).K) \mid s5(m,x) \rangle_z = [\text{SUM7}]$$

$$\Sigma(d1:D, \Sigma(n:\text{bit}, r2(d1,n).(i.s3(d1,n) + i.s3).K \mid s5(m,x) \rangle_z = [\text{ABP32}]$$

$$\Sigma(d1:D, \Sigma(n:\text{bit}, \delta)) \rangle_z = [\text{SUM1}]$$

$$\langle \Sigma(d1:D, \delta) \rangle_z = [\text{SUM1}]$$

$$\delta \rangle_z = [\text{GEN2}]$$

$\delta;$

[ABP133]

$$\begin{bmatrix} [1] \\ n = m \end{bmatrix}$$

$$\triangleright (r5(n) \mid s5(m)).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = [1]$$

$$\underline{(r5(m) \mid s5(m)).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = [CF1]}$$

$$c5(m).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = [1]$$

$$c5(m).((y1.s6(m) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = [A3]$$

$$c5(1).((y1.s6(m) + y2).y3ix)$$

] [→I]

$$n = m \rightarrow (r5(n) \mid s5(m)).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = c5(m).((y1.s6(m) + y2).y3ix);$$

[ABP134]

[[1]

$$\neg(n = m)$$

$$\underline{(r5(n) \mid s5(m)).z1 + z2 = [→E 1 CF2]}$$

$$\underline{\delta.z1 + z2 = [A7]}$$

$$\underline{\delta + z2 = [GEN1]}$$

$$z2$$

]]

$$\neg(n = m) \rightarrow (r5(n) \mid s5(m)).z1 + z2 = z2;$$

[ABP135]

[[1]

$$\neg((r5(n) \mid s5(m)).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = c5(m).((y1.s6(m) + y2).y3ix))$$

\triangleright [2]

[[3]

$$n = m$$

\triangleright [4]

$$(r5(n) \mid s5(m)).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = [→E [3] [ABP133]]$$

$$c5(m).((y1.s6(m) + y2).y3ix)$$

$$\perp [→E [4] [1]]$$

] [→I]

$$\neg(n = m);$$

[5]

$$\underline{(r5(n) \mid s5(m)).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = [→E [2] [ABP134]]}$$

$$c5(m).((y1.s6(m) + y2).y3ix);$$

$$\perp [→E [5] [1]]$$

] [RAA]

$$(r5(n) \mid s5(m)).((y1.s6(n) + y2).y3ix) + c5(m).((y1.s6(m) + y2).y3ix) = c5(m).((y1.s6(m) + y2).y3ix);$$

[ABP136]

$$(s5(m).x \mid \underline{\perp}) \mid z = [ABP7]$$

$$(s5(m).x \mid \underline{\Sigma(n:\text{bit}, r5(n).(i.s6(n) + i.s6(L))}) \mid z = [SC3]$$

$$\underline{\Sigma(n:\text{bit},r5(n).(i.s6(n) + i.s6).L) \lfloor s5(m).x \rfloor}_z = [\text{SUM7}]$$

$$\Sigma(n:\text{bit},r5(n).(i.s6(n) + i.s6).L \lfloor s5(m).x \rfloor)_z = [\text{CM7}]$$

$$\underline{\Sigma(n:\text{bit},(r5(n) \lfloor s5(m) \rfloor).(i.s6(n) + i.s6).L \lfloor x \rfloor)}_z = [\text{SUM3}]$$

$$(\Sigma(n:\text{bit},(r5(n) \lfloor s5(m) \rfloor).(i.s6(n) + i.s6).L \lfloor x \rfloor) + (r5(m) \lfloor s5(m) \rfloor).(i.s6(m) + i.s6).L \lfloor x \rfloor))_z = [\text{CF1}']$$

$$(\Sigma(n:\text{bit},(r5(n) \lfloor s5(m) \rfloor).(i.s6(n) + i.s6).L \lfloor x \rfloor) + c5(m).(i.s6(m) + i.s6).L \lfloor x \rfloor))_z = [\text{SUM1}]$$

$$(\Sigma(n:\text{bit},(r5(n) \lfloor s5(m) \rfloor).(i.s6(n) + i.s6).L \lfloor x \rfloor) + \Sigma(n:\text{bit},c5(m).(i.s6(m) + i.s6).L \lfloor x \rfloor))_z = [\text{SUM4}]$$

$$\Sigma(n:\text{bit},(r5(n) \lfloor s5(m) \rfloor).(i.s6(n) + i.s6).L \lfloor x \rfloor + c5(m).(i.s6(m) + i.s6).L \lfloor x \rfloor))_z = [\text{ABP135}]$$

$$\underline{\Sigma(n:\text{bit},c5(m).(i.s6(m) + i.s6).L \lfloor x \rfloor)}_z = [\text{SUM1}]$$

$$c5(m).(i.s6(m) + i.s6).L \lfloor x \rfloor)_z = [\text{GEN3}]$$

$$c5(m).(x \lfloor i.s6(m) + i.s6 \rfloor.L)_z = [\text{CM3}]$$

$$c5(m).(x \lfloor i.s6(m) + i.s6 \rfloor.L)_z = [\text{GEN4}]$$

$$c5(m).(x \lfloor i.s6(m) + i.s6 \rfloor.L)_z;$$

[ABP137]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, S(m).x \lfloor R(n).y \lfloor K \rfloor L)) = [\text{EXP4}]$$

$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$S(m).x \lfloor (R(n).y \lfloor K \rfloor L) + R(n).y \lfloor (S(m).x \lfloor K \rfloor L) + K \lfloor (S(m).x \lfloor R(n).y \rfloor L) + L \lfloor (S(m).x \lfloor R(n).y \rfloor K) + (S(m).x \lfloor R(n).y \rfloor \lfloor (K \rfloor L) + (S(m).x \lfloor K \rfloor \lfloor (R(n).y \rfloor L) + (S(m).x \lfloor L \rfloor \lfloor (K \rfloor R(n).y) + (R(n).y \lfloor K \rfloor \lfloor (S(m).x \rfloor L) + (R(n).y \rfloor L \lfloor (S(m).x \rfloor K) + (K \rfloor L \lfloor (S(m).x \rfloor R(n).y)) = [\text{ABP101}]$$

$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$S(m).x \lfloor (R(n).y \lfloor K \rfloor L) + R(n).y \lfloor (S(m).x \lfloor K \rfloor L) + K \lfloor (S(m).x \lfloor R(n).y \rfloor L) + L \lfloor (S(m).x \lfloor R(n).y \rfloor K) + (S(m).x \lfloor R(n).y \rfloor \lfloor (K \rfloor L) + (S(m).x \lfloor K \rfloor \lfloor (R(n).y \rfloor L) + (S(m).x \lfloor L \rfloor \lfloor (K \rfloor R(n).y) + (R(n).y \lfloor K \rfloor \lfloor (S(m).x \rfloor L) + (R(n).y \rfloor L \lfloor (S(m).x \rfloor K) + \delta) = [\text{A6}]$$

$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$S(m).x \lfloor (R(n).y \lfloor K \rfloor L) + R(n).y \lfloor (S(m).x \lfloor K \rfloor L) + K \lfloor (S(m).x \lfloor R(n).y \rfloor L) + L \lfloor (S(m).x \lfloor R(n).y \rfloor K) + (S(m).x \lfloor R(n).y \rfloor \lfloor (K \rfloor L) + (S(m).x \lfloor K \rfloor \lfloor (R(n).y \rfloor L) + (S(m).x \lfloor L \rfloor \lfloor (K \rfloor R(n).y) + (R(n).y \lfloor K \rfloor \lfloor (S(m).x \rfloor L) + (R(n).y \rfloor L \lfloor (S(m).x \rfloor K)) = [\text{ABP84}]$$

