*)

$$\int_{-\pi}^{\pi} \sin^2 kx \, dx = \left[\frac{x}{2} - \frac{\sin(2kx)}{4k} \right]_{-\pi}^{\pi}$$

$$= \frac{\pi}{2} - \frac{\sin(2k\pi)}{4k} - \left(\frac{-\pi}{2} - \frac{\sin(-2k\pi)}{4k} \right)$$

$$= \pi$$

**) Complex numbers $\mathbb{C}$:

1) $\mathbb{R} \subset \mathbb{C}$; sum and products for these numbers $\in \mathbb{R} \subset \mathbb{C}$ as before.

2) $\exists$ complex number i such that $i^2 = -1$

3) Every complex number can be uniquely expressed as $a + bi$, with $a, b \in \mathbb{R}$

4) $\alpha, \beta, \gamma \in \mathbb{C}$, then

$$(\alpha\beta)\gamma = \alpha(\beta\gamma)$$
$$(\alpha+\beta)+\gamma = \alpha + (\beta+\gamma)$$
$$\alpha(\beta+\gamma) = \alpha\beta + \alpha\gamma$$
$$(\beta+\gamma)\alpha = \beta\alpha + \gamma\alpha$$
$$\alpha\beta = \beta\alpha$$
$$\alpha+\beta = \beta+\alpha$$

If $1 \in \mathbb{R}$ then $1\alpha = \alpha$

If $0 \in \mathbb{R}$ then $0\alpha = 0$

Furthermore, $\alpha + (-1)\alpha = 0$.

$\alpha = a + bi$, $\beta = c + di$, then

$$\alpha + \beta = (a+bi) + (c+di) = (a+c) + (b+d)i$$

and

$$\alpha \cdot \beta = (a+bi)(c+di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$$

, and if $\lambda \in \mathbb{R}$ $\lambda\alpha = \lambda(a+bi) = \lambda a + \lambda bi$

$\bar{\alpha} = \overline{a+bi} = a - bi$ $\Rightarrow \alpha\bar{\alpha} = a^2 + b^2 \in \mathbb{R}$

$\bar{\alpha}$ is <u>conjugate</u> of α $\Rightarrow \alpha^{-1} = \dfrac{\bar{\alpha}}{\alpha\bar{\alpha}}$