Natural Numbers and Proofs by Induction

References:

F. van der Blij, J. van Tiel, Infinitesimaalrekening, Prisma-Technica, 2e druk,1975

Natural Numbers

Axiom

A set of Natural numbers that contains 1 and with every number n also its successor n+1 consists of all the Natural numbers.

Let P(n) a logical expression that contains the natural number n.

Theorem (Induction)

Assume P(1) is true and that for all Natural numbers n, if P(n) is true, then P(n+1) is true. Then P(n) is true for every Natural number n.

Proofs by Induction

Let P(n) an expression that contains the natural number n.

Theorem (Induction I)

Assume P(1) is true and that, for all Natural numbers n, if P(n) is true, then P(n+1) is true. Then P(n) is true for every Natural number n.

Theorem (Induction II)

Assume for a certain n_0 $P(n_0)$ is true and that, for all Natural numbers $n \geq n_0$, if P(n) is true, then P(n+1) is true. Then P(n) is true for every Natural number $n \geq n_0$.