$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$S(m).x \lfloor (R(n).y \lfloor K \rfloor L) + R(n).y \lfloor (S(m).x \lfloor K \rfloor L) + K \lfloor (S(m).x \lfloor R(n).y \rfloor L) + L \lfloor (S(m).x \lfloor R(n).y \rfloor K) + (S(m).x \lfloor R(n).y \rfloor \lfloor (K \rfloor L) + (S(m).x \lfloor K \rfloor \lfloor (R(n).y \rfloor L) + (S(m).x \lfloor L \rfloor \lfloor (K \rfloor R(n).y) + (R(n).y \lfloor K \rfloor \lfloor (S(m).x \rfloor L) + \delta) = [\text{A6}]$$

$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$S(m).x \lfloor (R(n).y \lfloor K \rfloor L) + R(n).y \lfloor (S(m).x \lfloor K \rfloor L) + K \lfloor (S(m).x \lfloor R(n).y \rfloor L) + L \lfloor (S(m).x \lfloor R(n).y \rfloor K) + (S(m).x \lfloor R(n).y \rfloor \lfloor (K \rfloor L) + (S(m).x \lfloor K \rfloor \lfloor (R(n).y \rfloor L) + (S(m).x \lfloor L \rfloor \lfloor (K \rfloor R(n).y) + (R(n).y \lfloor K \rfloor \lfloor (S(m).x \rfloor L)) = [\text{ABP83}]$$

$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$S(m).x \lfloor (R(n).y \lfloor K \rfloor L) + R(n).y \lfloor (S(m).x \lfloor K \rfloor L) + K \lfloor (S(m).x \lfloor R(n).y \rfloor L) + L \lfloor (S(m).x \lfloor R(n).y \rfloor K) + (S(m).x \lfloor R(n).y \rfloor \lfloor (K \rfloor L) + (S(m).x \lfloor K \rfloor \lfloor (R(n).y \rfloor L) + (S(m).x \lfloor L \rfloor \lfloor (K \rfloor R(n).y) + \delta) = [\text{A6}]$$

$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},$

$$S(m).x \lfloor (R(n).y \lfloor K \rfloor L) + R(n).y \lfloor (S(m).x \lfloor K \rfloor L) + K \lfloor (S(m).x \lfloor R(n).y \rfloor L) + L \lfloor (S(m).x \lfloor R(n).y \rfloor K) +$$

$$(S(m).x | R(n).y) \ll (K_i L) + (S(m).x | K) \ll (R(n).y_i L) + (S(m).x | L) \ll (K_i | R(n).y)) = [ABP82]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ \frac{S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L) + K \ll (S(m).x_i R(n).y_i L) + L \ll (S(m).x_i R(n).y_i K) + (S(m).x | R(n).y) \ll (K_i L) + (S(m).x | K) \ll (R(n).y_i L) + \delta}{}) = [A6]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L) + K \ll (S(m).x_i R(n).y_i L) + L \ll (S(m).x_i R(n).y_i K) + (S(m).x | R(n).y) \ll (K_i L) + (S(m).x | K) \ll (R(n).y_i L)) = [ABP81]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ \frac{S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L) + K \ll (S(m).x_i R(n).y_i L) + L \ll (S(m).x_i R(n).y_i K) + (S(m).x | R(n).y) \ll (K_i L) + \delta}{}) = [A6]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L) + K \ll (S(m).x_i R(n).y_i L) + L \ll (S(m).x_i R(n).y_i K) + (S(m).x | R(n).y) \ll (K_i L)) = [ABP80]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L) + K \ll (S(m).x_i R(n).y_i L) + L \ll (S(m).x_i R(n).y_i K) + \delta) = [A6]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ \frac{S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L) + K \ll (S(m).x_i R(n).y_i L) + L \ll (S(m).x_i R(n).y_i K)}{}) = [D3]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L) + K \ll (S(m).x_i R(n).y_i L)) + \\ \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, L \ll (S(m).x_i R(n).y_i K)) = [ABP27]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L) + K \ll (S(m).x_i R(n).y_i L)) + \delta = [A6]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L) + K \ll (S(m).x_i R(n).y_i L)) = [D3]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L)) + \\ \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, K \ll (S(m).x_i R(n).y_i L)) = [ABP26]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L)) + \delta = [A6]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(m).x \ll (R(n).y_i K_i L) + R(n).y \ll (S(m).x_i K_i L)) = [D3]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(m).x \ll (R(n).y_i K_i L)) + \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, R(n).y \ll (S(m).x_i K_i L)) = [ABP25]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(m).x \ll (R(n).y_i K_i L)) + \delta = [A6]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(m).x \ll (R(n).y_i K_i L)) = [ABP24]$$

$$\Sigma(d:D, r_1(d)). \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(d, m).x_i R(n).y_i K_i L);$$

[ABP138]

\underline{X} = [REC]

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S_i R_i K_i L) = [REC]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(0).S(1).S_i R_i K_i L) = [REC]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(0).S(1).S_i R(1).R(0).R_i K_i L) = [ABP137]$$

$$\Sigma(d:D, r_1(d)). \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, S(d, 0).S(1).S_i R(1).R(0).R_i K_i L) = [REC]$$

[ABP141]

$$X1(d) = [\text{REC}]$$

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S(d, 0).S(1).S_i R_i K_i L) = [\text{REC}]$$

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S(d, 0).S(1).S_i R(1).R(0).R_i K_i L) = [\text{ABP140}]$$

$$c2(d, 0). \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, ((r6(\text{invert}(0)) + r6).S(d, 0) + r6(0)).S(1).S_i R(1).R(0).R_i(i.s3(d, 0) + i.s3).K_i L);$$

[ABP142]

$$Y1(d) = [\text{REC}]$$

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S(d, 1).S_i R(0).R_i K_i L) = [\text{ABP140}]$$

$$c2(d, 1). \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, ((r6(\text{invert}(1)) + r6).S(d, 1) + r6(1)).S_i R(0).R_i(i.s3(d, 1) + i.s3).K_i L);$$

[ABP143]

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, ((r6(m1) + r6).x1 + r6(m2)).x2 R(n).y_i(i.z1 + i.z2).z3_i L) = [\text{EXP4}]$$

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\},$$

$$\begin{aligned} & ((r6(m1) + r6).x1 + r6(m2)).x2 \parallel (R(n).y_i(i.z1 + i.z2).z3_i L) + \\ & R(n).y_i \parallel (((r6(m1) + r6).x1 + r6(m2)).x2(i.z1 + i.z2).z3_i L) + \\ & (i.z1 + i.z2).z3_i \parallel (((r6(m1) + r6).x1 + r6(m2)).x2 R(n).y_i L) + \\ & L \parallel (((r6(m1) + r6).x1 + r6(m2)).x2 R(n).y_i(i.z1 + i.z2).z3) + \\ & (((r6(m1) + r6).x1 + r6(m2)).x2 \parallel R(n).y_i \parallel (i.z1 + i.z2).z3_i L) + \\ & (((r6(m1) + r6).x1 + r6(m2)).x2 \parallel (i.z1 + i.z2).z3) \parallel (R(n).y_i L) + \\ & (((r6(m1) + r6).x1 + r6(m2)).x2 \parallel L) \parallel (R(n).y_i(i.z1 + i.z2).z3) + \\ & (R(n).y_i \parallel (i.z1 + i.z2).z3) \parallel (((r6(m1) + r6).x1 + r6(m2)).x2_i L) + \\ & (R(n).y_i \parallel L) \parallel (((r6(m1) + r6).x1 + r6(m2)).x2(i.z1 + i.z2).z3) + \\ & \underline{(i.z1 + i.z2).z3_i L \parallel (((r6(m1) + r6).x1 + r6(m2)).x2 R(n).y_i)}) = [\text{ABP114}] \end{aligned}$$

