Tutor Statistics

Lectures

Partly from slides by Mark Huiskes.

Mathematical Statistics and Data Analysis

by J.A. Rice,
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- Probability (Chapter 1)
- Random Variables (Chapter 2)
- Joint Distributions (Chapter 3)
- Expected Values (Chapter 4)
- Limit Theorems (Chapter 5)
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Lecture 1: Sets

Definition:

A set is an unordered collection of zero or more distinct objects.

The objects that make up a set are called elements or members of the set.

Specifying Sets (simplified)

Two main ways:

1. Listing the members of the set.

   Notation: $A = \{ a_1, a_2, \ldots, a_n \}$ or $A = \{ a_1, a_2, \ldots \}$

   Examples: $A = \{ a, d, s, t, z \}$

   $N = \{ 1, 2, 3, 4, \ldots \}$

2. State the properties that characterize the members in the set.

   Notation: $B = \{ x : x \text{ satisfies } \wp(x) \}$

   Example: $B = \{ x : x \text{ is an even integer and } x > 0 \}$

   "B is the set of elements x such that x is an even integer and x is greater than zero."

Note: Sets are often denoted by capitals, and elements are usually lower case.

Some Properties of Sets

- The order in which the elements are presented in a set is not important:

  A = { a, e, i, o, u } and B = { e, o, u, a, I } both define the same set

- The members of a set can be anything, even sets.

- In a set the same member does not appear more than once.

  F = { a, e, i, o, a, u } is incorrect since the element ‘a’ repeats.

“Element in/Member of” Notation

Consider the set A = { a, e, i, o, u } then

- We write “ ‘a’ is a member of ‘A’ ” as: $a \in A$

- We write “ ‘b’ is not a member of ‘A’ ” as: $b \notin A$

Universal Set and Empty Set

- The members of all the investigated sets in a particular problem usually belongs to some fixed large set. In probability theory this is usually the sample space Ω.
- The set that has no elements is called the empty set and is denoted by $\emptyset$ or $\{\}$.

e.g. $\{ x : x^2 = 4 \text{ and } x \text{ is an odd integer} \} = \emptyset$

Cardinality of a Set

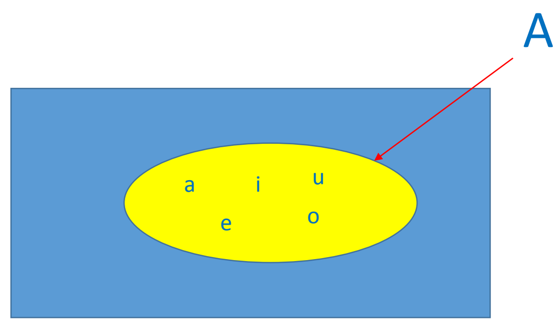
- The number of elements in a set is called the cardinality of a set. Let 'A' be any set then its cardinality is denoted by $|A|$.

e.g. $A = \{ a, e, i, o, u \}$ then $|A| = 5$.

Venn Diagrams

- A pictorial way of representing sets.
- The universal set is represented by the interior of a rectangle and the other sets are represented by disks lying within the rectangle.

e.g. A = { a, e, i, o, u }



Subsets

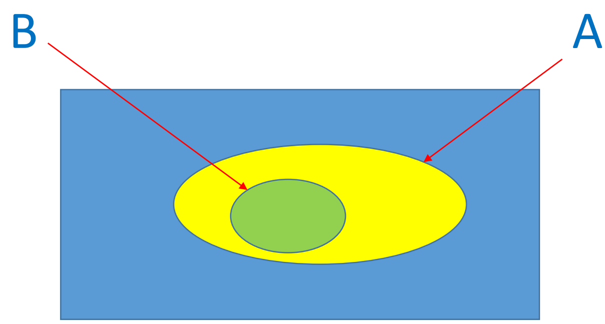
Definition:

Set ‘A’ is called a **subset** of set ‘B’ if and only if every element of set ‘A’ is also an element of set ‘B’. We also say that ‘A’ *is contained in* ‘B’ or that ‘B’ *contains* ‘A’.

It is denoted by $A \subseteq B$ or $B \supseteq A$.

Venn Diagram for a Proper Subset

Note that if $B \subset A$ then the Venn diagram depicting those sets is as follows:



If $B \subseteq A$ then the disc showing 'B' may overlap with the disc showing 'A'.

Power Set

- The set of all subsets of a set 'S' is called the power set of 'S'. It is denoted by P(S) or 2^S.

So: $P(S) = \{ x : x \subseteq S \}$

E.g. $A = \{ 1, 2, 3 \}$ then
$P(A) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\} \}$

Note that $|P(S)| = 2^{|S|}$

E.g. $|P(A)| = 2^{|A|} = 2^3 = 8$.

Set Operations - Complement

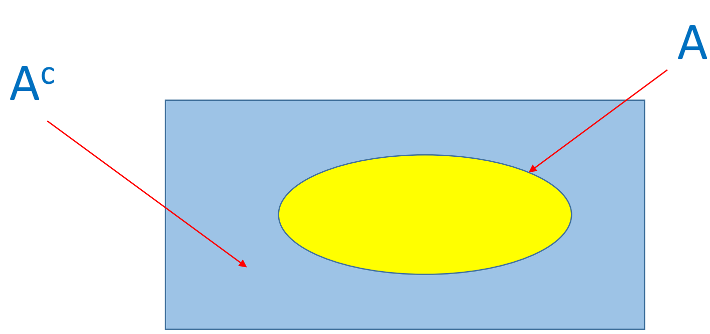
The (absolute) complement of a set ‘A’ is the set of elements which belong to Ω but which do not belong to A.