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\},$$

$$\begin{aligned} & ((r6(m1) + r6).x1 + r6(m2)).x2 \parallel (R(n).y_i(i.z1 + i.z2).z3_i L) + \\ & R(n).y_i \parallel (((r6(m1) + r6).x1 + r6(m2)).x2(i.z1 + i.z2).z3_i L) + \\ & (i.z1 + i.z2).z3_i \parallel (((r6(m1) + r6).x1 + r6(m2)).x2 R(n).y_i L) + \\ & L \parallel (((r6(m1) + r6).x1 + r6(m2)).x2 R(n).y_i(i.z1 + i.z2).z3) + \\ & (((r6(m1) + r6).x1 + r6(m2)).x2 \parallel R(n).y_i \parallel (i.z1 + i.z2).z3_i L) + \\ & (((r6(m1) + r6).x1 + r6(m2)).x2 \parallel (i.z1 + i.z2).z3) \parallel (R(n).y_i L) + \\ & (((r6(m1) + r6).x1 + r6(m2)).x2 \parallel L) \parallel (R(n).y_i(i.z1 + i.z2).z3) + \\ & (R(n).y_i \parallel (i.z1 + i.z2).z3) \parallel (((r6(m1) + r6).x1 + r6(m2)).x2_i L) + \\ & \underline{(R(n).y_i \parallel L) \parallel (((r6(m1) + r6).x1 + r6(m2)).x2(i.z1 + i.z2).z3) + \delta)}) = [\text{A6}] \end{aligned}$$

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\},$$

$$\begin{aligned} & ((r6(m1) + r6).x1 + r6(m2)).x2 \parallel (R(n).y_i(i.z1 + i.z2).z3_i L) + \\ & R(n).y_i \parallel (((r6(m1) + r6).x1 + r6(m2)).x2(i.z1 + i.z2).z3_i L) + \\ & (i.z1 + i.z2).z3_i \parallel (((r6(m1) + r6).x1 + r6(m2)).x2 R(n).y_i L) + \\ & L \parallel (((r6(m1) + r6).x1 + r6(m2)).x2 R(n).y_i(i.z1 + i.z2).z3) + \\ & (((r6(m1) + r6).x1 + r6(m2)).x2 \parallel R(n).y_i \parallel (i.z1 + i.z2).z3_i L) + \\ & (((r6(m1) + r6).x1 + r6(m2)).x2 \parallel (i.z1 + i.z2).z3) \parallel (R(n).y_i L) + \\ & (((r6(m1) + r6).x1 + r6(m2)).x2 \parallel L) \parallel (R(n).y_i(i.z1 + i.z2).z3) + \\ & (R(n).y_i \parallel (i.z1 + i.z2).z3) \parallel (((r6(m1) + r6).x1 + r6(m2)).x2_i L) + \\ & \underline{(R(n).y_i \parallel L) \parallel (((r6(m1) + r6).x1 + r6(m2)).x2(i.z1 + i.z2).z3)}) = [\text{ABP84}] \end{aligned}$$

$$\partial(\{r2, r3, r5, r6, s2, s3, s5, s6\},$$

$$\underline{((r6(m1) + r6).x1 + r6(m2)).x2 \parallel (R(n).y_i(i.z1 + i.z2).z3_i L) +$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 0), (s_4(d), s_5(0), R(0), R_i(z|L))((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2) = [\text{GEN4}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 0), ((s_4(d), s_5(0), R(0), R_i(z|L))((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2) = [\text{GEN3}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 0), (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_4(d), s_5(0), R(0), R_i(z|L) = [\text{D4}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 0)).$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_4(d), s_5(0), R(0), R_i(z|L) = [\text{D1}]$$

$$c_3(d, 0), \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_4(d), s_5(0), R(0), R_i(z|L);$$

[ABP146]

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2; R(0), y; s_3(d, 1), z|L) = [\text{ABP144}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, (R(0), y; s_3(d, 1), z|L))(((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2; L) = [\text{ABP92}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 1), (s_4(d), s_5(1), y; z|L))(((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2; L) = [\text{GEN3}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 1), (s_4(d), s_5(1), y; z|L))(((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2) = [\text{GEN4}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 1), (s_4(d), s_5(1), y; z|L))(((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2) = [\text{GEN4}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 1), ((s_4(d), s_5(1), y; z|L))(((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2) = [\text{GEN3}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 1), (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_4(d), s_5(1), y; z|L) = [\text{D4}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_3(d, 1)).$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2; s_4(d), s_5(1), y; z|L) = [\text{D1}]$$

$$c_3(d, 1), \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2; s_4(d), s_5(1), y; z|L);$$

[ABP147]

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2; R(n), y; s_3, z|L) = [\text{EXP4}]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\},$$

$$\begin{aligned} & ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_1; (R(n), y; s_3, z|L) + R(n), y; (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_3, z|L) + \\ & s_3, z|L; (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y; L) + L; (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y; s_3, z) + \\ & (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y; (s_3, z|L) + (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_3, z; (R(n), y; L) + \\ & (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); L; (R(n), y; s_3, z) + (R(n), y; s_3, z); (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); L) + \\ & (R(n), y; L); (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_3, z) + (s_3, z; L); (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y)) = [\text{ABP129}] \end{aligned}$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\},$$

$$\begin{aligned} & ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_1; (R(n), y; s_3, z|L) + R(n), y; (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_3, z|L) + \\ & s_3, z|L; (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y; L) + L; (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y; s_3, z) + \\ & (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y; (s_3, z|L) + (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_3, z; (R(n), y; L) + \\ & (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); L; (R(n), y; s_3, z) + (R(n), y; s_3, z); (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); L) + \\ & (R(n), y; L); (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_3, z) + \delta) = [\text{A6}] \end{aligned}$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\},$$

$$\begin{aligned} & ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_1; (R(n), y; s_3, z|L) + R(n), y; (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_3, z|L) + \\ & s_3, z|L; (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y; L) + L; (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y; s_3, z) + \\ & (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); R(n), y; (s_3, z|L) + (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_3, z; (R(n), y; L) + \\ & (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); L; (R(n), y; s_3, z) + (R(n), y; s_3, z); (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); L) + \\ & (R(n), y; L); (((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2); s_3, z)) = [\text{ABP84}] \end{aligned}$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\},$$

$$\begin{aligned} & \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ & \quad \frac{((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel (R(n).y_i K_{i s_6}.w) + R(n).y_i \parallel ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel K_{i s_6}.w) + \\ & \quad K \parallel ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel R(n).y_i s_6.w)}{((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel s_6.w)} \parallel (R(n).y_i K)) = [D3] \end{aligned}$$

$$\begin{aligned} & \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ & \quad ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel (R(n).y_i K_{i s_6}.w) + R(n).y_i \parallel ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel K_{i s_6}.w)) + \\ & \quad \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, K \parallel ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel R(n).y_i s_6.w)) + \\ & \quad \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel s_6.w) \parallel (R(n).y_i K)) = [ABP26] \end{aligned}$$

$$\begin{aligned} & \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ & \quad \frac{((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel (R(n).y_i K_{i s_6}.w) + R(n).y_i \parallel ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel K_{i s_6}.w)) + \delta}{((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel s_6.w)} \parallel (R(n).y_i K)) = [A6] \end{aligned}$$

$$\begin{aligned} & \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ & \quad \frac{((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel (R(n).y_i K_{i s_6}.w) + R(n).y_i \parallel ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel K_{i s_6}.w)) +}{((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel s_6.w)} \parallel (R(n).y_i K)) = [D3] \end{aligned}$$

$$\begin{aligned} & \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel (R(n).y_i K_{i s_6}.w)) + \\ & \quad \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, R(n).y_i \parallel ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel K_{i s_6}.w)) + \\ & \quad \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel s_6.w) \parallel (R(n).y_i K)) = [ABP25] \end{aligned}$$

$$\begin{aligned} & \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel (R(n).y_i K_{i s_6}.w)) + \delta + \\ & \quad \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel s_6.w) \parallel (R(n).y_i K)) = [A6] \end{aligned}$$

$$\begin{aligned} & \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel (R(n).y_i K_{i s_6}.w)) + \\ & \quad \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel s_6.w) \parallel (R(n).y_i K)) = [ABP29] \end{aligned}$$

$$\delta + \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel s_6.w) \parallel (R(n).y_i K)) = [GEN1]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(m_1) + r_6).x_1 + r_6(m_2)).x_2 \parallel s_6.w) \parallel (R(n).y_i K)) = [ABP127]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_6.(x_1.x_2 \parallel w \parallel R(n).y_i K)) = [GEN3]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_6.(x_1.x_2 \parallel (R(n).y_i K) \parallel w)) = [GEN4]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_6.(x_1.x_2 \parallel R(n).y_i K \parallel w)) = [D4]$$

$$\partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, c_6) \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x_1.x_2 \parallel R(n).y_i K \parallel w) = [D1]$$

$$c_6 \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, x_1.x_2 \parallel R(n).y_i K \parallel w);$$

[ABP155]

$$X1(d) = [ABP141]$$

$$\begin{aligned} & c_2(d, 0) \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \\ & \quad \frac{((r_6(\text{invert}(0)) + r_6).S(d, 0) + r_6(0)).S(1).S_i R(1).R(0).R_i(i.s_3(d, 0) + i.s_3).K_i L)}{((r_6(\text{invert}(0)) + r_6).S(d, 0) + r_6(0)).S(1).S_i R(1).R(0).R_i s_3(d, 0).K_i L)} = [ABP143] \end{aligned}$$

c2(d, 0).

$$\begin{aligned} & (i \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(\text{invert}(0)) + r_6).S(d, 0) + r_6(0)).S(1).S_i R(1).R(0).R_i s_3(d, 0).K_i L) + \\ & \quad i \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(\text{invert}(0)) + r_6).S(d, 0) + r_6(0)).S(1).S_i R(1).R(0).R_i s_3.K_i L)) = [ABP145] \end{aligned}$$

c2(d, 0).

$$\begin{aligned} & (i \cdot c_3(d, 0) \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, \frac{((r_6(\text{invert}(0)) + r_6).S(d, 0) + r_6(0)).S(1).S_i s_4(d).s_5(0).R(0).R_i K_i L}{((r_6(\text{invert}(0)) + r_6).S(d, 0) + r_6(0)).S(1).S_i R(1).R(0).R_i s_3.K_i L)} + \\ & \quad i \cdot \partial(\{r_2, r_3, r_5, r_6, s_2, s_3, s_5, s_6\}, ((r_6(\text{invert}(0)) + r_6).S(d, 0) + r_6(0)).S(1).S_i R(1).R(0).R_i s_3.K_i L)) = [ABP148] \end{aligned}$$

c2(d,0).

$$(i.c3(d,0).s4(d).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{i5}(0).R(0).R_{iK}L) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{iR}(1).R(0).R_{iS3}.K_{iL})) = [\text{REC}]$$

c2(d,0).

$$(i.c3(d,0).s4(d).X2(d) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{iR}(1).R(0).R_{iS3}.K_{iL})) = [\text{ABP147}]$$

c2(d,0).

$$(i.c3(d,0).s4(d).X2(d) + i.c3.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{iS5}(1).R(1).R(0).R_{iK}L)) = [\text{ABP149}]$$

c2(d,0).

$$(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{iR}(1).R(0).R_{iK}(i.s6(1) + i.s6).L)) = [\text{ABP150}]$$

c2(d,0).

$$(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{iR}(1).R(0).R_{iK}S6(1).L) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{iR}(1).R(0).R_{iK}S6.L)) = [\text{ABP152}]$$

c2(d,0).

$$(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S(d,0).S(1).S_{iR}(1).R(0).R_{iK}L) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{iR}(1).R(0).R_{iK}S6.L)) = [\text{REC}]$$

c2(d,0).

$$(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S(d,0).S(1).S_{iR}K_{iL}) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{iR}(1).R(0).R_{iK}S6.L)) = [\text{REC}]$$

c2(d,0).

$$(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).(i.c6(1).X1(d) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_{iR}(1).R(0).R_{iK}S6.L)) = [\text{ABP154}]$$

c2(d,0).

$$(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).(i.c6(1).X1(d) + i.c6.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S(d,0).S(1).S_{iR}(1).R(0).R_{iK}L)) = [\text{REC}]$$

c2(d,0).

$$(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).(i.c6(1).X1(d) + i.c6.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S(d,0).S(1).S_{iR}K_{iL})) = [\text{REC}]$$

$$c2(d,0).(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).(i.c6(1).X1(d) + i.c6.X1(d))) = [\text{ABP2}]$$

$$c2(d,0).(i.c3(d,0).s4(d).X2(d) + i.c3.c5(1).(i.c6(1) + i.c6).X1(d)) = [\text{A1}]$$

$$c2(d,0).(i.c3.c5(1).(i.c6(1) + i.c6).X1(d) + i.c3(d,0).s4(d).X2(d)) = [\text{ABP2}]$$

$$c2(d,0).(i.c3.c5(1).(i.c6(1).X1(d) + i.c6.X1(d)) + i.c3(d,0).s4(d).X2(d));$$

[ABP156]