This is denoted by A^c or Ω or Ã.

In other words we can say:

$$A^c = \{ x : x \in \Omega \land x \notin A \}$$

Venn Diagram for the Complement



Set Operations - Union

Definition:

The **union** of two sets 'A' and 'B' is the set of all elements which belong to either 'A' or 'B' or both. This is denoted by $A \cup B$.

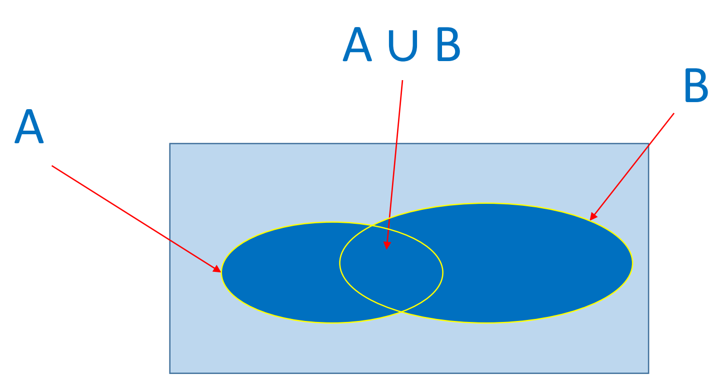
In other words we can say:

$A \cup B = \{ x : x \in A \vee x \in B \}$

E.g. $A = \{ 3, 5, 7 \}$, $B = \{ 2, 3, 5 \}$

$A \cup B = \{ 3, 5, 7, 2, 3, 5 \} = \{ 2, 3, 5, 7 \}$

Venn Diagram Representation for Union



Set Operations - Intersection

Definition:

Intersection of two sets 'A' and 'B' is the set of all elements which belong to both 'A' and 'B'.

This is denoted by $A \cap B$.

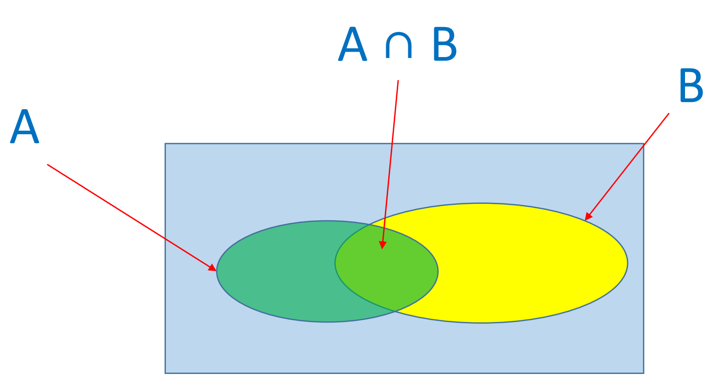
In other words we can say:

$A \cap B = \{ x : x \in A \wedge x \in B \}$

E.g. $A = \{ 3, 5, 7 \}$, $B = \{ 2, 3, 5 \}$

$A \cap B = \{ 3, 5 \}$

Venn Diagram Representation for Intersection



Set Operations - Difference

Definition:

The **difference** or the **relative complement** of a set 'B' with respect to a set 'A' is the set of elements which belong to 'A' but which do not belong to 'B'.

This is denoted by A – B or A \ B.

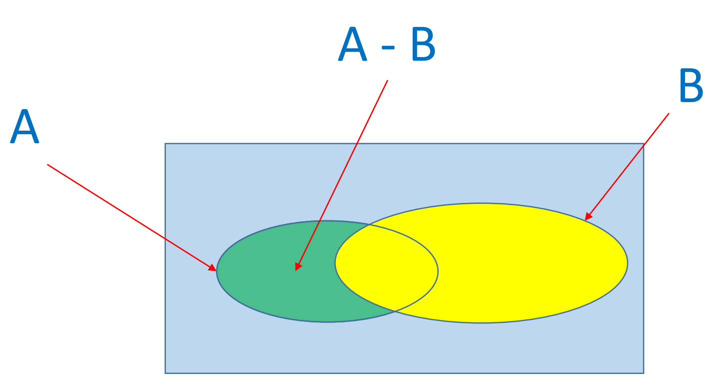
In other words we can say:

$A - B = \{ x : x \in A \wedge x \notin B \}$

E.g. $A = \{ 3, 5, 7 \}$, $B = \{ 2, 3, 5 \}$

$A - B = \{ 3, 5, 7 \} - \{ 2, 3, 5 \} = \{ 7 \}$

Venn Diagram Representation for Difference



Natural Numbers and Proofs by Induction

References:

F. van der Blij, J. van Tiel, Infinitesimaalrekening, Prisma-Technica, 2e druk, 1975

Natural Numbers

Axiom

A set of Natural numbers that contains 1 and with every number n also its successor n+1 consists of all the Natural numbers.

Let P(n) a logical expression that contains the natural number n.

Theorem (Induction)

Assume P(1) is true and that for all Natural numbers n, if P(n) is true, then P(n+1) is true. Then P(n) is true for every Natural number n.

Proofs by Induction

Let P(n) an expression that contains the natural number n.

Theorem (Induction I)

Assume P(1) is true and that, for all Natural numbers n, if P(n) is true, then P(n+1) is true. Then P(n) is true for every Natural number n.

Theorem (Induction II)

Assume for a certain n_0 $P(n_0)$ is true and that, for all Natural numbers $n \geq n_0$, if P(n) is true, then P(n+1) is true. Then P(n) is true for every Natural number $n \geq n_0$.