 $Y1(d) = [ABP142]$

$$c2(d,1). \partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, ((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri(i.s3(d,1) + i.s3).KiL)) = [ABP143]$$

$$c2(d,1). (i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.s3(d,1).KiL) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.s3.KiL)) = [ABP146]$$

$$c2(d,1). (i.c3(d,1).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\}, ((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).Si.s4(d).s5(1).Ri.KiL) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.s3.KiL)) = [ABP148]$$

$$c2(d,1). (i.c3(d,1).s4(d).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).Si.s5(1).Ri.KiL) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.s3.KiL)) = [REC]$$

$$c2(d,1). (i.c3(d,1).s4(d).Y2(d) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.s3.KiL)) = [ABP147]$$

$$c2(d,1). (i.c3(d,1).s4(d).Y2(d) + i.c3.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).Si.s5(0).R(0).Ri.KiL)) = [ABP149]$$

$$c2(d,1). (i.c3(d,1).s4(d).Y2(d) + i.c3.c5(0).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.Ki(i.s6(0) + i.s6).L)) = [ABP150]$$

$$c2(d,1). (i.c3(d,1).s4(d).Y2(d) + i.c3.c5(0). (i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.Ki.s6(0).L) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.Ki.s6.L)) = [ABP153]$$

$$c2(d,1). (i.c3(d,1).s4(d).Y2(d) + i.c3.c5(0). (i.c6(0).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S(d,1).SiR(0).Ri.KiL) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.Ki.s6.L)) = [REC]$$

$$c2(d,1). (i.c3(d,1).s4(d).Y2(d) + i.c3.c5(0). (i.c6(0).Y1(d) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(1)) + r6).S(d,1) + r6(1)).SiR(0).Ri.Ki.s6.L)) = [ABP154]$$

$$c2(d,1). (i.c3(d,1).s4(d).Y2(d) + i.c3.c5(0). (i.c6(0).Y1(d) + i.c6.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S(d,1).SiR(0).Ri.KiL)) = [REC]$$

$$c2(d,1).(i.c3(d,1).s4(d).Y2(d) + i.c3.c5(0).(i.c6(0).Y1(d) + i.c6.Y1(d))) = [ABP2]$$

$$c2(d,1).(i.c3(d,1).s4(d).Y2(d) + i.c3.c5(0).(i.c6(0) + i.c6).Y1(d)) = [A1]$$

$$c2(d,1).(i.c3.c5(0).(i.c6(0) + i.c6).Y1(d) + i.c3(d,1).s4(d).Y2(d)) = [ABP2]$$

$$c2(d,1).(i.c3.c5(0).(i.c6(0).Y1(d) + i.c6.Y1(d)) + i.c3(d,1).s4(d).Y2(d));$$

[ABP157]

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(m1) + r6).S(d,m2) + r6(m3)).x_i s5(n1).R(n2).y_i K_i L) = [ABP149]$$

$$c5(n1).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(m1) + r6).S(d,m2) + r6(m3)).x_i R(n2).y_i K_i(i.s6(n) + i.s6).L) = [ABP150]$$

c5(n1).

$$(i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(m1) + r6).S(d,m2) + r6(m3)).x_i R(n2).y_i K_i s6(n).L) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(m1) + r6).S(d,m2) + r6(m3)).x_i R(n2).y_i K_i s6.L)) = [ABP154]$$

c5(n1).

$$(i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(m1) + r6).S(d,m2) + r6(m3)).x_i R(n2).y_i K_i s6(n).L) + i.c6.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S(d,m2).x_i R(n2).y_i K_i L)) = [ABP140]$$

c5(n1).

$$(i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(m1) + r6).S(d,m2) + r6(m3)).x_i R(n2).y_i K_i s6(n).L) + i.c6.c2(d,m2).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(m2)) + r6).S(d,m2) + r6(m2)).x_i R(n2).y_i(i.s3(d,m2) + i.s3).K_i L)) = [ABP143]$$

c5(n1).

$$(i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(m1) + r6).S(d,m2) + r6(m3)).x_i R(n2).y_i K_i s6(n).L) + i.c6.c2(d,m2). (i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(m2)) + r6).S(d,m2) + r6(m2)).x_i R(n2).y_i s3(d,m2).K_i L) + i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(m2)) + r6).S(d,m2) + r6(m2)).x_i R(n2).y_i s3.K_i L))) = [ABP147]$$

c5(n1).

$$(i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(m1) + r6).S(d,m2) + r6(m3)).x_i R(n).y_i K_i s6(n2).L) + i.c6.c2(d,m2). (i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(m2)) + r6).S(d,m2) + r6(m2)).x_i R(n2).y_i s3(d,m2).K_i L) + i.c3.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(m2)) + r6).S(d,m2) + r6(m2)).x_i s5(n2).R(n2).y_i K_i L))) = [ABP144]$$

c5(n1).

$$(i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(m1) + r6).S(d,m2) + r6(m3)).x_i R(n).y_i K_i s6(n2).L) + i.c6.c2(d,m2). (i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},(R(n2).y_i s3(d,m2).K_i L)((r6(\text{invert}(m2)) + r6).S(d,m2) + r6(m2)).x_i L)) + i.c3.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(m2)) + r6).S(d,m2) + r6(m2)).x_i s5(n2).R(n2).y_i K_i L))) = [ABP151]$$

c5(m1).

$$(i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},(((r6(m1) + r6).S(d,m2) + r6(m3)).x_i s6(n).L)\|(R(n2).y_i K_i) + i.c6.c2(d,m2). (i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},(R(n2).y_i s3(d,m2).K_i L)((r6(\text{invert}(m2)) + r6).S(d,m2) + r6(m2)).x_i L)) + i.c3.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(m2)) + r6).S(d,m2) + r6(m2)).x_i s5(n2).R(n2).y_i K_i L)));$$

[ABP158]

$$\underline{X2(d)} = [REC]$$

$$\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_i s5(0).R(0).R_i K_i L) = [ABP157]$$

c5(0).

$$(i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},(((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_i s6(0).L)\|(R(0).R_i K_i)) + i.c6.c2(d,0). (i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},(R(0).R_i s3(d,0).K_i L)((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_i L)) + i.c3.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},((r6(\text{invert}(0)) + r6).S(d,0) + r6(0)).S(1).S_i s5(0).R(0).R_i K_i L))) = [REC]$$

c5(0).

c5(0).

$$(i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},c_6(0).(S(1).S_i\llcorner R(0).R_iK)) + i.c_6.c_2(d,0).(i.c_3(d,0) + i.c_3).X_2(d)) = [\text{GEN3}]$$

c5(0).

$$(i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},c_6(0).(S(1).S_i\llcorner(R(0).R_iK)\llcorner L)) + i.c_6.c_2(d,0).(i.c_3(d,0) + i.c_3).X_2(d)) = [\text{GEN4}]$$

c5(0).

$$(i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},c_6(0).(S(1).S_iR(0).R_iK\llcorner L)) + i.c_6.c_2(d,0).(i.c_3(d,0) + i.c_3).X_2(d)) = [\text{D4}]$$

c5(0).

$$(i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},c_6(0)).\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},S(1).S_iR(0).R_iK\llcorner L) + i.c_6.c_2(d,0).(i.c_3(d,0) + i.c_3).X_2(d)) = [\text{REC}]$$

$$c_5(0).(i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},c_6(0)).Y + i.c_6.c_2(d,0).(i.c_3(d,0) + i.c_3).X_2(d)) = [\text{D1}]$$

$$c_5(0).(i.c_6(0).Y + i.c_6.c_2(d,0).(i.c_3(d,0) + i.c_3).X_2(d)) = [\text{A1}]$$

$$c_5(0).(i.c_6.c_2(d,0).(i.c_3(d,0) + i.c_3).X_2(d) + i.c_6(0).Y) = [\text{ABP2}]$$

$$c_5(0).(i.c_6.c_2(d,0).(i.c_3(d,0).X_2(d) + i.c_3.X_2(d)) + i.c_6(0).Y);$$

[ABP159]

$$Y_2(d) = [\text{REC}]$$

$$\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S_i s_5(1).R_iK\llcorner L) = [\text{REC}]$$

$$\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S_i s_5(1).R(1).R(0).R_iK\llcorner L) = [\text{ABP157}]$$

c5(1).

$$(i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},(((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S \llcorner s_6(1).L)\llcorner(R(1).R(0).R_iK)) + i.c_6.c_2(d,1). (i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},(R(1).R(0).R \llcorner s_3(d,1).K)\llcorner(((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S_iL)) + i.c_3.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S_i s_5(1).R(1).R(0).R_iK\llcorner L))) = [\text{REC}]$$

c5(1).

$$(i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},(((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S \llcorner s_6(1).L)\llcorner(R(1).R(0).R_iK)) + i.c_6.c_2(d,1). (i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},(R(1).R(0).R \llcorner s_3(d,1).K)\llcorner(((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S_iL)) + i.c_3.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S_i s_5(1).R_iK\llcorner L))) = [\text{REC}]$$

c5(1).

$$(i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},(((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S \llcorner s_6(1).L)\llcorner(R(1).R(0).R_iK)) + i.c_6.c_2(d,1). (i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},(R(1).R(0).R \llcorner s_3(d,1).K)\llcorner(((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S_iL)) + i.c_3.Y_2(d))) = [\text{ABP97}]$$

c5(1).

$$(i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},(((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S \llcorner s_6(1).L)\llcorner(R(1).R(0).R_iK)) + i.c_6.c_2(d,1). (i.\partial(\{r_2,r_3,r_5,r_6,s_2,s_3,s_5,s_6\},c_3(d,1).(s_5(1).R(1).R(0).R_iK\llcorner(((r_6(\text{invert}(1)) + r_6).S(d,1) + r_6(1)).S_iL)) + i.c_3.Y_2(d))) = [\text{GEN3}]$$

c5(1).

c5(1).

$$(\quad i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},c6(1)).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S_iR_i(1),R(0),R_iK_iL) + \\ i.c6.c2(d,1).(i.c3(d,1) + i.c3).Y2(d)) = [REC]$$

c5(1).

$$(\quad i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},c6(1)).\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S_iR_iK_iL) + \\ i.c6.c2(d,1).(i.c3(d,1) + i.c3).Y2(d)) = [REC]$$

$$c5(1).(i.\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},c6(1)).X + i.c6.c2(d,1).(i.c3(d,1) + i.c3).Y2(d)) = [D1]$$

$$c5(1).(i.c6(1).X + i.c6.c2(d,1).(i.c3(d,1) + i.c3).Y2(d)) = [A1]$$

$$c5(1).(i.c6.c2(d,1).(i.c3(d,1) + i.c3).Y2(d) + i.c6(1).X) = [ABP4]$$

$$c5(1).(i.c6.c2(d,1).(i.c3(d,1).Y2(d) + i.c3.Y2(d)) + i.c6(1).X);$$

[ABP160]

$$\tau.\tau(\{c2,c3,c5,c6,i\},X1(d)) = [CFAR($$

$$\{ \quad Z0(d) = c2(d,0).Z1(d) \\ Z1(d) = i.Z2(d) + i.c3(d,0).s4(d).X2(d) \\ Z2(d) = c3.Z3(d) \\ Z3(d) = c5(1).Z4(d) \\ Z4(d) = i.Z5(d) + i.Z6(d) \\ Z5(d) = c6(1).Z0(d) \\ Z6(d) = c6.Z0(d) \},$$

$$\{ \quad \lambda d.X1(d)/Z0 \\ \lambda d.(i.c3.c5(1).(i.c6(1).X1(d) + i.c6.X1(d)) + i.c3(d,0).s4(d).X2(d))/Z1, \\ \lambda d.c3.c5(1).(i.c6(1).X1(d) + i.c6.X1(d))/Z2, \\ \lambda d.c5(1).(i.c6(1).X1(d) + i.c6.X1(d))/Z3, \\ \lambda d.(i.c6(1).X1(d) + i.c6.X1(d))/Z4, \\ \lambda d.c6(1).X1(d)/Z5, \\ \lambda d.c6.X1(d)/Z6 \})$$

$$\{ \quad [ABP155] \\ [REFL(i.c3.c5(1).(i.c6(1).X1(d) + i.c6.X1(d)) + i.c3(d,0).s4(d).X2(d))] \\ [REFL(c3.c5(1).(i.c6(1).X1(d) + i.c6.X1(d)))] \\ [REFL(c5(1).(i.c6(1).X1(d) + i.c6.X1(d)))] \\ [REFL(i.c6(1).X1(d) + i.c6.X1(d))] \\ [REFL(c6(1).X1(d))] \\ [REFL(c6.X1(d))] \}]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},i.c3(d,0).s4(d).X2(d)) = [TI4]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},i).\tau(\{c2,c3,c5,c6,i\},c3(d,0).s4(d).X2(d)) = [TI2]$$

$$\tau.\tau.\tau(\{c2,c3,c5,c6,i\},c3(d,0).s4(d).X2(d)) = [A5]$$

$$(\tau.\tau).\tau(\{c2,c3,c5,c6,i\},c3(d,0).s4(d).X2(d)) = [T1]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},c3(d,0).s4(d).X2(d)) = [TI4]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},c3(d,0)).\tau(\{c2,c3,c5,c6,i\},s4(d).X2(d)) = [TI2]$$

$$\tau.\tau.\tau(\{c2,c3,c5,c6,i\},s4(d).X2(d)) = [A5]$$

$$(\tau.\tau).\tau(\{c2,c3,c5,c6,i\},s4(d).X2(d)) = [T1]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},s4(d).X2(d)) = [T14]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},s4(d)).\tau(\{c2,c3,c5,c6,i\},X2(d)) = [T11]$$

$$\tau.s4(d).\tau(\{c2,c3,c5,c6,i\},X2(d));$$

[ABP161]

$$\tau.\tau(\{c2,c3,c5,c6,i\},Y1(d)) = [CFAR ($$

$$\{ Z0(d) = c2(d,1).Z1(d)$$

$$Z1(d) = i.Z2(d) + i.c3(d,1).s4(d).Y2(d)$$

$$Z2(d) = c3.Z3(d)$$

$$Z3(d) = c5(0).Z4(d)$$

$$Z4(d) = i.Z5(d) + i.Z6(d)$$

$$Z5(d) = c6(0).Z0(d)$$

$$Z6(d) = c6.Z0(d) \},$$

$$\{ \lambda d.Y1(d)/Z0$$

$$\lambda d.(i.c3.c5(0).(i.c6(0).Y1(d) + i.c6.Y1(d)) + i.c3(d,1).s4(d).Y2(d))/Z1,$$

$$\lambda d.c3.c5(0).(i.c6(0).Y1(d) + i.c6.Y1(d))/Z2,$$

$$\lambda d.c5(0).(i.c6(0).Y1(d) + i.c6.Y1(d))/Z3,$$

$$\lambda d.(i.c6(0).Y1(d) + i.c6.Y1(d))/Z4,$$

$$\lambda d.c6(0).Y1(d)/Z5,$$

$$\lambda d.c6.Y1(d)/Z6 \})$$

$$\{ [ABP156]$$

$$[REFL(i.c3.c5(0).(i.c6(0).Y1(d) + i.c6.Y1(d)) + i.c3(d,1).s4(d).Y2(d))]$$

$$[REFL(c3.c5(0).(i.c6(0).Y1(d) + i.c6.Y1(d)))]$$

$$[REFL(c5(0).(i.c6(0).Y1(d) + i.c6.Y1(d)))]$$

$$[REFL(i.c6(0).Y1(d) + i.c6.Y1(d))]$$

$$[REFL(c6(0).Y1(d))]$$

$$[REFL(c6.Y1(d))] \}]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},i.c3(d,1).s4(d).Y2(d)) = [T14]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},i).\tau(\{c2,c3,c5,c6,i\},c3(d,1).s4(d).Y2(d)) = [T12]$$

$$\tau.\tau.\tau(\{c2,c3,c5,c6,i\},c3(d,1).s4(d).Y2(d)) = [A5]$$

$$(\tau.\tau).\tau(\{c2,c3,c5,c6,i\},c3(d,1).s4(d).Y2(d)) = [T1]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},c3(d,1).s4(d).Y2(d)) = [T14]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},c3(d,1)).\tau(\{c2,c3,c5,c6,i\},s4(d).Y2(d)) = [T12]$$

$$\tau.\tau.\tau(\{c2,c3,c5,c6,i\},s4(d).Y2(d)) = [A5]$$

$$(\tau.\tau).\tau(\{c2,c3,c5,c6,i\},s4(d).Y2(d)) = [T1]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},s4(d).Y2(d)) = [T14]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},s4(d)).\tau(\{c2,c3,c5,c6,i\},Y2(d)) = [T11]$$

$$\tau.s4(d).\tau(\{c2,c3,c5,c6,i\},Y2(d));$$

[ABP162]

$$\tau.\tau(\{c2,c3,c5,c6,i\},X2(d)) = [CFAR ($$

$$\{ Z0(d) = c5(0).Z1(d)$$

$$Z1(d) = i.Z2(d) + i.c6(0).Y$$

$$\begin{aligned} Z2(d) &= c6.Z3(d) \\ Z3(d) &= c2(d,0).Z4(d) \\ Z4(d) &= i.Z5(d) + i.Z6(d) \\ Z5(d) &= c3(d,0).Z0(d) \\ Z6(d) &= c3.Z0(d) \}, \\ \{ &\lambda d.X2(d)/Z0, \\ &\lambda d.(i.c6.c2(d,0).(i.c3(d,0).X2(d) + i.c3.X2(d)) + i.c6(0).Y)/Z1, \\ &\lambda d.c6.c2(d,0).(i.c3(d,0).X2(d) + i.c3.X2(d))/Z2, \\ &\lambda d.c2(d,0).(i.c3(d,0).X2(d) + i.c3.X2(d))/Z3, \\ &\lambda d.(i.c3(d,0).X2(d) + i.c3.X2(d))/Z4, \\ &\lambda d.c3(d,0).X2(d)/Z5, \\ &\lambda d.c3.X2(d)/Z6 \}) \\ \{ &[ABP158] \\ &[REFL(i.c6.c2(d,0).(i.c3(d,0).X2(d) + i.c3.X2(d)) + i.c6(0).Y)] \\ &[REFL(c6.c2(d,0).(i.c3(d,0).X2(d) + i.c3.X2(d)))] \\ &[REFL(c2(d,0).(i.c3(d,0).X2(d) + i.c3.X2(d)))] \\ &[REFL((i.c3(d,0).X2(d) + i.c3.X2(d)))] \\ &[REFL(c3(d,0).X2(d))] \\ &[REFL(c3.X2(d))] \}] \end{aligned}$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},i.c6(0).Y) = [T14]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},i).\tau(\{c2,c3,c5,c6,i\},c6(0).Y) = [T12]$$

$$\tau.\tau.\tau(\{c2,c3,c5,c6,i\},c6(0).Y) = [A5]$$

$$(\tau.\tau).\tau(\{c2,c3,c5,c6,i\},c6(0).Y) = [T1]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},c6(0).Y) = [T14]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},c6(0)).\tau(\{c2,c3,c5,c6,i\},Y) = [T12]$$

$$\tau.\tau.\tau(\{c2,c3,c5,c6,i\},Y) = [A5]$$

$$(\tau.\tau).\tau(\{c2,c3,c5,c6,i\},Y) = [T1]$$

$$\tau.\tau(\{c2,c3,c5,c6,i\},Y);$$

[ABP163]

$$\tau.\tau(\{c2,c3,c5,c6,i\},Y2(d)) = [CFAR($$

$$\begin{aligned} \{ &Z0(d) = c5(1).Z1(d) \\ &Z1(d) = i.Z2(d) + i.c6(1).X \\ &Z2(d) = c6.Z3(d) \\ &Z3(d) = c2(d,1).Z4(d) \\ &Z4(d) = i.Z5(d) + i.Z6(d) \\ &Z5(d) = c3(d,1).Z0(d) \\ &Z6(d) = c3.Z0(d) \}, \\ \{ &\lambda d.Y2(d)/Z0, \\ &\lambda d.(i.c6.c2(d,1).(i.c3(d,1).Y2(d) + i.c3.Y2(d)) + i.c6(1).X)/Z1, \\ &\lambda d.c6.c2(d,1).(i.c3(d,1).Y2(d) + i.c3.Y2(d))/Z2, \\ &\lambda d.c2(d,1).(i.c3(d,1).Y2(d) + i.c3.Y2(d))/Z3, \\ &\lambda d.(i.c3(d,1).Y2(d) + i.c3.Y2(d))/Z4, \\ &\lambda d.c3(d,1).Y2(d)/Z5, \\ &\lambda d.c3.Y2(d)/Z6 \}) \\ \{ &[ABP159] \\ &[REFL(i.c6.c2(d,1).(i.c3(d,1).Y2(d) + i.c3.Y2(d)) + i.c6(1).X)] \end{aligned}$$

[REFL(c6.c2(d,1).(i.c3(d,1).Y2(d) + i.c3.Y2(d)))]
 [REFL(c2(d,1).(i.c3(d,1).Y2(d) + i.c3.Y2(d)))]
 [REFL(i.c3(d,1).Y2(d) + i.c3.Y2(d))]
 [REFL(c3(d,1).Y2(d))]
 [REFL(c3.Y2(d))]]

$\tau.\tau(\{c2,c3,c5,c6,i\},i.c6(1).X) = [T14]$

$\tau.\tau(\{c2,c3,c5,c6,i\},i).\tau(\{c2,c3,c5,c6,i\},c6(1).X) = [T12]$

$\tau.\tau.\tau(\{c2,c3,c5,c6,i\},c6(1).X) = [A5]$

$(\tau.\tau).\tau(\{c2,c3,c5,c6,i\},c6(1).X) = [T1]$

$\tau.\tau(\{c2,c3,c5,c6,i\},c6(1).X) = [T14]$

$\tau.\tau(\{c2,c3,c5,c6,i\},c6(1)).\tau(\{c2,c3,c5,c6,i\},X) = [T12]$

$\tau.\tau.\tau(\{c2,c3,c5,c6,i\},X) = [A5]$

$(\tau.\tau).\tau(\{c2,c3,c5,c6,i\},X) = [T1]$

$\tau.\tau(\{c2,c3,c5,c6,i\},X);$

[ABP164]

$\tau(\{c2,c3,c5,c6,i\},\underline{X}) = [ABP138]$

$\tau(\{c2,c3,c5,c6,i\},\Sigma(d:D,r1(d).X1(d))) = [SUM9]$

$\Sigma(d:D,\tau(\{c2,c3,c5,c6,i\},r1(d).X1(d))) = [T14]$

$\Sigma(d:D,\tau(\{c2,c3,c5,c6,i\},r1(d)).\tau(\{c2,c3,c5,c6,i\},X1(d))) = [T11]$

$\Sigma(d:D,r1(d).\tau(\{c2,c3,c5,c6,i\},X1(d))) = [T1]$

$\Sigma(d:D,(r1(d).\tau).\tau(\{c2,c3,c5,c6,i\},X1(d))) = [A5]$

$\Sigma(d:D,r1(d).\tau.\tau(\{c2,c3,c5,c6,i\},X1(d))) = [ABP160]$

$\Sigma(d:D,r1(d).\tau.s4(d).\tau(\{c2,c3,c5,c6,i\},X2(d))) = [A5]$

$\Sigma(d:D,(r1(d).\tau).s4(d).\tau(\{c2,c3,c5,c6,i\},X2(d))) = [T1]$

$\Sigma(d:D,r1(d).\underline{s4(d)}).\tau(\{c2,c3,c5,c6,i\},X2(d))) = [T1]$

$\Sigma(d:D,r1(d).(s4(d).\tau).\tau(\{c2,c3,c5,c6,i\},X2(d))) = [A5]$

$\Sigma(d:D,r1(d).s4(d).\tau.\tau(\{c2,c3,c5,c6,i\},X2(d))) = [ABP162]$

$\Sigma(d:D,r1(d).\underline{s4(d)}).\tau.\tau(\{c2,c3,c5,c6,i\},Y) = [A5]$

$\Sigma(d:D,r1(d).(s4(d).\tau).\tau(\{c2,c3,c5,c6,i\},Y)) = [T1]$

$\Sigma(d:D,r1(d).s4(d).\tau(\{c2,c3,c5,c6,i\},Y));$

[ABP165]

$$\tau(\{c2,c3,c5,c6,i\},Y) = [ABP139]$$

$$\tau(\{c2,c3,c5,c6,i\},\Sigma(d:D,r1(d),Y1(d))) = [SUM9]$$

$$\Sigma(d:D,\tau(\{c2,c3,c5,c6,i\},r1(d),Y1(d))) = [T14]$$

$$\Sigma(d:D,\tau(\{c2,c3,c5,c6,i\},r1(d)),\tau(\{c2,c3,c5,c6,i\},Y1(d))) = [T11]$$

$$\Sigma(d:D,r1(d),\tau(\{c2,c3,c5,c6,i\},Y1(d))) = [T1]$$

$$\Sigma(d:D,r1(d),\tau(\{c2,c3,c5,c6,i\},Y1(d))) = [A5]$$

$$\Sigma(d:D,r1(d),\tau(\{c2,c3,c5,c6,i\},Y1(d))) = [ABP161]$$

$$\Sigma(d:D,r1(d),\tau.s4(d),\tau(\{c2,c3,c5,c6,i\},Y2(d))) = [A5]$$

$$\Sigma(d:D,r1(d),\tau.s4(d),\tau(\{c2,c3,c5,c6,i\},Y2(d))) = [T1]$$

$$\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},Y2(d))) = [T1]$$

$$\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},Y2(d))) = [A5]$$

$$\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},Y2(d))) = [ABP163]$$

$$\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},X)) = [A5]$$

$$\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},X)) = [T1]$$

$$\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},X));$$

[ABP166]

$$\tau(\{c2,c3,c5,c6,i\},X) = [ABP164]$$

$$\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},Y)) = [ABP165]$$

$$\Sigma(d:D,r1(d),s4(d),\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},X)));$$

[ABP167]

$$\tau(\{c2,c3,c5,c6,i\},Y) = [ABP165]$$

$$\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},X)) = [ABP164]$$

$$\Sigma(d:D,r1(d),s4(d),\Sigma(d:D,r1(d),s4(d),\tau(\{c2,c3,c5,c6,i\},Y)));$$

[ABP168]

$$\underline{ABP} = [REC]$$

$$\tau(\{c2,c3,c5,c6,i\},\partial(\{r2,r3,r5,r6,s2,s3,s5,s6\},S|R|K|L)) = [REC]$$

$$\tau(\{c2,c3,c5,c6,i\},X) = [RSP]$$

$$\{Z = \Sigma(d:D,r1(d),s4(d),\Sigma(d:D,r1(d),s4(d),Z)),[\tau(\{c2,c3,c5,c6,i\},X)/Z],[\tau(\{c2,c3,c5,c6,i\},Y)/Z]\}$$

$$\{[ABP166]\}$$

$$\{[ABP167]\}$$

$$\tau(\{c2,c3,c5,c6,i\},Y) = [ABP165]$$

$\Sigma(d:D, r1(d).s4(d). \tau(\{c2, c3, c5, c6, i\}, X)) = [\text{REC}]$

$\Sigma(d:D, r1(d).s4(d). \tau(\{c2, c3, c5, c6, i\}, \partial(\{r2, r3, r5, r6, s2, s3, s5, s6\}, S; R; K; L))) = [\text{REC}]$

$\Sigma(d:D, r1(d).s4(d). \text{ABP});$

□

References

- [1] J.C.M Baeten and W.P.Weijland, *Process Algebra*, Cambridge tracts in Theoretical Computer Science 18, Cambridge University Press, Cambridge 1990.
- [2] K.A. Barlett, R.A. Scantlebury, P.T. Wilkinson, *A note on reliable full-duplex transmission over half-duplex lines*, Comm. of the ACM 12, pp. 260-261, 1969.
- [3] J.A. Bergstra, J.W. Klop, *Verification of an alternating bit protocol by means of process algebra*, Math. Methods of Spec. and Synthesis of Software Systems '85 (eds. W. Bibel and K.P. Jantke), Math. research 31, Akademie-Verlag Berlin, pp. 9-23, 1986. (Also LNCS 215, Springer Verlag, pp 9-23.)
- [4] M. Bezem and J.F. Groote, *A formal Verification of the Alternating Bit Protocol in the Calculus of Constructions*, Logic Group Preprint Series No. 88 - March 1993, Department of Philosophy Utrecht University, Utrecht 1993.
- [5] E.W. Dijkstra and W.H.J. Feijen, *Een methode van programmeren*, Academic service, Den Haag 1984
- [6] J.F. Groote and A. Ponse, *The syntax and semantics of μ CRL*, Report CS-R9076, Centre for Mathematics and Computer Science, Amsterdam 1990.
- [7] J.F. Groote and A. Ponse, *Proof theory for μ CRL*, Report CS-R9138, Centre for Mathematics and Computer Science, Amsterdam 1991.
- [8] H. Korver and J. Springintveld, *A computer-checked verification of Milners Scheduler*, Proceedings of TACS, Japan 1994. To appear in LNCS. Also appeared as Logic Group Preprint Series No. 101 - November 1993, Department of Philosophy Utrecht University, Utrecht 1993.
- [9] M.P.A. Sellink, *Verifying Process Algebra proofs in Type Theory*, Logic Group Preprint Series No. 87 - March 1993, Department of Philosophy Utrecht University, Utrecht 1993.
- [10] SPECS-semantics. *Definition of MR and CRL Version 2.1*, 1990.
- [11] F.W. Vaandrager, *Verification of two communication protocols by means of process algebra*, report CS-R8608, Centre for Mathematics and Computer Science, Amsterdam 1986.